Open Problems from CCCG 2009

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The following is a list of the problems presented on August 17, 2009 at the open-problem session of the *21st Canadian Conference on Computational Geometry* held in Vancouver, British Columbia, Canada.

Simply Developing Geodesic Polygons

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Let $\mathcal{P}$ be a simple piecewise-geodesic curve on a sphere, i.e., a curve composed of a finite sequence of arcs of great circles that do not intersect except at shared endpoints. A *development* of $\mathcal{P}$ is a polygonal chain in the plane whose segments have the same lengths as the arcs of $\mathcal{P}$, and whose angles correspond to the angles between the arcs on the sphere. A development can be viewed as created by rolling the sphere on the plane, starting at one end of $\mathcal{P}$, maintaining $\mathcal{P}$ in contact with the plane at all times, and rolling without twisting or slippage until reaching the other endpoint.

The goal is to establish that certain classes of (closed) geodesic polygons always develop without intersection, regardless of the starting point for the development. The only class known to develop without intersection are geodesic convex polygons: those with vertex angles (to one consistent side) at most π [OS89].

During the presentation of the problem, the poser illustrated an example of a simple open curve that develops to a crossing chain, and Jack Snoeyink observed that closing it to a simple geodesic polygon $\mathcal{P}$ shows that not all star-shaped geodesic polygons develop without intersection. The example is shown in Figure 1. Every arc from a to a point inside $\mathcal{P}$ is nowhere exterior to $\mathcal{P}$, so indeed this is a star-shaped geodesic polygon.

Any natural class of nonconvex geodesic polygons that always develop without intersection would be of interest. More ambitious would be to *characterize* those geodesic polygons that develop without intersection. More ambitiously still, generalize to geodesic polygons on convex polyhedra, or on smooth convex surfaces.

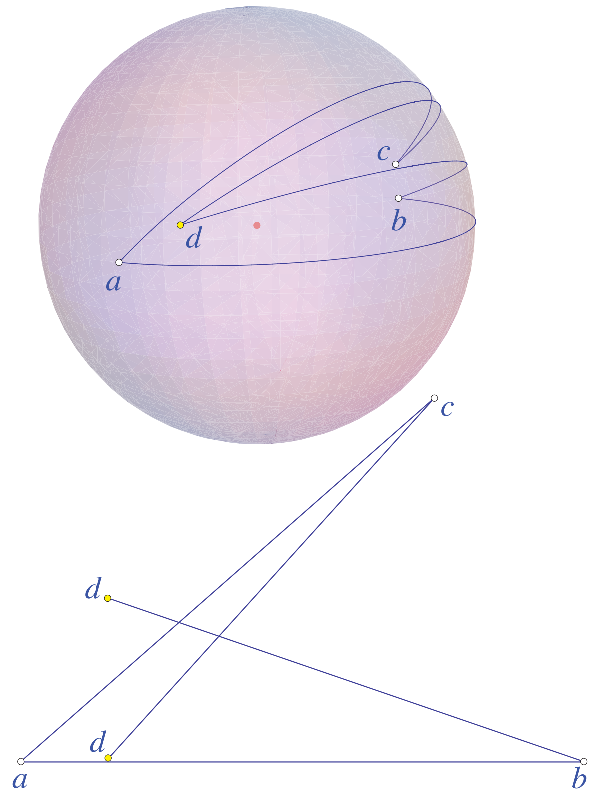

Figure 1: This polygon on the sphere is star-shaped from a. When cut open at d, it develops to a crossing chain in the plane.

References

[OS89] Joseph O'Rourke and Catherine Schevon. On the development of closed convex curves on 3-polytopes. *J. Geom.*, 13:152–157, 1989.

Random Graph Spanners

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Select n points uniformly at random from the unit square, and then form a graph $G = G(n, p)$

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by connecting points according to the Erdős–Renyi model, adding an edge between each pair of points with probability p. This is not a "random geometric graph" in the usual sense of that term, because points are connected without regard to their geometric distance. Rather, it is a random graph built on geometric points.

1. It is known that the threshold for the creation of a "giant component" $C \subseteq G$ is $p_1 = 1/n$. Is C (almost surely) a spanner for the points it connects, for $p = p_1 + \varepsilon$? My conjecture: NO.
2. It is known that the threshold for complete connection of the point set is $p_2 = \ln n/n$. Is $G(n, p_2 + \varepsilon)$ (almost surely) a spanner? My conjecture: YES.

Stretch Embedding in Labeled Grids
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Find two labelings Λ_1 and Λ_2 of the n^2 vertices in the $n \times n$ grid graph so as to maximize the minimum "stretch" between Λ_1 and Λ_2. Here the labelings must use the same n^2 labels, say $1, 2, \ldots, n^2$. Each such label λ defines one vertex x_1 according to Λ_1 and one vertex x_2 according to Λ_2. The *stretch* $d(\lambda)$ is the ℓ_1 distance between vertices x_1 and x_2. The goal is to find two labelings that maximize the minimum stretch over all labels λ.

This phrasing is the inverse of the original posed problem, in which we are given the two labelings Λ_1 and Λ_2 and wish to find a third labeling of vertices in the $n \times n$ grid graph that does not "stretch" either labeling too much, i.e., minimizes the maximum stretch. A large answer to the problem above provides a bad example for the latter problem.

An equivalent formulation of the problem is as follows. Define the *combined distance* between a pair of labels (λ_1, λ_2) as the sum of the ℓ_1 distances between the two vertices marked with those labels under Λ_1, and between the two vertices marked with those labels under Λ_2. Maximizing the minimum combined distance is an equivalent problem.

Shortly after the conference, two groups independently established that the optimal stretch is $\Theta(\sqrt{n})$. See [JPT10] in this proceedings.

References

[JPT10] Minghui Jiang, Vincent Pilaud, and Pedro J. Tejada. On a dispersion problem in grid labeling. In *Proc. 22nd Canad. Conf. Comput. Geom.*, August 2010.

Crowd Simulation
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Collision avoidance in crowd simulations leads to the following problem.

Given a floor plan represented as a polygon with holes, preprocess it for "limited visibility queries": expand a visibility region (disk or wedge) until it contains either enough agents or enough polygon complexity. The agents are points tracked by a kinetic data structure. Thus one seeks a relevant subset of the visibility polygon with bounded complexity (in terms of both agents and polygon features).

Compatible Tetrahedralizations
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Given two polyhedra P_1 and P_2 whose surfaces are combinatorially equivalent (in particular, their 1-skeletons are isomorphic graphs), do they necessarily have a compatible tetrahedralization with Steiner points? A *compatible tetrahedralization with Steiner points* of two polyhedra is a tetrahedralization of the interior of each polyhedron, allowing additional (Steiner) vertices in addition to the boundary vertices given by the polyhedron, such that the two tetrahedralizations are combinatorially equivalent: there is a bijection between the vertices of the two tetrahedralizations that preserves triangles and tetrahedra in both directions.

The analogous problem for 2D polygons is known to be always possible [ASS93], requiring $\Theta(n^2)$ Steiner points in the worst case, where n is the number of vertices of each polygon.

In a more general version of the problem (which skirts the issue of how to define "polyhedron"), we are given two tetrahedralizations whose two-dimensional boundary surfaces are combinatorially equivalent. Here we wish to find a tetrahedralization that refines each given tetrahedralization (i.e., includes all existing triangles), such that the two refined tetrahedralizations are combinatorially equivalent.

Compatible tetrahedralizations permit natural morphing between the geometries.

References

[ASS93] Boris Aronov, Raimund Seidel, and Diane Souvaine. On compatible triangulations of simple polygons. *Computational Geometry: Theory and Applications*, 3(1):27–35, June 1993.

Landmark Navigation
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The task is to navigate from point A to B, using distances to *landmark* points L_i.

For any point p, define $d_i(p) = d(p, L_i)$ and $d^2(p) = (1/n)\sum_{i=1}^{n} d_i^2(p)$. Define "coordinates" for A,

$$c(A) =< d_1^2(A) - d^2(A), \ldots, d_i^2(A) - d^2(A), \ldots >$$

and similarly for $c(B)$. Let

$$d^*(A, B) = ||c(A) - c(B)||^2$$

With a sufficient number of landmarks, it is known that following gradient descent of $d^*(A, B)$ will get you from A to B [FGG+05].

The question is this: what happens if there are obstacles? In a polygon with holes, one can get stuck. The poser conjectures that, in a simple polygon, with the vertices as landmarks, and distance defined by the geodesic distance within the polygon, gradient descent works: one never gets stuck. There is experimental support for this conjecture. Perhaps using art-gallery guards as the landmarks would also suffice.

References

[FGG+05] Q. Fang, J. Gao, L. Guibas, V. de Silva, and L. Zhang. GLIDER: Gradient landmark-based distributed routing for sensor networks. In *Proc. IEEE Conference on Computer Communications (INFOCOM)*, 2005.

Nearest Neighbors for Well-Separated Points
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Given n well-spaced points in $\mathbb{R}^d$, can they be preprocessed to achieve $O(\log n)$-time (exact) nearest-neighbor queries? Points are *well-spaced* if the ratio of the radii of the balls enclosing and enclosed in every point's Voronoi cell is $O(1)$. It is known that this condition implies that the Voronoi diagram has linear complexity, but what about the natural data structural extension? **[reference?]**

Hausdorff Core of a Polygon
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Given a simple polygon P, find a convex polygon contained in P that minimizes the maximum Hausdorff distance between the two polygons (in both directions). This problem is related to the "potato-peeling" problem, which instead asks for a contained convex polygon of maximum area. The latter problem can be solved by a dynamic-programming algorithm in $O(n^7)$ time. Is there a similar dynamic-programming algorithm for exact construction of this "Hausdorff core"? An approximation algorithm appeared in [DDF+09].

Joseph O'Rourke suggested a variant which removes the requirement that the convex polygon be enclosed in P, which the poser thought might be easier.

References

[DDF+09] Reza Dorrigiv, Stephane Durocher, Arash Farzan, Robert Fraser, Alejandro López-Ortiz, J. Ian Munro, Alejandro Salinger, and Matthew Skala. Finding a Hausdorff core of a polygon: On convex polygon containment with bounded Hausdorff distance. In *WADS '09: Proc. 11th Internat. Symp. Algorithms Data Structures*, pages 218–229, Berlin, Heidelberg, 2009. Springer-Verlag.