Celeste is PSPACE-hard

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Abstract

We investigate the complexity of the platform video game Celeste. We prove that navigating Celeste is PSPACE-hard in four different ways, corresponding to different subsets of the game mechanics.

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1 Introduction

Celeste is a 2D platform video game released in 2018 by Extremely OK Games. It won the Best Independent Game and Games for Impact awards at The Game Awards 2018 and sold over a million copies [7]. In Celeste, the player controls a single character, Madeline, who must navigate various hazards along her journey. We consider the following decision problem about Celeste:

Definition 1.1 (Celeste). Given a Celeste level, is it possible for Madeline to traverse from a designated start location to a designated end location?

We amend a previous result of Ahmed et al. [?] which attempted to show that Celeste is NP-complete. We give four proofs that Celeste is instead PSPACE-hard, each using a different set of game mechanics. All of these proofs involve constructing a polynomial-time reduction to Celeste from a motion-planning problem through a planar network of doors [2]. We make use of both open-close-traverse doors, as introduced in [1, 3, 4] and shown not to need crossovers in [2], and self-closing doors, as introduced in [2]. In all but one case we additionally show containment in PSPACE.

2 Main Results

Theorem 2.1. Celeste with spinners, seekers, barriers, and move blocks is PSPACE-complete.

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*https://exok.com/games/celeste/. Celeste and its sprites are the properties of Extremely OK Games. Sprites are used here under Fair Use for the educational purpose of illustrating mathematical theorems.
Figure 2: Part of the proof of Theorem 2.1: an open-close-traverse door constructed with spinners, a seeker, barriers, and move blocks. Currently in the “closed” state.

See Figure 2 for the main gadget in the proof of Theorem 2.1.

**Theorem 2.2.** Celeste with spinners, jellyfish, and barriers is PSPACE-complete.

**Theorem 2.3.** Celeste with spinners and pufferfish is PSPACE-complete.

**Theorem 2.4.** Celeste with spinners and Kevin blocks is PSPACE-hard.

Other subsets of mechanics remain to be studied. In particular, it would be interesting to find subsets that are harder than PSPACE (e.g., undecidable), NP-complete, or (nontrivially) polynomial time.

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Images of sprites and gadgets were composed and tested using the fan-made level editor Ahorn†. We thank Cruor, Vexatos, and Ahorn’s other contributors for creating this excellent tool. We additionally thank the Celeste speedrunning, modding, and Tool-Assisted Speedrunning community for extensively researching Celeste’s mechanics.

Finally, we thank Extremely OK Games for producing Celeste, a difficult and wonderful experience in many ways.

**References**


†[https://github.com/CelestialCartographers/Ahorn](https://github.com/CelestialCartographers/Ahorn)