Negative Instance for the Edge Patrolling Beacon Problem

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Suppose you have a metallic iron *ball* in some compact polygonal space P. To manipulate the ball, you can move a strong magnet or *beacon*. At all times, the ball moves maximally toward subject to staying within P. The goal is to design a movement strategy for the beacon so that the ball and beacon end up touching.

This type of model was introduced by Biro *et al.* [1] as a generalization of the art gallery problem. In their problem, they considered placing multiple stationary toggleable beacons in some guarded domain. Both the ball and the beacons are modeled as points within P. When beacon b is activated, the ball p moves toward it in a straight line until it reaches b or hits ∂P. In the second case, the ball continues gliding along ∂P as long as the distance to b decreases monotonically; see Figure 1.

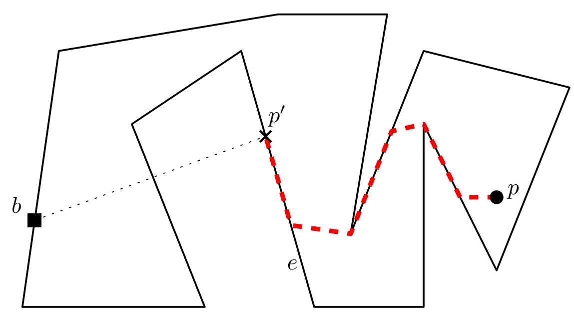

Figure 1: When the beacon (denoted by a square) is activated, it attracts the metal ball along the dashed red trajectory. The ball moves along the interior and boundary of P until it reaches a point p' (denoted by a cross) for which no local movement can reduce its distance to b. In particular, the segment $\overline{p'b}$ and the edge e of P form a right angle.

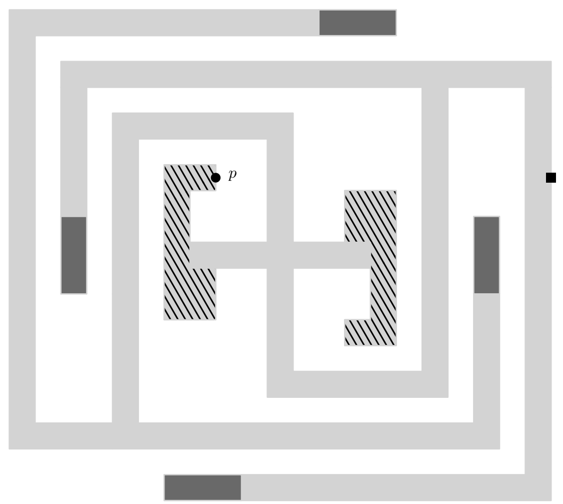

Figure 2: Counterexample to Kouhestani and Rappaport's conjecture. By the way the instance is created, the ball will be confined to the two *zones* (dashed black regions). The beacon may move into either of the zones, but unless we can pierce through P and avoid moving around any of the four *tentacles* (darker rectangular regions), the movement will push the ball to the opposite zone. An interactive version of this counterexample can be found at http://erikdemaine.org/attractor/.

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Biro *et al.* [1] studied the problem of placing as few beacons as possible so that the ball can be moved from any starting position to any target destination by turning on and off the beacons one at a time. Similar to the classic art gallery result, they showed that a smart placement of $\lfloor n/2 \rfloor - 1$ beacons is always sufficient (and sometimes necessary), where n is the number of corners of P. They also showed that it is NP-hard to minimize the number of beacons for a given problem instance.

In a follow-up paper, Kouhestani and Rappaport [4] considered the case in which we have a single beacon that is permanently activated and can move along the boundary of P. In addition to the polygonal boundary P, we are given the starting location of both the ball p (which could be in either the interior or boundary of P) and the beacon b (which can only be in the boundary). The aim is to give a beacon movement strategy that moves the beacon along the boundary of P so that the ball and beacon coincide at the same point.

Kouhestani and Rappaport [4] give an algorithm that, in $O(n^3 \log n)$ time determines if it is possible for the beacon to attract the ball. Whenever possible, the algorithm can even report the shortest way for both objects to meet (by measuring only the movement of the beacon, the ball, or both). Even though they can determine feasibility of any problem instance, they could never design an instance for which their algorithm would return a negative answer. Thus, they conjectured that in all polygon instances there should always be a way for the beacon to attract the ball [5].

In this paper, we disprove this conjecture by giving a problem instance in which the ball and the beacon can never be united. In fact, the same fact holds true even if we allow the beacon to move freely in the exterior of P in addition to the boundary ∂P and even if the polygon is restricted to be orthogonal. Figure 2 shows the counterexample, and an interactive version can be found at http://erikdemaine.org/attractor/. We verified the correctness of the solution using Dijkstra's algorithm, where each vertex is a state represented by the positions of the beacon and the ball. The algorithm explores all states reachable from the initial state, none of which reach a state where the beacon and the ball are in the same (or adjacent) position. We note that our example is tight in the sense that no more freedom to the movement of the beacon can be added; if we allow the ball to move in the interior of P, then the ball can be easily captured by walking along the geodesic from the starting positions (a fact observed in [3]).

References

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