Erratum for "Disjoint Segments have Convex Partitions with 2-Edge Connected Dual Graphs"

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A set of n disjoint line segments in the plane and a permutation π of the $2n$ segment endpoints define a partition of the plane into convex faces: extend the segments beyond their endpoints one-by-one in the order given by π until they hit another segment, a previous extension, or infinity. If no three segment endpoints are collinear, then every permutation π produces $n+1$ convex faces.

For convex partition, the *dual graph* is defined where the $n+1$ convex faces correspond to the vertices, and every segment endpoint corresponds to an edge between the two incident faces on opposite sides of the segment.

In [1], we presented a partition algorithm (see below) that, for a set S of n disjoint line segments, computes a nonempty subset $S' \subseteq S$ and a convex partition P' of S' such that each remaining segment in $S \setminus S'$ lies in the interior of a face of P'. We claimed that the dual graph of P' is 2-edge connected. This claim is false. Sometimes the dual graph of P' has a bridge (Fig 1).

Partition Algorithm. Input S.

- Pick a segment $s_0 = a_0 b_0$ with an endpoint b_0 along $\mathrm{conv}(\cup S)$. Set $s := s_0$, $p := a_0$, $\gamma := 0$, $S' := \{s_0\}$, and $i := 1$.
- Repeat while $p \neq b_0$:

  Extend s beyond p into a ray $\overrightarrow{r}$ until it hits another segment, a previous extension, or to infinity.

  - If $\overrightarrow{r}$ hits a segment in $S \setminus S'$, denote it by $s_i = a_i b_i$ such that $\angle(\overrightarrow{r}, \overrightarrow{a_i b_i}) < 0 < \angle(\overrightarrow{r}, \overrightarrow{b_i a_i})$, let $\gamma_i = \gamma + \angle(\overrightarrow{r}, \overrightarrow{a_i b_i})$, put $S' := S' \cup \{s_i\}$, $s := s_i$, $p := a_i$, $\gamma := \gamma_i + \pi$, and $i := i+1$.
  - Else, over all integers j, $0 \leq j < i$, such that $s_j \in S'$ has not been extended beyond b_j, pick one where the turning angle γ_j is maximal. Set $s := s_j$, $p := b_j$, and $\gamma := \gamma_j$.

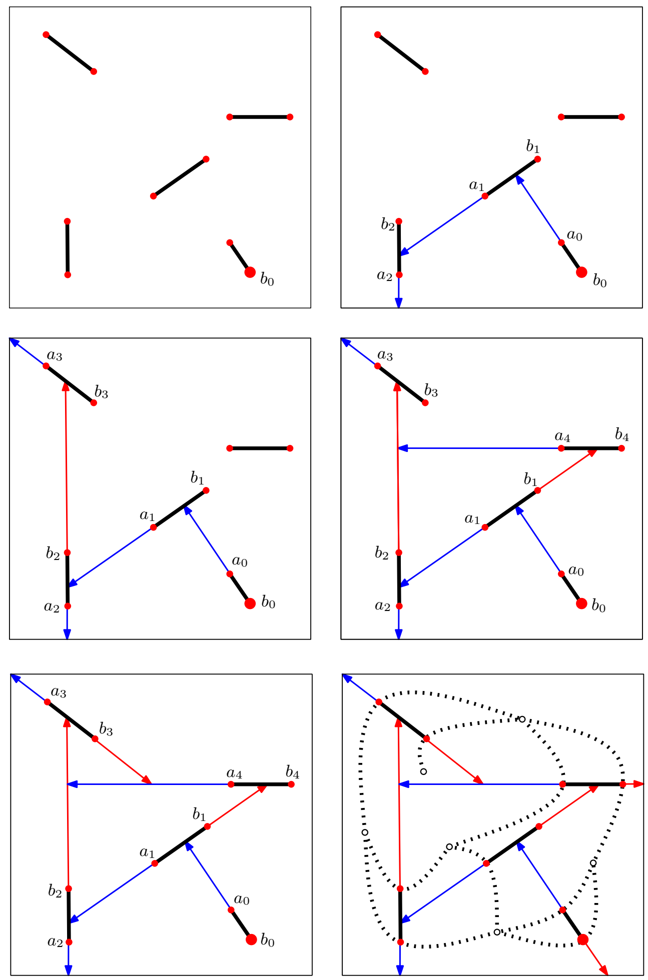

Figure 1: Steps of our partition algorithm for five input segments, and the resulting dual graph.

Specifically, the last phrase in the "proof" for our Lemma 4 is false. It is not true that after a ray $\overrightarrow{r}$ hits a segment $a_i b_i$, no extension can hit $\overrightarrow{r}$ from the right before the extension $a_i b_i$ beyond b_i is drawn.

References

[1] N. M. Benbernou, E. D. Demaine, M. L. Demaine, M. Hoffmann, M. Ishaque, D. L. Souvaine, and Cs. D. Tóth, Disjoint segments have a convex partition with a 2-edge connected dual graph, in *Proc. 19th Canadian Conf. Comp. Geom.*, 2007, Ottawa, ON, pp. 13–16.

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