

Spiral Galaxies Font

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Abstract

We present 36 Spiral Galaxies puzzles whose solutions form the 10 numerals and 26 letters of the alphabet.

1 Introduction

In a Spiral Galaxies puzzle, you are given a grid of unit squares and a collection of *centers* (points, drawn as dots, typically at square centers or edge midpoints). The goal is to decompose the grid into regions called *galaxies*, one containing each center, so that each galaxy is a polyomino (an edge-to-edge joining of unit squares) and 180° rotationally symmetric about its center. In general, the solution may not be unique, as shown in Figure 1, but most puzzles are designed to have a unique solution.

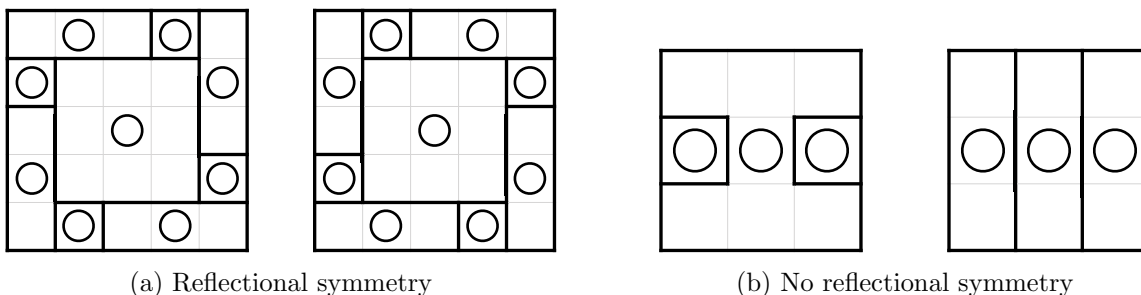


Figure 1: Examples of Spiral Galaxies puzzles with multiple solutions.

Spiral Galaxies or Tentai Show (天体ショー in Japanese) is one of the many pencil-and-paper puzzles designed for the Japanese puzzle magazine and publisher *Nikoli* [Nik12]. Like most Nikoli puzzles, Spiral Galaxies is *NP-complete* [Fri02], meaning that there is no efficient algorithm to solve them, assuming a famous conjecture in computational complexity theory ($P \neq NP$). See [DH09, HD09] for related complexity results about other puzzles.

In this paper, we design a typeface featuring 36 Spiral Galaxies puzzles, one for each numeral and letter of the alphabet. Figure 2 presents the unsolved font, which shows just the grid and centers. Like the Spiral Galaxies puzzles published by Nikoli, the centers are drawn with dots of two different colors, black and white, with the intent that the corresponding galaxy polyomino should be filled the same color. In our puzzles, the black galaxies will form a letter or numeral. Figure 5 illustrates these (unique) solutions, so do not look if you want to solve the puzzles on your own. You can write custom messages in both fonts using a free web app: <http://erikdemaine.org/fonts/spiralgalaxies/>.

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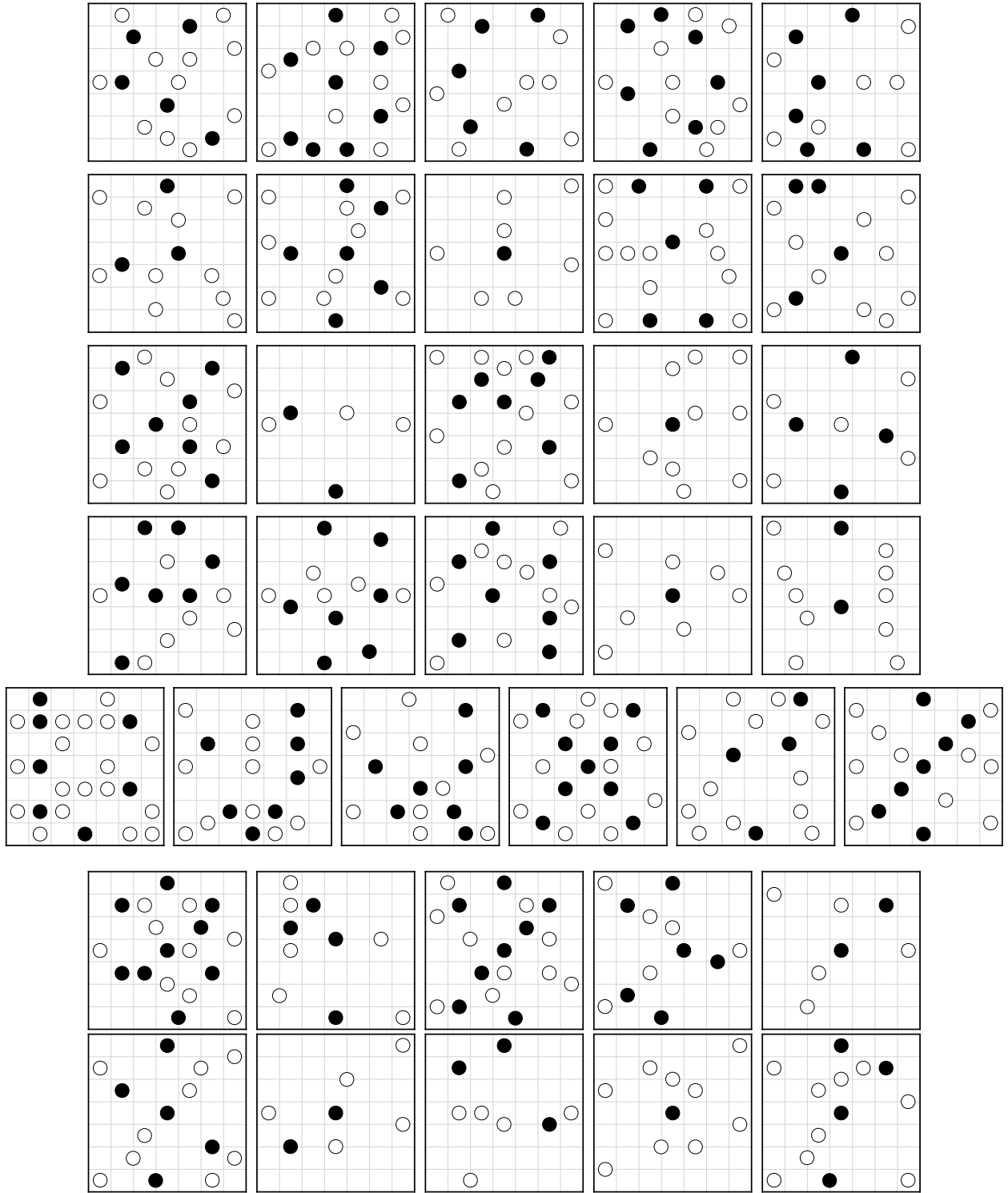


Figure 2: Unsolved font.

This mathematical puzzle typeface is part of a series of fonts by the second and third authors and their collaborators. These fonts aim to illustrate mathematical ideas, theorems, and open problems to the general public, inviting the reader to engage with mathematics through solving

puzzles to read the letters. See <http://erikdemaine.org/fonts/> for more examples.

2 Solving Spiral Galaxies Puzzles

Before we describe how we designed our Spiral Galaxies font puzzles, we describe in general how you might solve a Spiral Galaxies puzzle. This approach will let you try solving the font puzzles yourself, and it is also central to how we design puzzles with guaranteed unique solutions.

The general approach is to follow logical deductions from the basic rules of the puzzle and the resulting properties of the solution:

1. **Membership:** Every grid square belongs to exactly one galaxy.
2. **Centers:** Every center is contained in one galaxy, so its incident grid squares belong to that galaxy.
3. **Symmetry:** Every galaxy is 180° rotationally symmetric about its center. This symmetry applies to both the constituent grid squares and the boundary of the galaxy.

Figure 3 walks through a sample 5×5 Spiral Galaxies puzzle. We start with the puzzle centers in Figure 3a.

The first rule we apply (Figure 3b) is the centers rule, which tells us that whenever a center touches multiple grid squares, those squares must all be part of the same galaxy. We draw these same-galaxy relations with gray lines. This rule helps us get started, but we will not need it any further.

It is also helpful to keep track of the boundaries of the galaxies (Figure 3c). Whenever two adjacent grid squares must belong to different galaxies (being connected to different centers via gray edges, say), we can draw a black boundary line between them.

Next we apply the symmetry rule to the galaxy marked with a star in Figure 3d. This galaxy's upper-left square has a boundary edge to its left. Because the region is rotationally symmetric, its lower-right square must have a boundary edge to its right. Can you see how similar logic can be used to place all of the dotted-line borders in Figure 3e?

Next we apply the membership rule to the starred grid square in Figure 3f. This square must belong to some galaxy, but it is surrounded by three boundary edges. Therefore we can draw a gray edge in the fourth (downward) direction. Applying the symmetry rule again (Figure 3g), we can complete the lower-left polyomino.

Next we would like to apply the membership rule to the starred grid square in Figure 3g. Which of the two remaining incomplete galaxies does it belong to? By the symmetry rule, the square cannot belong to the right galaxy, because its then-symmetric square is located outside the grid. By the membership rule, it must then belong to the left galaxy, as in Figure 3h.

Applying the symmetry rule one last time to the left polyomino, we solve the puzzle (Figure 3i).

3 Designing a Spiral Galaxies Font

Each puzzle was designed by hand. Here we describe the process, using the Y puzzle as an example; refer to Figure 4.

First, we designed a black/white pixel rendering of the letter in question; see Figure 4a. Our goal is to make the black pixels decompose into the black galaxy polyominoes, and the white pixels decompose into the white galaxy polyominoes.

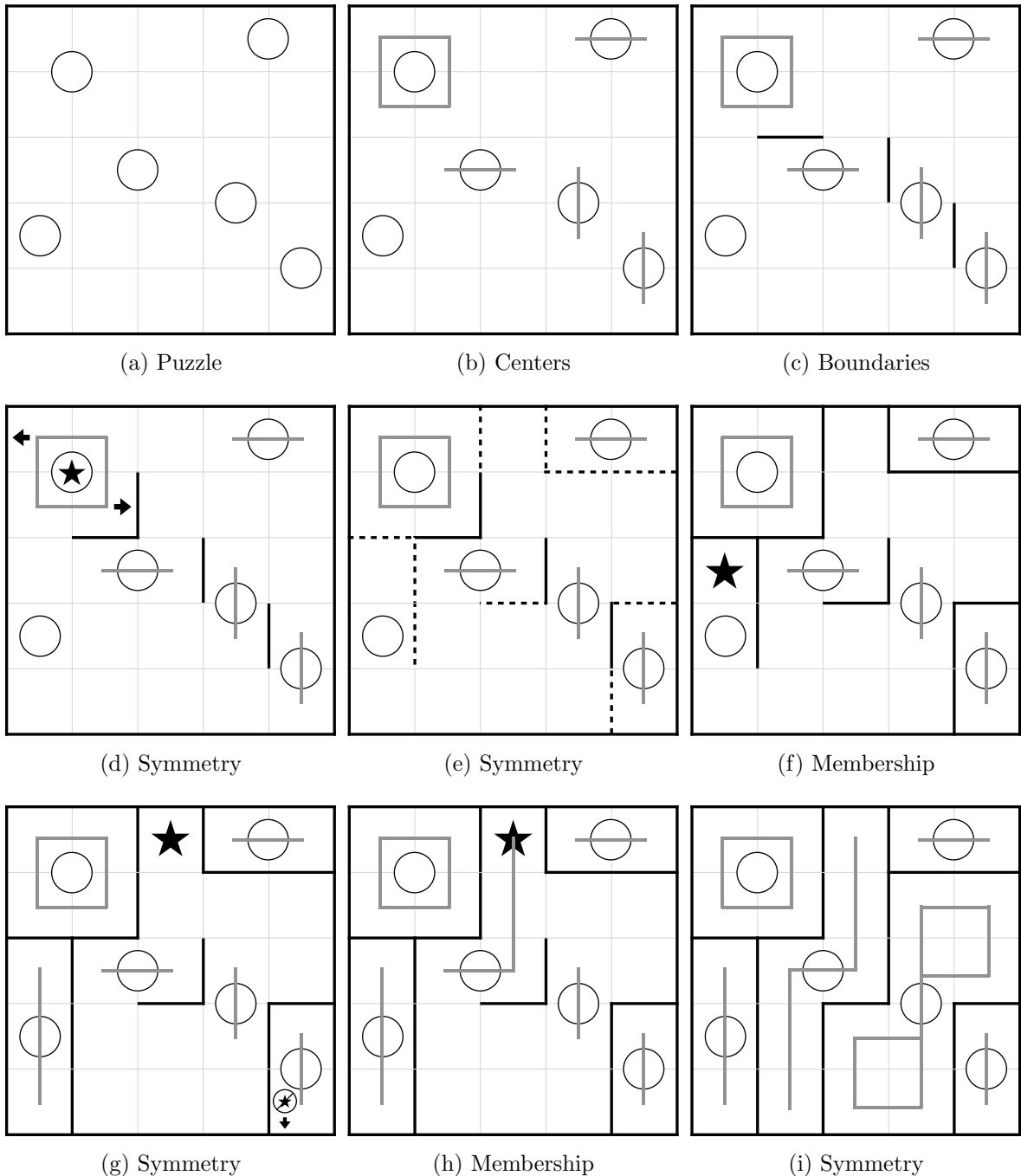


Figure 3: A possible sequence of steps for solving a 5×5 Spiral Galaxies puzzle. Gray lines connect grid squares that must be in the same galaxy, while black lines separate grid squares that must be in different galaxies.

In each successive step, we remove a rotationally symmetric polyomino from the not-yet-decomposed black or white region. We tried to choose polyominoes that would be fun to discover, either with an unusual shape or having a large size. In Figure 4b, we chose a large polyomino.

Before proceeding to the next step, we try to make sure that this polyomino is necessary/forced.

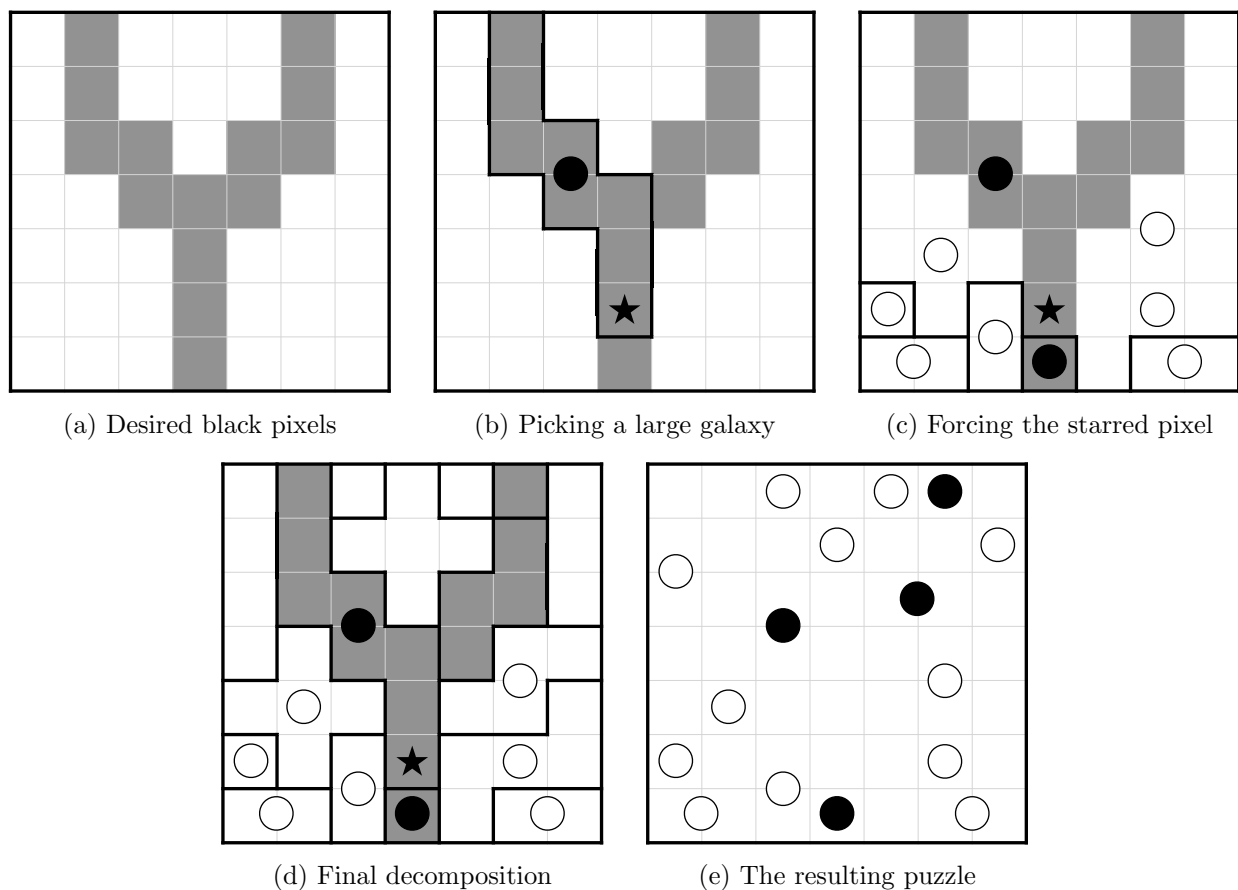


Figure 4: Design walkthrough.

One simple way to do this is to pick a square and set up clues so that the square can only be part of the desired polyomino. For example, consider the starred square in Figure 4b. Once we add the white centers in Figure 4c, the starred square must be part of the polyomino for the already placed black center, because including it in any other polyomino would force that (rotationally symmetric) polyomino to extend past the border of the grid or into squares already occupied by a different polyomino.

Once we are near the end, we simply fill in the remaining regions with rotationally symmetric polyominoes (Figure 4d) and place the corresponding centers (Figure 4e).

Finally, we need to test the puzzle to ensure that the solution is unique. To do so, we solved the puzzle in the traditional way, only writing something into the grid if we could logically justify that it had to be the case. If the solution ended up not unique, we would go back and tweak a few of the polyominoes to reach uniqueness.

We encourage the reader to design their own puzzles — for example, another font of a different grid size, or lower-case letters, or a different language — and send them to us!

Acknowledgments

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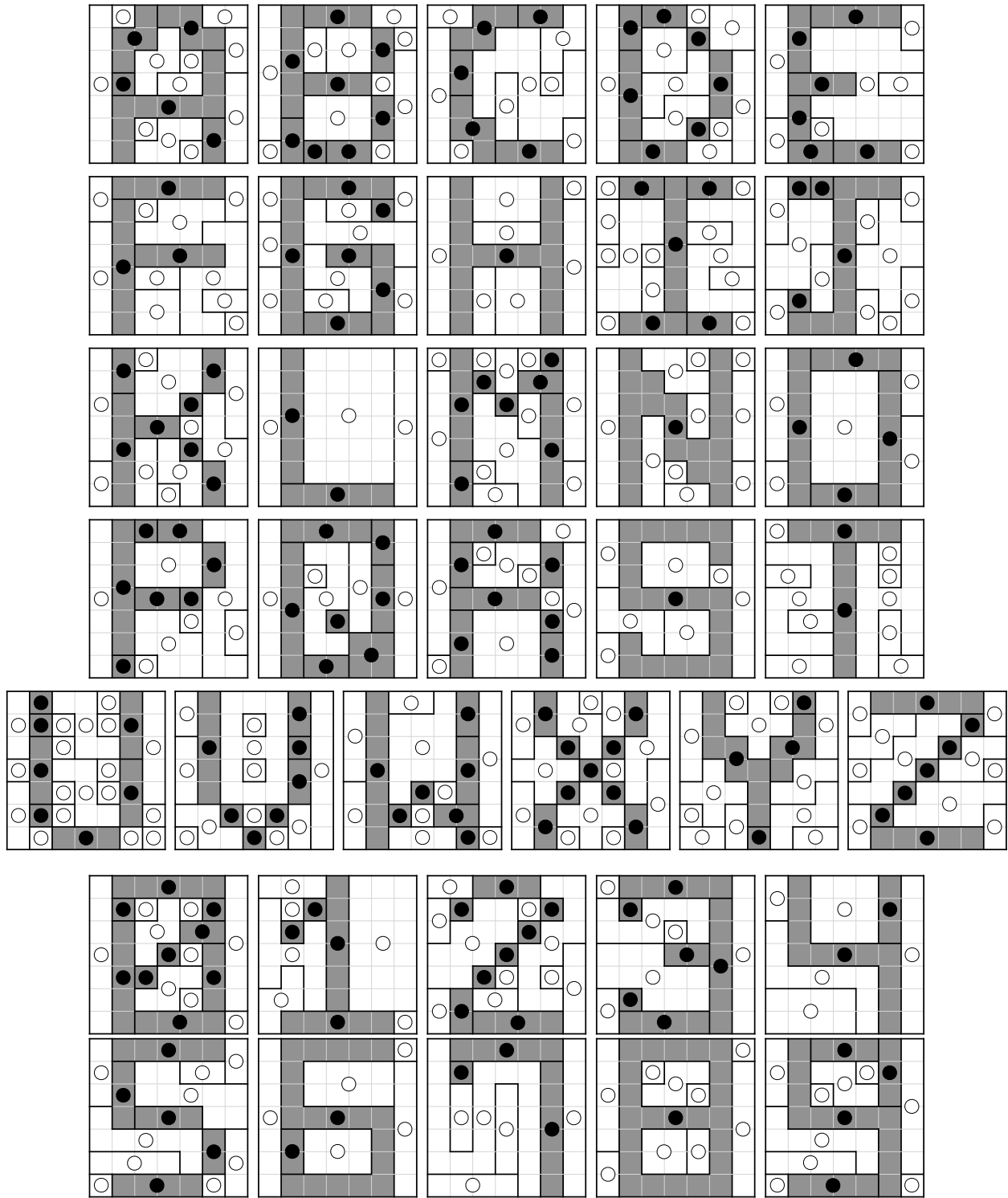


Figure 5: Solved font.

References

- [DH09] Erik D. Demaine and Robert A. Hearn. Playing games with algorithms: Algorithmic combinatorial game theory. In *Games of No Chance 3*, pages 3–56. Cambridge University

Press, 2009. arXiv:cc.CC/0106019.

[Fri02] Erich Friedman. Spiral galaxies puzzles are NP-complete. Unpublished manuscript, March 2002. <http://www.stetson.edu/~efriedma/papers/spiral/spiral.html>.

[HD09] Robert A. Hearn and Erik D. Demaine. *Games, Puzzles, and Computation*. A K Peters, July 2009.

[Nik12] Nikoli Co., Ltd. Tentai show. http://nikoli.co.jp/en/puzzles/astronomical_show.html, 2012.