Computational Geometry through the Information Lens

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A central issue in computational geometry is the discrepancy between the idealized geometric view of points and lines with infinite precision, and the realistic computational view that everything is represented by (finitely many) bits. The geometric view is inspired by Euclidean geometric constructions from circa 300 BC. The computational view matches the reality of digital computers as we know them today and as set forth by Turing in 1936. This discrepancy is traditionally seen as negative: theoretically simple algorithms with infinite precision become difficult to implement in practice with finite precision. A new body of research views the finite-precision reality to be a feature, not a bug, and analyzes the extent to which it can be exploited to obtain faster algorithms than possible for infinite precision.

The bounded-precision / information-theoretic viewpoint has proved extremely successful in the field of (one-dimensional) data structures, reaching tight upper and lower bounds for many fundamental problems. A simple example, hashing, tells us that searching for an exact copy of a query item in a set of n items requires around $\lg n$ bits of information about the query, regardless of the domain. A more complex example, fusion trees, tells us that we need only b bits of information to search for the one-dimensional nearest neighbor among b numbers each b bits long. The information-theoretic view has led to solutions to long-standing open problems in data structures for both finite- and infinite-precision problems, as well as a better understanding of practical uses of finite precision such as radix sort.

The past year has seen the first exploitation of bounded precision in two-dimensional, nonorthogonal problems. We now have data structures for static planar point location and dynamic convex hulls with sublogarithmic query time, and algorithms for constructing Voronoi diagrams in near-linear time. This work starts an exciting new line of research that is far more challenging than classic one-dimensional problems. Our goal is to elucidate the fundamental ways in which geometric information such as points and lines can be decomposed in algorithmically useful ways, enabling a deeper understanding of the relative difficulty of geometric problems.