

Reconstructing David Huffman's Origami Tessellations

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David A. Huffman (1925–1999) is best known in computer science for his work in information theory, particularly Huffman codes, and best known in origami as a pioneer of curved-crease folding. But during his early paper folding in the 1970s, he also designed and folded over a hundred different straight-crease origami tessellations. Unlike most origami tessellations designed in the past twenty years, Huffman's straight-crease tessellations are mostly three-dimensional, rigidly foldable, and have no locking mechanism. In collaboration with Huffman's family, our goal is to document all of his designs by reverse-engineering his models into the corresponding crease patterns, or in some cases, matching his models with his sketches of crease patterns. Here we describe several of Huffman's origami tessellations that are most interesting historically, mathematically, and artistically.

1 Introduction

David A. Huffman was a brilliant mathematician, computer scientist, and artist. Sadly, he wrote only one paper devoted to mathematical paper folding, in 1976 [1]. His paper describes fundamentals of both straight and curved creases, in particular using a dual diagram to analyze both local behavior and interactions among creases. Beyond his paper, Huffman designed and folded hundreds of models and sculptures, and took copious notes on his ideas and designs, but never published and only twice exhibited his work (at U. C. Santa Cruz in 1977 and at Xerox PARC in 1998). More recently, a 2012 exhibit [2] revealed several of his pieces to the public for the first time.

In this paper, we focus on Huffman's straight-crease designs, particularly his work in the area now known as *origami tessellations*. (Huffman's pioneering curved-crease designs are the study of other ongoing work [3].)

An origami tessellation is a folding design where both the crease pattern and the folded state use repeated elements to form a two-dimensional pattern, typically accord-

ing to the symmetries of one of the 17 periodic “wallpaper groups”. Origami historian David Lister [4, 5] attributes the origin of origami tessellations to Shuzo Fujimoto, a Japanese schoolteacher of mathematics who self-published the first origami tessellation book *Twist Origami* in 1976. In 1966, however, artist Ronald Resch patented several origami tessellation designs [6]. Huffman met Resch in 1968, and they talked more extensively at University of Utah in 1973, while Huffman was on sabbatical and Resch was a professor there. Both Huffman and Resch may have been influenced by these discussions, though we have no clear documentation to this effect. Huffman's tessellation designs spanned the 1970s while he was a professor at University of California, Santa Cruz.

An explosion in artistic origami tessellations over the past two decades includes Chris Palmer's work in fabric, Joel Cooper's representational sculpture, and Eric Gjerde's book [7]. On the mathematics side, Lang and Bateman [8] recently characterized a general family of flat-folded origami tessellations, while another recent result [9] can be interpreted as algorithmic design of a family of 3D origami tessellations.

Huffman's tessellations are remarkably different in style from most modern tessellations as well as Fujimoto's historical tessellations. The latter tessellations typically fold flat and are locked (not rigidly foldable): the finished piece is two-dimensional and, from the folded form, it is impossible to flatten back out into its original square without bending along non-crease lines. Most of Huffman's origami tessellations, on the other hand, are three-dimensional and not locked: they can be rigidly folded into their final form by bending the material just at the crease lines.

Our ongoing work, in collaboration with the Huffman family, aims to document, reverse engineer, and analyze David Huffman's origami tessellation designs, of which there are over one hundred. In this paper, we detail the reverse-engineering process for one representative example, and detail (our reconstructions of) one family of closely related models.

The authors are listed in alphabetical order. A preliminary version of this paper appears in the ASME 2013 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (IDETC/CIE 2013).

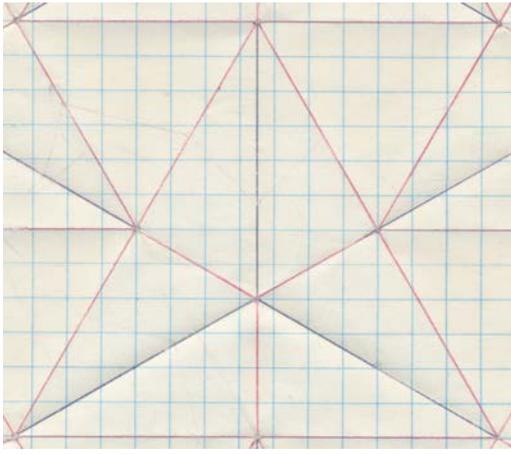


Figure 1: David Huffman’s drawing of an origami tessellation crease pattern (date unknown) in which the triangle is nearly equilateral after being rounded to have corners on a square grid. Used with permission of the Huffman family.

2 Reconstruction Process

Our starting point is Huffman’s physical models, sculptures, and handwritten notes, which are being photographed, scanned, and organized into an archive as part of the curved-crease analysis [3]. In some cases, we have both Huffman’s hand drawing of a crease pattern as well as his folded model, but often we have only one or the other. Our reconstruction process differs depending on whether we work mainly from a (scanned) hand-drawn crease pattern or (a photograph of) a folded model. In either case, our goal is to produce an accurate vector drawing of the crease pattern, print it out (using a Graphtec cutting plotter to score the crease lines), and fold it to ensure that we correctly reconstructed the design.

When starting from a crease pattern, our process is fairly simple. We estimate the exact positions of the crease lines by determining the underlying grid system—most patterns follow a regular square or triangular/hexagonal lattice—and measuring lengths of creases as accurately as possible. Sometimes the resulting folding is not accurate enough, so we adjust the creases and repeat.

One interesting detail is that, while Huffman’s designs often appeared to have triangular or hexagonal symmetry, he drew his crease patterns on (square) graph paper, so he often rounded the triangular grid onto the square grid. Figure 1 shows a simple example. Mathematically, this rounding removes some of the symmetry in the design, but in practice the difference is imperceptible. Our reconstructions treat this rounding as an artifact of Huffman’s process and not part of the mathematical design, and thus use an exact triangular grid.

Reconstructing a design from a photograph is much more complicated. Huffman often explored many variations of a design, and we exploit this fact by modifying previous reconstructions to determine many reconstructions, often by guessing and trying small variations until producing the desired one. For the first design in each family, however, we needed a more precise strategy. First we broke each tessella-



Figure 2: Our photograph of David Huffman’s design “Three Axis Woven Design” (date unknown), which was the basis of our reconstruction.

tion into tileable units, which are repeated to form the overall model. From there, we reconstructed a crease pattern for each unit using a mixture of techniques, and then tiled a page with those units.

3 Reconstructing “Three Axis Woven Design”

For sake of example, we detail the reconstruction process for one of Huffman’s most stunning origami tessellations. Figure 2 shows our starting point, and Figure 6 shows our finished reconstruction. This was one of our first reconstructions from a photographed model. In fact, after further searching through the Huffman archives, we discovered Huffman’s crease pattern for the model, shown in Figure 5, which turns out to be identical to our reconstruction, aside from Huffman’s rounding of equilateral triangles to a square grid. Nonetheless, it serves as an instructive example of our reconstruction process.

Refer to Figures 3 and 4 for notation and step-by-step partial reconstructions.

Step 1. The inset hexagon is formed by a hexagon of creases: $A_1, A_2, A_3, A_4, A_5, A_6$.

Step 2. The little orthogonal valley fold is crease B_1 . From looking at the interior wall of the hexagon, we see that said interior wall is made out of right triangles (angles unknown) so we know that segment B_1 is perpendicular to segment A_1 .

Step 3. In the intersection of A_1 and B_1 is segment C_1 , which runs behind the triangle that makes up the outer face of the interior hexagon wall. We cannot actually see C_1 , but because of the symmetry of the hexagon, we can infer that it exists. The angle between B_1 and A_1 is 90 degrees. The angle between B_1 and C_1 is also 90 degrees. Thus, the angle

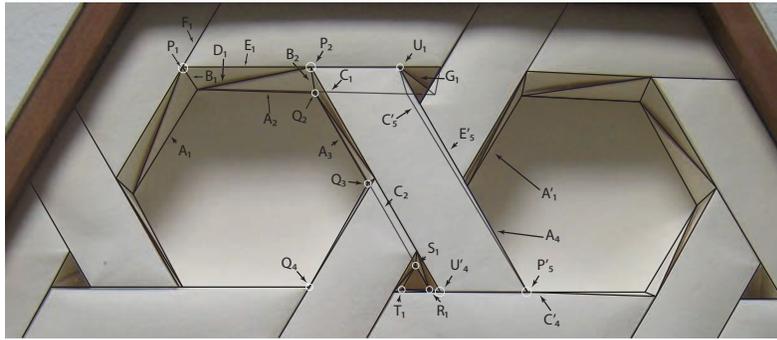


Figure 3: Points and lines of interest in Figure 2 for reconstructing Huffman's "Three Axis Woven Design".

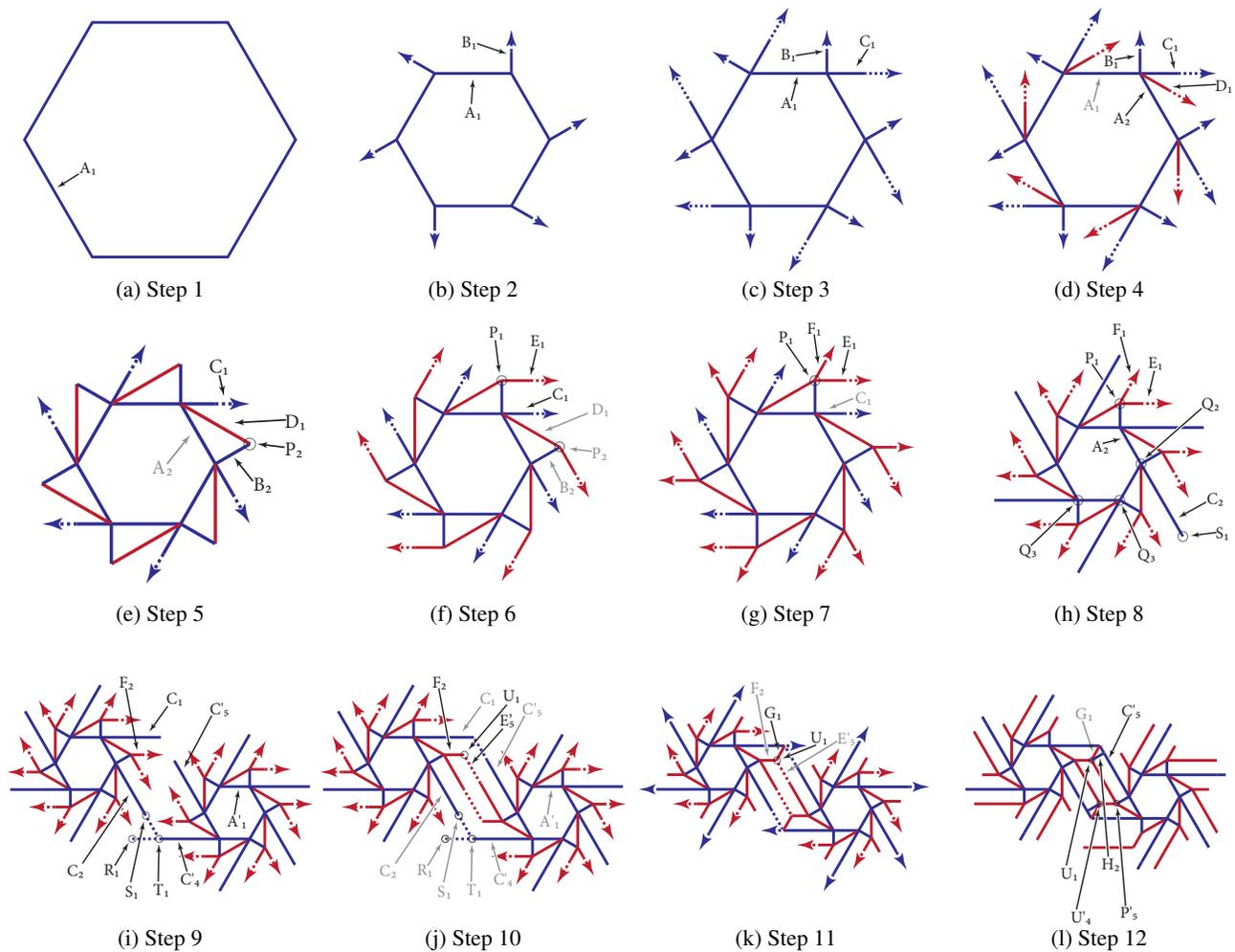


Figure 4: Step-by-step partial reconstructions of Huffman's "Three Axis Woven Design".

between A_1 and C_1 is 180 degrees, meaning that A_1 and C_1 must be the same line.

Step 4. We know that A_2 and C_1 end up collinear in the folding. The only way for them to be collinear is for the angle between them to be bisected. That angle is 60 degrees, thus D_1 must be 30 degrees away from both of them.

Step 5. D_1 and B_2 meet at point P_2 .

Step 6. E_1 is a valley fold radiating from point P_1 . In the folded form, it is parallel to A_2 . Because C_1 lies along the same line as A_2 and there are no fold lines running between E_1 and C_1 , we know E_1 and C_1 are parallel.

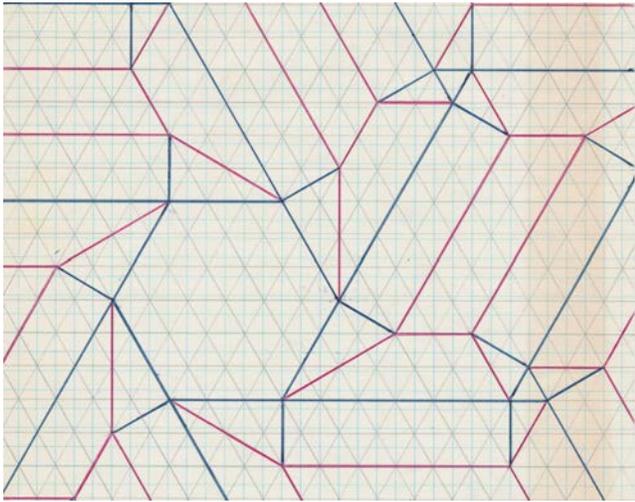


Figure 5: David Huffman's hand-drawn crease pattern for his design "Three Axis Woven Design" shown in Figure 2. Used with permission of the Huffman family.

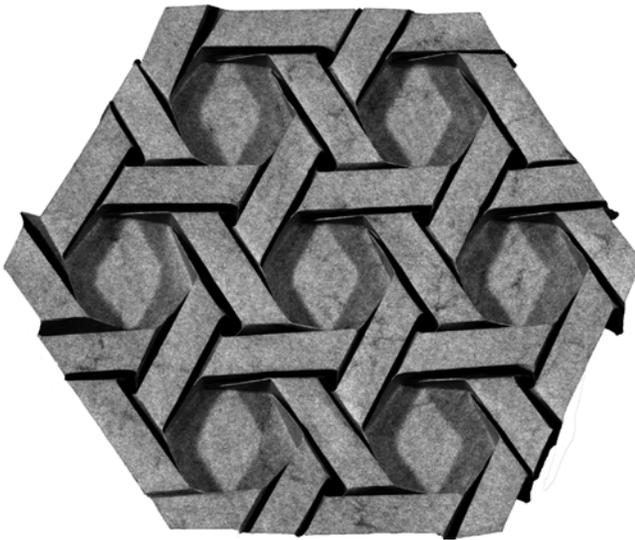


Figure 6: Our reconstruction of David Huffman's "Three Axis Woven Design".

Step 7. F_1 is a valley fold which also comes from P_1 , and is 60 degrees away from E_1 .

Step 8. The section of C_2 between Q_2 and Q_3 is the same length as A_2 . Points Q_3 , Q_4 , and R_1 define an equilateral triangle, so the section of C_2 between Q_3 and R_1 is also the same length as A_2 . Thus, the total length of C_2 is twice the length of A_2 .

Step 9. At this point, it looks like we can put together two tiles. However, there is still a problem: C_2 appears to terminate at R_1 . However, C'_4 , which must meet C_2 , seems to terminate at T_1 . Thus, we know that C_2 cannot end at R_1 . Instead, A_2 meets C'_4 , and R_1 is left as part of the smaller mini triangles. On closer inspection, A'_2 wraps around and termi-

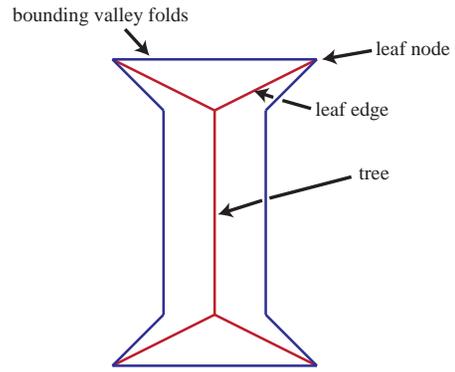


Figure 7: The components of a repeating unit.

nates at T_1 , while C'_4 continues on the same path and finishes at S_1 . If A_2 were to meet R_1 , then there would be no miniature triangles, and we would have an inaccurate reconstruction.

Step 10. When we extend the lines we have so far, F_2 and E'_5 intersect at point U_1 .

Step 11. G_1 runs from point U_1 to the intersection of C_1 and C'_5 .

Step 12. H_2 runs from point U_1 , and intersects C'_5 two A lengths away from its start. We know it is two A lengths away because the parallelogram defined by P_1 , T_1 , U'_4 , and P'_5 has sides of length $2A$. Each side is defined by A_3 and C'_2 . C'_2 is equal to A_4 , so each side is the sum of those segments, and thus is of length $2A$. This locks all proportions into place, and our reconstruction is complete.

4 Families

Many of Huffman's origami tessellation designs are closely related to each other. We call two designs *directly related* if there is a relatively simple transformation that turns one design into the other, and call two designs *related* or *belonging to the same family* if there is a sequence of such transformations.

In many origami tessellation designs, the repeating unit of the crease pattern can be characterized as in Figure 7 to have *bounding polygon* of valley folds, and an interior *tree* (or nearly tree) structure of mountains. To compare (or transform) two tessellations, we focus on just this repeating unit, and ignore other auxiliary creases.

In these settings, the transformations we allow are symmetric shifts of the angles of the tree; point enlarging; and shears, stretches, compressions, and twists of the crease pattern, so long as they preserve symmetry. The *stretches* and *compressions* dilate or contract either all or parts of the tree, subject to preserving tree symmetry. A *twist* is comprised of fixing the center of the tree and then stretching and rotating the rest of the tree and boundary around that point. We can also *enlarge* points into polygons, as long as there are at least three incident lines, two or more of which must be leaves of the tree; the expanded polygon must have a number

of vertices between 2 and the number of creases incident to the point.

In a transformation, we allow changing the bounding polygon, to enable different clipping the tree to different shapes for tiling. We also allow modifying other auxiliary creases. We forbid changing the mountain-valley assignment of individual creases. (We can allow flipping all creases, though.)

5 The Vanes Family

Figure 10 illustrates one family of Huffman designs we have identified, their crease patterns, and their direct relations. We call the family “Vanes” after Huffman’s own name for one of the designs: “Raised Vanes, Both Vertical and Horizontal”. Figures 6 and 11–19 give photographs of our folded reconstructions of these designs.

Next we describe some of the direct relations between members of the Vanes family.

Redrawing the bounding polygon. “Stars-Triangles” and “Raised Vanes, Both Vertical and Horizontal” share a tree. To transform from one to the other, we simply redraw the bounding polygon.

Tree angle shifts. To go from “Raised Vanes, Both Vertical and Horizontal” to “Crippled Vanes”, we merely enlarge the angle of the two leaves of the end of the tree in “Raised Vanes”.

Shears. “Three Axis Woven Design” is a sheared version of “Rectangular Woven Design”, sheared from a square grid to an equilateral triangle grid.

Stretches and compressions. “Rectangular Woven Design” is a stretched version of “Exdentented Boxes”, stretching the square center into a rectangle.

Twists. To make “Pinwheels”, we fix the center of the square in “Exdentented Boxes” and rotate the leaves 22.5° around the center, and then redraw the bounding polygon.

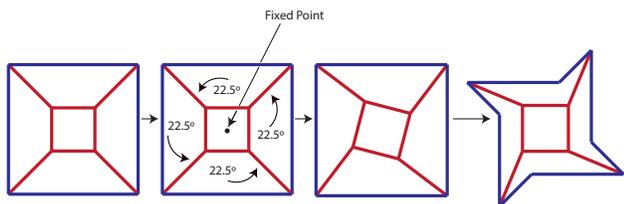


Figure 8: The twist to make “Pinwheels”.

Point to polygon. To make “Rectangular Woven Design” from “Extruded Boxes”, we enlarge both nodes into polygons, forming the rectangle of “Rectangular Woven Design”.

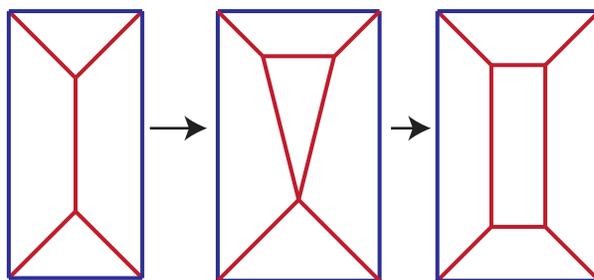


Figure 9: Enlarging the nodes of “Extruded Boxes”.

6 Conclusion

The family of reconstructions presented here just scratch the surface of Huffman’s origami tessellations, of which there are over a hundred. In ongoing work, we are reconstructing, analyzing, and categorizing all of his designs. We expect there to be roughly twenty different families, most of which should be loosely linked. By reconstructing his tessellations, we aim to open the world of three-dimensional tessellations. Most work today is centered on tightly locked, two-dimensional tessellations, and we look forward to encouraging further exploration of Huffman’s three-dimensional, rigidly foldable tessellation style.

7 Acknowledgments

The authors would like to thank the Huffman family for graciously allowing us access to David Huffman’s foldings and notes, for permission to display selected notes, and for inviting all of us to visit and see his models in person. We also thank Duks Koschitz for his invaluable help in navigating through the Huffman archives.

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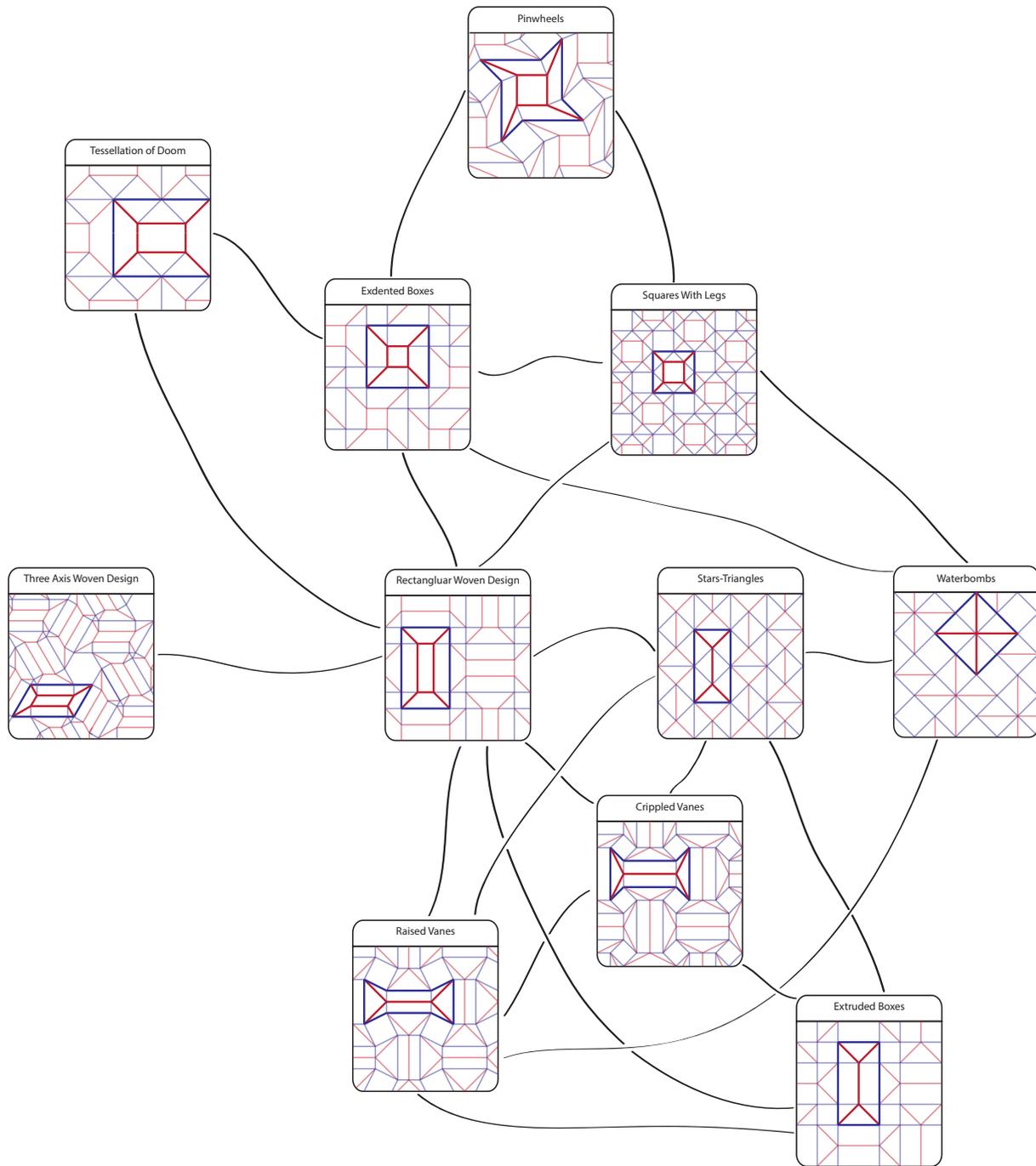


Figure 10: The Vanes Family Tree

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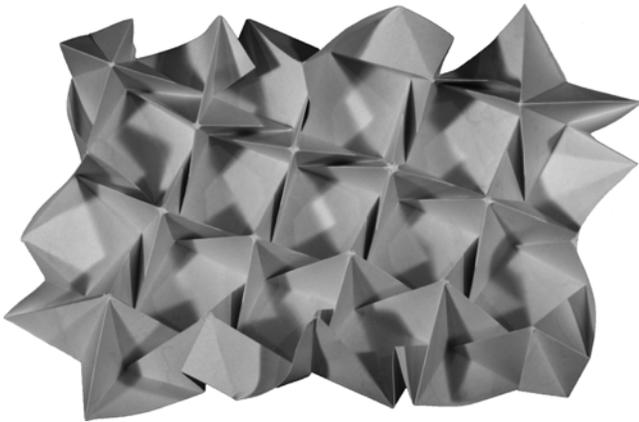


Figure 11: Our reconstruction of David Huffman's origami tessellation (title and date unknown), which we call "Water-bombs".

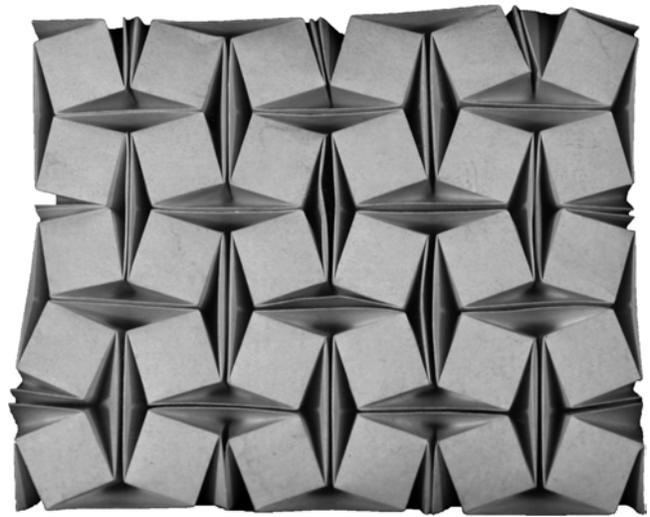


Figure 13: Our reconstruction of David Huffman's origami tessellation (title and date unknown), which we call "Ex-truded Boxes".

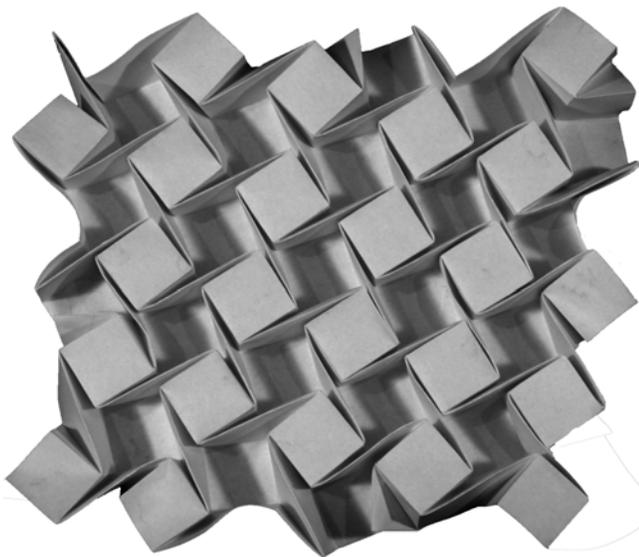


Figure 12: Our reconstruction of David Huffman's origami tessellation (title and date unknown), which we call "Ex-dented Boxes".

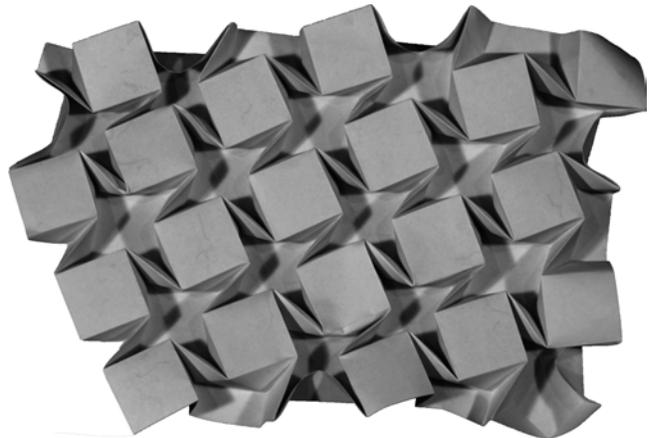


Figure 14: Our reconstruction of David Huffman's origami tessellation (title and date unknown), which we call "Pin-wheels".



Figure 15: Our reconstruction of David Huffman's "Raised Vanes, Both Vertical and Horizontal" (date unknown).

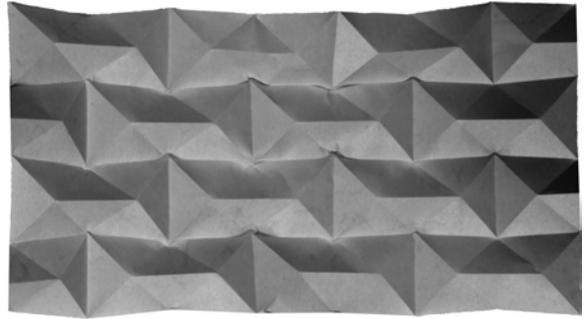


Figure 18: Our reconstruction of David Huffman's origami tessellation (title and date unknown), which we call "Stars-Triangles".

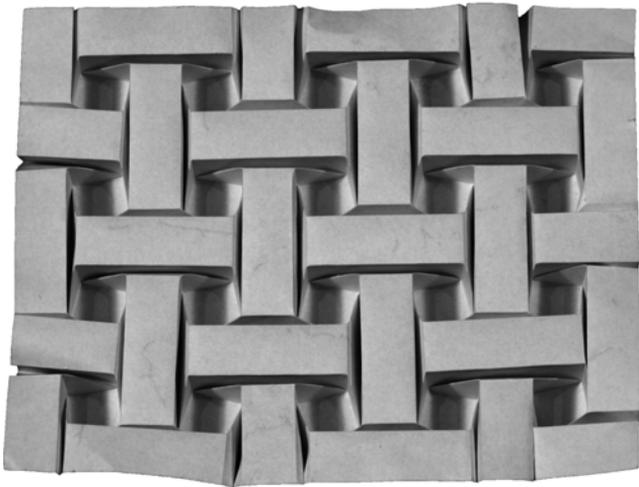


Figure 16: Our reconstruction of David Huffman's "Rectangular Woven Design" (date unknown)

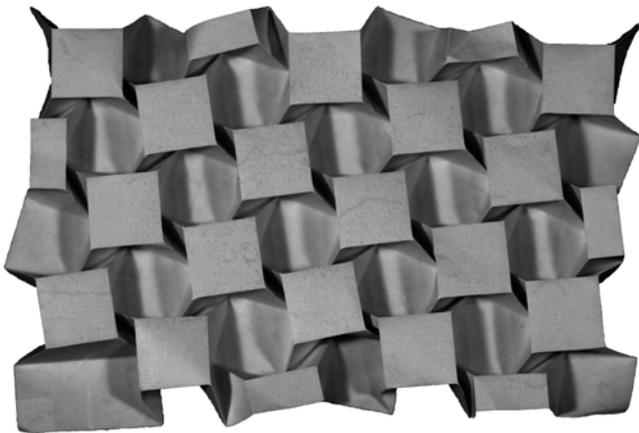


Figure 17: Our reconstruction of David Huffman's "Squares With Legs" (date unknown).



Figure 19: Our reconstruction of David Huffman's origami tessellation (title and date unknown), which we call "Tessellation of Doom".