Computational Design with Curved Creases: David Huffman's Approach to Paperfolding

by

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Submitted to the Department of Architecture in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in Architecture: Design and Computation

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ABSTRACT

This dissertation provides a new framework for defining design approaches with curved creases by investigating the work of David A. Huffman, famous computer scientist and pioneer of curved-crease paperfolding.

The history of curved folding has diverse cultural origins. I outline the boundaries of curved-crease paperfolding as a field of knowledge beyond geometry by identifying its role in art, design and pedagogy.

I document Huffman's entire curved-crease work, including approximately 2000 notes on geometry and 150 unique designs, and analyze it by means of a taxonomy divided into geometric categories. I abstract his designs into gadgets, or small sets of curves with specific three-dimensional folding consequences. The taxonomy begins with reflected cylinders and cones which are well understood mathematically. The bulk consists of gadgets Huffman invents for himself which use conics and refraction. The gadgets control the behavior of the paper around the crease, providing an opportunity for discrete representation. More general curve types conclude the taxonomy. This structure allows me to present Huffman's varied oeuvre in a coherent way, raise questions for future research in geometry and describe Huffman's ways of designing with constraints, making the work accessible to mathematicians as well as designers.

The dissertation strives to expand the field of design and computation to include curved-crease paperfolding. I create a new framework that formalizes digital and analog design approaches, expanding on the works of Huffman and others such as Josef Albers, Roy Iwaki and Ron Resch. I compare and evaluate each approach in terms of the necessary tacit versus a priori knowledge of geometry, whether they are material-driven or digitally-driven, and their use of digital representation. Since a discrete representation is available for simulation I propose a computational approach that provides real-time feedback. I present results of seminars and workshops I have conducted.

Curved creases have become increasingly attractive for designers and architects as manufacturing of furniture and building parts with them has become possible. Folding flat materials provides economic and energy efficiencies compared to stamping. Making design approaches for curved creases accessible provides material-based approaches for design pedagogy and allows designers to expand their canon of forms.

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4. A taxonomy of DAH's curved-crease paperfolding work

1. Introduction

This dissertation provides a new framework for defining design approaches with curved creases that allow designers to work with this underexplored subset of geometry. Investigating and abstracting the works of artists, designers, and specifically the expansive work of computer scientist David A. Huffman allows for the definition of such approaches in digital and analog ways. The computer scientist, famous for his compression procedure, was a pioneer of curved-crease paperfolding and has left behind several thousand notes and over 150 unique designs, which I converted into an archive that I use as the major case study.
Curved-crease paperfolding has cultural origins and historical records exist in diverse fields from art and craft to pedagogy to mathematical origami. Telling its story can delineate the boundary of this field of knowledge and identify its role in the disciplines in which it has been used.

The geometry has been deployed by designers and artists throughout history and it is becoming increasingly attractive to architects as manufacturing curved creases for furniture and building components has become possible [Gla 07]. Many building materials rely on the practicality of flat surfaces as they provide ease of handling and shipping. Developable surfaces can be shipped flat and formed into a curved configuration in fabrication facilities via the use of curved creases. This alternative to stamping provides economic and energy efficiencies as no mold and far smaller forces are necessary. Making design approaches for such applications more accessible allows artists, designers and architects to expand their canon of forms that can be manufactured efficiently.

One hurdle consists of the poor mathematical understanding of curved creases and their behavior, which remains underexplored. This results in a lack of adequate digital tools for designers and no formalized design approaches have been made available yet. Artists and designers have developed workarounds. Huffman developed a methodical approach. Both are valid as design approach, but need to be evaluated differently.

The dissertation strives to expand the field of design and computation to include curved-crease paperfolding. As the geometry of curved creases needs further mathematical understanding defining computational ways of designing with them has posed challenges. I evaluate analog (Fig 5.5.3) and digital approaches (Fig 5.3.4) in order to formalize structured design approaches.

The main case study consists of Huffman's curved-crease paperfolding work. His oeuvre may look random or idiosyncratic, but in fact there are methods or principles underlying the work which I categorize in order to present the work in a structured way.

By abstracting Huffman's work I define design approaches that integrate digital tools to aid the process of designing with curved creases. In addition I propose to expand analog alternatives to designing with curved creases that enable design exploration, which relies on tactile tacit knowledge.

I present Huffman's paperfolding work in the form of a taxonomy that follows geometric principles. The categories of the taxonomy elucidate how Huffman worked and which subsets of geometry he investigated. I abstract his work into small set of creases or 'gadgets' that determine how the paper behaves around them. Huffman often uses these gadgets in regular repetitive patterns. Within the large categories of the taxonomy the gadgets allow for further structuring of the work into smaller sections.

The gadgets also provide a possible discrete representation of paper that consists of small line segments. This representation can be used for software simulation, which allows me to formulate a computational design approach that uses digital tools with real-time feedback. This means that a designer can observe how a flat design folds on the computer.

In order to provide an evaluation of Huffman's design approaches I compare them to approaches by other artists and designers. Some of these more expressive design approaches are based on alternative analog techniques.

Road map of the dissertation

The dissertation is structured in chapters that are divided into numbered sections and further subsections. This introduction chapter 1 introduces motivations and the structure of the dissertation. The following chapter elucidates the history and background of curved creases.

<u>2. Historical background of paperfolding:</u>

As paperfolding has origins in many places and disciplines, the second chapter presents a history of paperfolding that helps to define areas of investigation in which the geometry has played a role. One goal consists of defining the term 'paperfolding', and to delineate the boundaries of curved-crease folding on a cultural level and as a form of knowledge.

The history investigates the beginnings of straight crease paperfolding in Asia and Europe over the past 400 years and later segues into curved creases. Despite the lack of tools for designing with curved creases, artists, educators and mathematicians have used this geometry intermittently over the course of the past three centuries. Art historian George Kubler provides a definition of history as a linked successions of works distributed over time in different fields [Kub 62]. He uses the art of enamel as an example, which has a discontinuous history across different regions. Regarding curved creases we encounter exactly that, scattered events across the previously mentioned disciplines, which relate to one another only via a specific geometry that is bound to the constraints of paper.

Curved napkin folding provides a possible point of departure in the 17th century. Friedrich Fröbel introduced rule-based systems in his Kindergarten and his disciples taught straight crease paperfolding as one of his so called 'educational occupations' in the 19th century [Hei 03]. Josef Albers asked his students to perform curved-crease paperfolding exercises as part of the German Bauhaus foundation curriculum. Their work represents an early example of this special geometry in the context of architectural pedagogy [Adl 04].

More recent works by David Huffman and Ron Resch in the 1960s and 1970s provide substantial case studies for the field. In their works one can think of 'the paper being the computer', in the sense that the paper's materiality necessitates the definition of rules for computational design approaches.

A short introduction of relevant mathematical work follows in order to demonstrate what is known in the field today.

3. David Albert Huffman (DAH):

This chapter studies Huffman and strives to describe his ways of interpreting visual problems, ways of relating mathematical concepts to physical artifacts, and ways of designing. I hence present an abbreviated biographical history by focusing on visual and design related endeavors in his life.

A pioneer in curved-crease paperfolding, David Albert Huffman created hundreds of designs, at first as a hobby, and later as part of his research at the University of California Santa Cruz (Fig 1.1). He only published 2 mathematical papers and did not publicize his curved-crease work beyond a few exhibitions he participated in. His death in 1999 left us without his deep understanding, but his many models and notes provide a glimpse into his thinking.

While his work on curved creases is largely unknown, the invention of lossless compression, namely 'The Huffman Coding Procedure', represents his most famous contribution to computer science. His background in machine vision, an evolving field during the 1960s at MIT, led him to create a notational standard for the recognition of shapes. The system would enable machines to infer geometrical shapes in an image. The computer science culture at MIT privileged a 'divide and conquer' approach. It is a means to solve a large problem by first dividing it into smaller, possibly simplified parts, each of which is solved and then put together again to solve the larger problem.

Huffman's approach to designing with curved creases is structured in a similar way. He

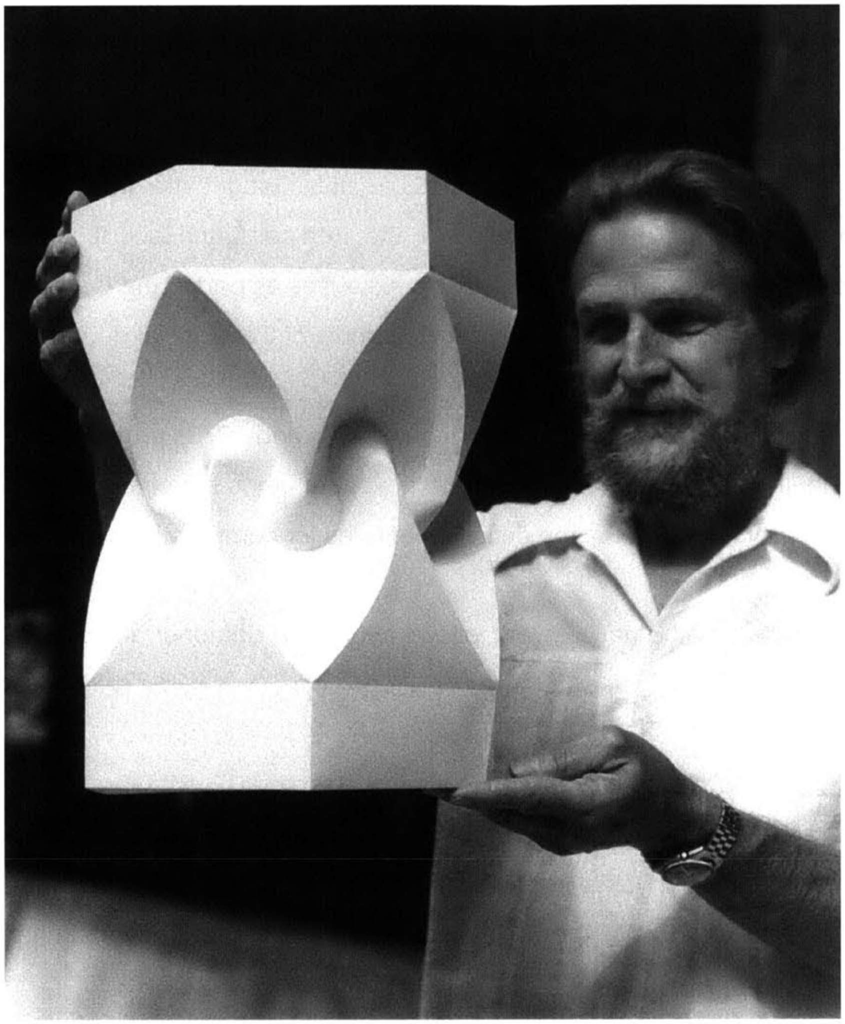

Fig 1.1 David A. Huffman and his 'hexagonal tower with cusps' at UCSC

meticulously assembled small sets of curves and points, the gadgets, and repeated them in regular patterns. He used mostly curves with a special property that allowed him to predict the location of rulings, the straight linear moments in the paper.

Huffman ventured to discover the relationship between a flattened curve and its 3d configuration, and received a National Science Foundation grant by arguing for the importance of paperfolding in the field of machine vision. He published a paper on the local behavior of a curved crease in 1976, which is regarded as one of the seminal texts on this kind of geometry still today.

Accompanied by his set of tools, curve plotting and borrowed concepts of straight crease paperfolding, Huffman designed several hundred models. He did not regard himself as an artist, but believed that beauty in mathematical relationships could provide visually aesthetic results.

The chapter ends with a description of his mathematical and physical aids that enabled him to design his vast body of work.

4. A taxonomy of DAH's curved-crease paperfolding work:

The beginning of the taxonomy defines terminology and discusses methods of reconstruction. The taxonomy itself describes Huffman's work and evaluates it on several levels. It consists of reproductions and reconstructions that are based on my interpretation of the archival work that was available to me. The taxonomy serves as a mathematical description of the designs as well as an aesthetic evaluation of the work. The descriptions of the physical artifacts elucidate Huffman's sense of aesthetics.

A large part of this dissertation is based on reconstructions of Huffman's pristine vinyl models and his sketch models. He took meticulous notes that explain how he approached his designs. I present the work in the form of a taxonomy with several geometric categories that I order by curve type and gadget. Every design is presented individually in the form of a geometric description of the crease pattern and an analysis of the folded design.

Huffman wanted to determine where the rulings are located in his designs before folding the paper, so he could predict what the creases would fold into. To do so, he resorted to conic sections, because he could exploit a special feature, namely their refractive properties. We know the phenomenon and its role in optics and communication technology. Satellite antennas have parabolic shapes, because reflected beams converge in a focal point. If we consider transparent lenses, we can observe how light beams get refracted. Huffman categorized the cases he exploited for his designs in a diagram he created for himself (Fig 1.2). The left examples show how rule lines get refracted if they start in the focal point of a hyperbola, parabola or ellipse. The cases

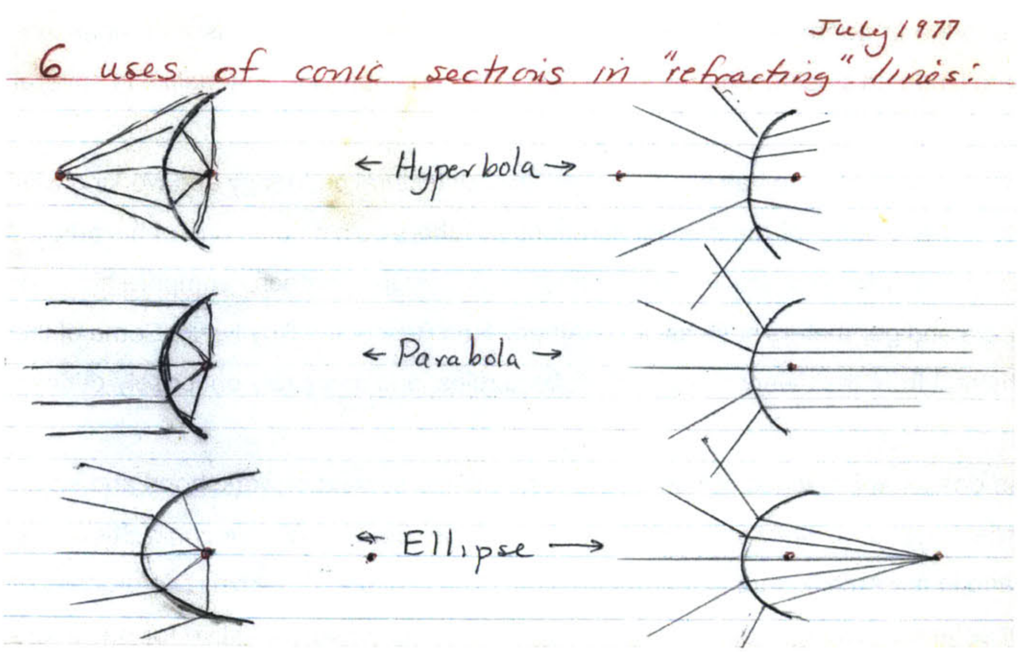

Fig 1.2 Ray refraction of conic sections that form the basis for refraction gadgets (1977, DAH [DK])

on the right assume converging lines for a hyperbola, parallel lines for a parabola, and lines starting in the second focal point for an ellipse. The gadgets in the taxonomy follow these refractive properties and the description of every example starts with the crease pattern.

Huffman took great interest in tilings, the regular repetition of gadgets. The combinatorics of his tilings provide further opportunities for categorizations of his work. I evaluate folded models and Huffman's assumptions on the location of the rulings in 3d. In some cases Huffman might have had a wrong conjecture. The last part of the individual presentation of a design consists of possible sources for inspiration and a description of the work in terms of craft, aesthetics, and method of physical construction.

The first sections of the taxonomy are concerned with reflections of cylinders and cones, which are well understood mathematically. The gadgets in this section are called reflection gadgets. The second part addresses the work designed with refraction gadgets and the taxonomy further divides the designs into sections by curve type and tiling. The third part consists of a single design in its own section. Its gadget does not rely on a specific curve type. The last sections explore designs that have no gadgets as the rulings for spirals, vortexes, and sinks are unknown.

I conclude the taxonomy with a summary of general findings and opportunities for future research in geometry.

5. Design approaches for curved creases in comparison:

I propose to expand several design approaches to include digital tools for curved creases. The approaches range from known geometric descriptions to ways of designing that do not require any a priori knowledge in geometry. The proposed design approaches serve as a comparison, and help to situate Huffman's achievements in the field as his contributions materialize in several of the presented sections.

Constraint based designing has been common in architectural pedagogy and working with paper represents fertile ground for material explorations as Albers points out in one of his articles on design pedagogy. In order to critically evaluate Huffman's design approach I compare his work to artists, designers and geometers such as Josef Albers, Ron Resch and Roy Iwaki. Some of the approaches I abstract from their work follow well-defined rules, and some rely on loosely-defined concepts.

I propose design approaches, which I have tested in the context of workshops and seminars at the Massachusetts Institute of Technology, Cambridge, Southern Polytechnic State University, Atlanta and in the Architecture Department at the Pratt Institute, Brooklyn. The process of drawing and subsequently folding paper along curved creases is not very intuitive for designers and the approaches presented at the end of this dissertation aspire to make designing with this geometry understandable to a wider audience.

The chapter begins with the use of reflection followed by closed curved creases that are based on the work of Albers's students. The third design approach uses Huffman's reflection and refraction gadgets. Step by step evaluations inspired by Iwaki structure the fourth section. Finally, I suggest to expand Resch's crinkle analysis to include curved creases via the use of flexible materials. The approach includes recording instructions with a notational system.

6. General conclusion

The final chapter concludes with general remarks, contributions and next steps.

2. Historical background of paperfolding

Sources and academic challenges

Writing a history of folding paper necessitates relying on a variety of sources such as religious writing, folding manuals, texts on pedagogy, art works and even cook books. The eclectic nature of the references poses significant difficulty in articulating an appropriate history of paperfolding.

Art historian George Kubler provides a fitting framework for a not-continuous and geographically disparate series of events by offering an alternative way of writing history. He describes the possibility of history as linked successions of works distributed over time in different fields [Kub 62, p106] and proposes to distinguish between continuous and intermittent classes in history. Intermittent classes can lapse inside the same cultural grouping or span different cultures. Enameled jewelry serves as an example of such a class and he traces its history across cultures with vast gaps in time. The construction of a history disparate parts and events allows him to arrive at the definition of a field of knowledge. Similarly, paperfolding can be found in craft, pedagogy, art and mathematics, and curved-crease paperfolding occurs at very intermittent times in history. In order to provide a comprehensive view of specifically curved-crease paperfolding, I first examine the general history of straight crease paperfolding with a geographical focus on Europe and Asia. I begin with general accounts of paperfolding in chronological order and distinguishing them by discipline. I segue into curved crease paperfolding in the following chapter.

Another problem within the history of paperfolding lies in the incoherence of sources as it emerged in part as a craft, which has been sparsely documented. Many accounts have been collected by non academic folding enthusiasts, which has led to the propagation of errors in popular literature. However, the work of David Lister and Joan Sallas i Campmany, who co-founded the International Association for Documentation and Research on Paperfolding (PADORE) in 2004, provides the necessary rigor. They recognize the demand for an academic investigation of the history of paperfolding and I refer to much of their work in this chapter. Sallas has collected over 2000 books on folding from the 16th century until today and recreates many examples as part of his research. Lister who has passed away in 2013 never compiles his knowledge into a single publication. His collection of articles, e-mail responses and web postings, available on the British Origami Societey's website point to several misconceptions and discrepancies, which Koshiro Hatori, another expert on the history and culture of origami, agrees with [Hat 11]. The following compilation of historic events owes much to Lister, Sallas and Hatori's efforts in correcting facts on the history of paperfolding and strives to show the many different origins and trajectories of paperfolding across disciplines.

The history of folding paper can help to differentiate the definitions for paperfolding and origami. I return to the distinction between the two terms, which is partially based on geography and partially rooted in the types of designs that appear throughout history, at the end of the chapter.

2.1 A history of straight crease paperfolding

Paperfolding in Europe

The beginnings of paperfolding are difficult to trace and early original material can be found in Europe and Asia. I believe it is appropriate to separate the traditions geographically as much of the knowledge transfer is historically unaccounted for. Sparse evidence of paperfolding in Europe can be found in Spain, Germany, Italy, France and England. The development appeared uniformly across borders, but specific incidents were unique to Germany, a country only united as one in 1871.

Vicente Palacios, author of many origami books, claims that Spanish paperfolding is indigenous to Spain without any Moorish contribution [Pal 02] and that Spanish paperfolding predates Eastern paperfolding. Lister disputes this as paper was invented in Asia and introduced to Europe much later. European paperfolding may have originated in complete isolation from Eastern paperfolding, but its knowledge may also have been brought to Europe along the silk route, sea routes or even via the technology of paper-making by the Arabs. Palacios's claim that paperfolding spread from the West to the East seems unlikely [Lis a].

In the 4th century A.D. depictions of popes included elaborate ceremonial fans or 'flabelli', usually made from ostrich feathers, and Palacios discovered that they are made from folded parchment in later centuries. He points to similarities with the folded pattern of the 'Pajarita', the pattern of the 'Astrological Square'. The square design, invented by the Italian Gerardo Cremone, was used for casting horoscopes starting in the 12th century. It was replaced by circular drawings in the 19th century. No evidence exists that these horoscopes were ever folded up into Pajarita or multiform patterns. The common crease patterns serve designers as a base, the most reduced version of a crease pattern that can be used as a point of departure for other models. A possible connection between the astrological square and paperfolding in France, Germany and Austria could be the custom of folding certificates of baptism into a double blintz, a square with folded in corners. However, further evidence is necessary to confirm this theory [Lis 03].

Another of Palacios's conjectural discoveries quotes the 'Tractatus de Sphaera Mundi' ('Treatise on the Sphere of the World'), by John Holywood, who is also known as Johannes de Sacro Bosco, an English mathematician and astronomer of the 13th century. An edition published in Venice in 1490 shows a picture of a town and two boats at sea, which appear to resemble simple paper boats. The depictions do have similarities with folded paper boats, but it is unclear if the artist really had paper boats in mind [Lis a].

Further sources exist in the 1600s, namely paper contraptions made by children as prisons for flies mentioned in a play by John Webster in 1614 and a basket made out of paper in a diary of Samuel Pepys around 1660 [Lis a].

Much of paperfolding is part of the history of pleating cloth or napkins as illustrations of folded and pleated clothing date back to Egyptian, Classical and Byzantine eras. Pleated clothing may not seem very related to paperfolding, but the 16th century brought about new ways of folding cloth. The topic has been elaborated upon exhaustively by Joan Sallas i Campmany. He mentions Italy as the country of origin for artistically pleated napkins, where this art was taught as part of the 'Tranchierausbildung', literally translated as the education of carving meat, meaning the education of table decor and presentation of food, in 1609 [Sal 10]. Italy saw an increased use of napkins during meals at a time before individual forks became a universal part of the equipment at every civilized table. Opulent displays and table decorations became fashionable at the many courts of Italy during the Baroque and a new fashion for centerpieces on the table spread from Italy to Northern Europe [Tre 08]. Starched napkins transformed into animals, birds, sailing ships and other impressive models. Napkins were pleated and sometimes cross pleated at 90° to form a pliable sheet of tessellated quadrilaterals, which was shaped into its desired configuration. Several napkins were often stitched together with a red thread, which is a significant deviation from folding a single sheet of paper. [Lis a] These publications also include depictions of designs with curved creases, which I elaborate on in a later section.

Mattia Giegher, an Italian whose name was phonetically translated from Matthias Jaeger, published a treatise in 1629. His book, 'Li Tre Trattati' contains very elaborate table decorations (Fig 2.1.1 left). 'Trattato delle piegature' (Treatise of folding), one part of the publication, may well represent the first publication on folding with instructions worldwide (Fig 2.1.1 right). It includes a few designs for the folding of individual napkins in a way still familiar today such as the folding of a waterbomb base, a square folded along the center lines and the diagonals. In 1657 in Munich Georg Philipp Harsdörffer expanded Giegher's book and translated it into German [Sal 10].

Two paperfolding references from the 1700s exist. One, discovered by Vicente Palacios, consists of a report by Guillermo Pen in 1757, who describes kites, boats, ships, birds and many other things made of paper. The second, mentions a blind man in Greenwich Park, outside London, in about 1790 who was able to make many shapes out of paper [Lis a].

Friedrich Fröbel, the inventor of the Kindergarten was born in 1782. His disciples used paperfolding as part of his pedagogy, which I discuss in detail in the section on education. One of his followers, Eleanore Heer, mentioned that before Fröbel's time folding was a common craft in nurseries and in the homes of rich and poor. Paperfolding was most probably generally known in Europe by the 18th century.

Elsje van der Ploeg discovered a drawing in Holland from 1806 depicting a little boy sailing a Chinese junk, which used to be the first dated and unambiguous illustration of paperfolding in Europe [Lis a]. It is now preceded by Sallas's sources. A collection of folded soldiers on horses, folded from variants of the blintz fold, another base, represents another recognized account of paperfolding in the 19th century. The work is part of the permanent collection of the German National Museum in Nürnberg. Around 1880 the 'flapping bird' was introduced to Europe allegedly by Japanese jugglers. The 'jumping frog' also appeared around the same time. The English speaking world saw a publication with a few paperfoldings in 1881 in 'Cassell's Book of Indoor Amusements, Card Games and Fireside Fun', an early incident of recreational folding.

Another figure worth mentioning is Miguel Unamuno, a Spanish poet and philosopher. He wrote about paperfolding in 1902 in his 'Amor y Pedagogia', a satirical novel on the excesses of positivism. The publication includes engravings by Gérard Angiolini. Unamuno makes significant modifications to the bird base and is the first to devise new models of animals, such as birds, similar to the work of Akira Yoshizawa, but thirty years earlier. Dr. Vicente Solorzano, Ligia Montoya, and the Italian Giordano Lareo are credited for further experimentation on the bird base [Lis a].

In Germany, paperfolding developed at the Bauhaus school of design, founded by Walter Gropius in 1919. Its ideas transformed European design despite its closure in 1933. Many of the faculty members left for the USA to teach at Harvard University and Black Mountain College. Among them was Josef Albers who strongly believed in the pedagogy of paperfolding [Lon 72].

Lister describes developments of rapid dissemination in the 1950s by attributing Gershon Legman, Robert Harbin and Lillian Oppenheimer, who have great influence on him [Lis a]. From then on paperfolding gained such popularity on a global level that a further history would demand a very specific focus [New 73].


LI TRE TRATTATI
DI MESSER MATTIA GIEGHER
BAVARO DI MOSBVRC,
TRINCIANTE DELL'ILL.MA NATIONE
ALEMANNA IN PADOVA.

Nel primo ſi moſtra con facilità grande il modo di piegare ogni ſorte di panni lini, cioè, ſaluiette, e touaglie, e d'apparecchiare vna tauola, con certe altre galanterie degne d'eſſer ſapute; lequali coſe tutte ſon rappreſentate in figure di rame: e queſta è inuenzion del tutto nuoua dell'autore, ne mai più per addietro veduta.

Nel ſecondo, intitolato lo Scalco, s'inſegna, oltr'al conoſcere le ſtagioni di tutte le coſe, che ſi mangiano, la maniera di mettere in tauola le viuande; il che, con tre differenti figure di tauole intagliate in rame, chiaramente ſi moſtra

Nel terzo, detto il Trinciante, s'inſegna il modo di trinciare ogni ſorte di viuande, con le ſue figure medeſimamente in rame; opera rinuouata, e di molte coſe accreſciuta.

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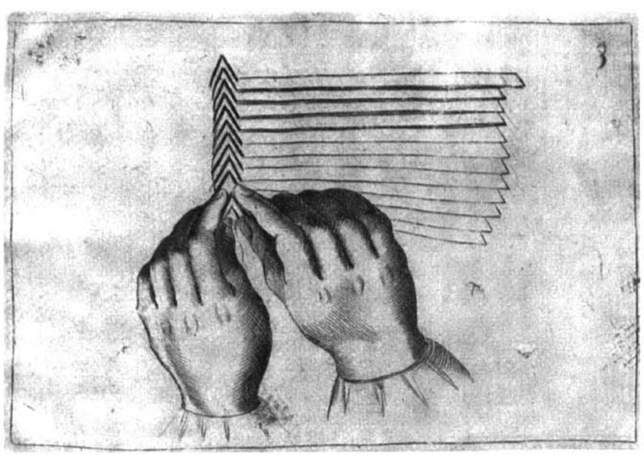


Fig 2.1.1 'Li Tre Trattati' Mattia Giegher, 1639 (likely similar to 1629 edition), folding instruction

Origami in Asia

Early scholarly work on origami in Japan was conducted by Satoshi Takagi [Tak 93] and by Masao Okamura [Oka 02]. Both authors represent two of the few reliable sources for Lister [Lis b] and Koshiro Hatori . Lister argues that no real evidence has surfaced to point to the origin of paperfolding and begins his rationale with early instances of rough paper making in China around 200 BC with refinements over the next century. Hatori however lists several incidents of paper inventions in Southeast Asia to as far back as 5000 BC [Hat 11]. The technology or craft of making paper was introduced to Japan around 550 AD, and further improvements took place right away. Folding might have occurred, but Lister and Hatori agree that evidence of the technology does not qualify as evidence of folding.

A story suggests that origami has its origins in Japan, which is based on a legend that Abe-no Seimei made a paper bird and brought it to life in the Heian period (794 to 1185). Hatori dismisses this and other very early accounts such as the ocho and mecho butterflies, used to decorate sake bottles, as insignificant. It appears that the earliest records of what is understood as origami today stem from the 17th century in Japan. Since it was developed as a craft it may very well have had an earlier history reaching to the 16th century. Whether folding in the East influenced folding in Europe or vice versa or whether the two regions developed their own paperfolding independently appears to be unclear [Eng 94].

The documentation of origami in a poem by Ihara Saikaku in 1680, which refers to the previous kind of butterfly models, was much later than the disputed accounts mentioned above. 'Ramma Zushiki', a book from 1734, shows pictures of familiar models and according to 'Ise Sadatake's Book of Wrapping' in 1764 origami was also taught in samurai classes. The most known printed book on recreational origami in Japanese is 'Sembazuru Orikata', one thousand cranes, 1797 (sometimes spelled Senbazuru). It gives instructions on how to fold multiple cranes from a single sheet of paper, cut into smaller squares, and connected at the corners [Bro 61]. This kind of paperfolding has little to do with ordinary recreational folding according to Lister, due to its almost unique characteristic of many classic cranes. The two printed sheets of paper known as 'Chushingura Orikata', also from 1797, show instructions for folding simple human figures, which are much closer to ordinary recreational folding than Senbazuru Orikata. Another Japanese book of ceremonial paper folding and knotting appears to be often overlooked. 'Tsutsumi musubi no ki" by Sadatake Ise was originally published in 1764 [Mit 09a].

Lister points to the discovery of Kayaragusa (or "Kan no mado"), a Japanese paperfolding classic from around 1845, which discloses that the Japanese had an advanced style of cut-and-fold paperfolding in the mid 19th century. Many early versions of Japanese paper art did not

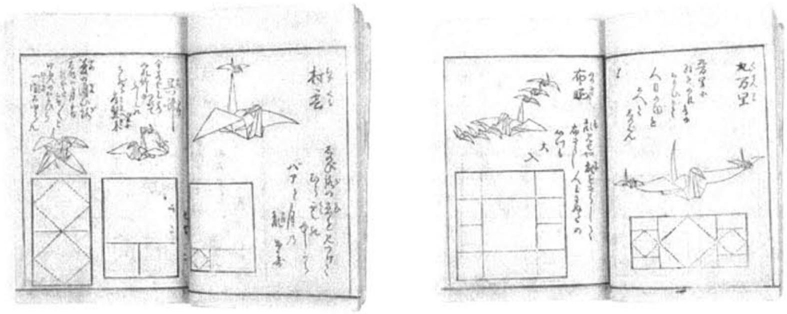


Fig 2.1.2 'Sembazuru Orikata', 1797

adhere to the 'fold no cut' rule and often consisted of multiple pieces of paper, sometimes even painted.

The 20th century is full of examples of knowledge transfer between East and West and in the 1950s Japanese books in English by Isao Honda , Florence Sakade, Tokinobu and Hideko Mihara contribute to the dissemination of origami in the USA and Europe.

Modern origami

Akira Yoshizawa's work, specifically the designs included in his Japanese publication in 1952, is often quoted as initiating a new era in origami, called 'modern origami'. He was born in 1911 and started folding at the age of three. Lister says, 'I firmly believe that although Unamuno started folding creatively at the end of the 19th century, Yoshizawa was the principal originator of modern creative origami' [Lis c]. He contributed to the field by emphasizing uncut paper, creating new bases, supporting a creative approach to origami and wet folding (Fig 2.1.3). He is also known for introducing his system of diagrams with lines and arrows. Robert Harbin and Samuel Randlett developed the now called Yoshizawa–Randlett notation from Yoshizawa's notation in 1961 (Fig 2.1.3).

Yoshizawa's work became an inspiration for Japan and the West in general. Modern origami introduced several other aspects of paperfolding, one of which was acknowledging the author of a design and shifting the focus from 'well executed' paperfolding to 'original design'. It is worth mentioning that Uchiyama Koko went so far to patent his designs. Folders began to value published diagrams of a design or 'crease patterns' as well as the design itself. The idea of a crease pattern attained a new set of values.

In the 1950s and 1960s folders such as Takahama Toshie, Honda Isao, Robert Harbin, Gershon, Legman, Lillian Oppenheimer, Samuel Randlett and Vicente Solorzano-Sagredo established an international circle, which contributed to the popularization of origami globally [Smi 09]. Robert Lang states that 50 years ago all origami designs could have been catalogued on a

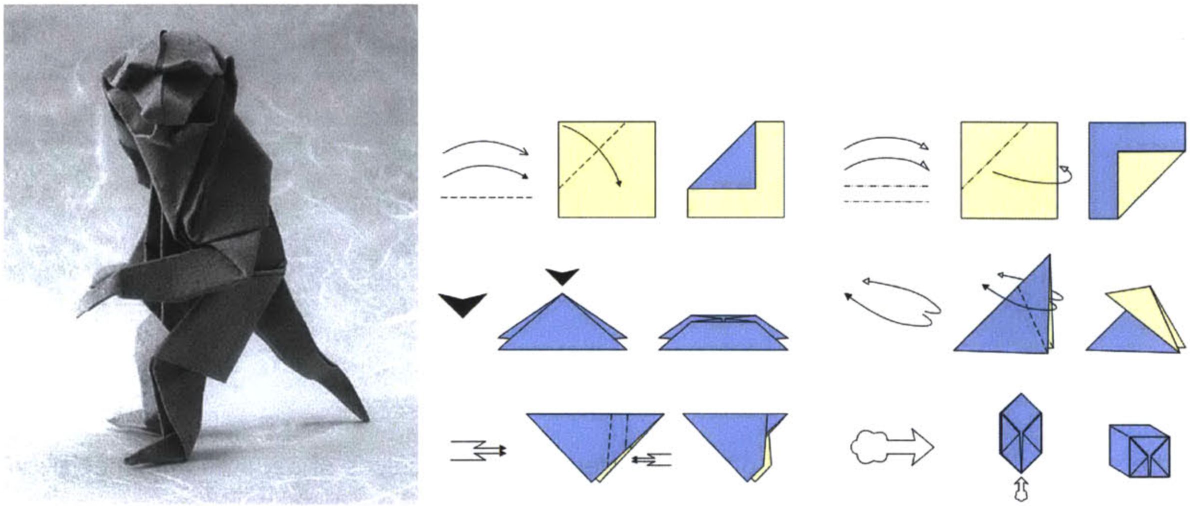


Fig 2.1.3 Model by Yoshizawa, Yoshizawa–Randlett notation

single typed sheet of paper [Lan 03]. Since then the numbers have exploded in part due to the development of modern and mathematical origami. I present work from the 1960s until today in later chapters.

Computational and mathematical origami

Many folding sequences of paperfolding designs start with the same set of steps, which are called the 'base'. Such a base can be obtained computationally and the pioneers in that area are Jun Maekawa, more recently Kawahata Fumaki, Robert Lang and Erik Demaine and among others. Using computational origami can be described as a shift from 'folding as a design process' to 'designing a base for a model'. Robert Lang, author of 'Origami Design Secrets', a large collection of work completed over a period of 20 years, reveals a scientific view on designing origami. The chapter on tree theory informs Lang's software 'TreeMaker', intended for designing such origami bases. The user can draw a stick figure of the base on the computer and each stick in the stick figure (the tree) gets represented by a flap on the base. After various constraints are posed on the flaps, TreeMaker computes the full crease pattern for the correlating base. The printed crease pattern is ready for folding and can be shaped into its final shape (Fig 2.1.4).

Computational or mathematical origami has evolved into an interdisciplinary field that encompasses computer science, mathematics and engineering. I discuss further work in chapter 2.3 that elaborates on several geometric issues. 'Geometric Folding Algorithms for Linkages, Origami and Polyhedra' by Erik Demaine and Joseph O'Rourke, designed in the form of a text book, provides an encyclopedic overview of problems and mathematical proofs of what can be achieved with folding in 2d and 3d [Dem 07]. The work has implications for applications in areas such as protein folding, cancer research and programmable matter. Two folding experts in Japan, Jun Mi-

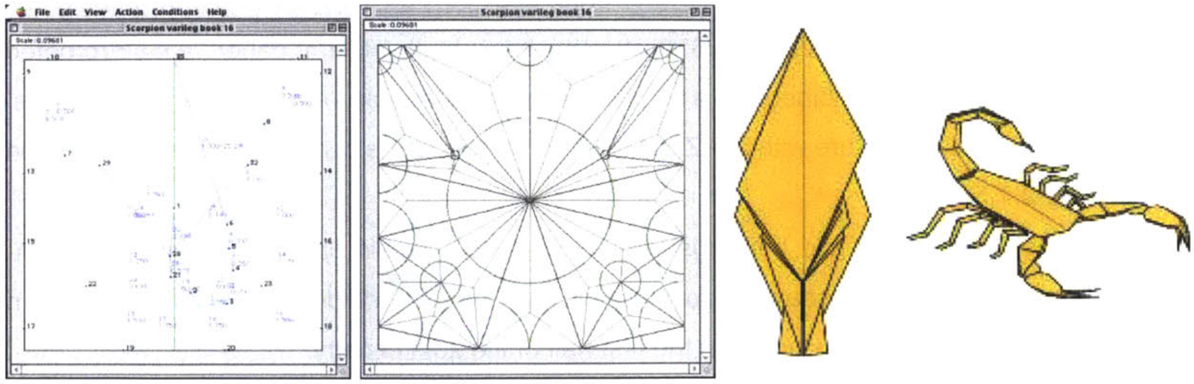


Fig 2.1.4 Tree, circle packing and crease pattern, base, model by Robert Lang

tani, computer scientist, and Tomohiro Tachi, trained architect with a strong passion in computer science and mathematics, create software tools for simulation and design, which I will elaborate on in later chapters.

Paperfolding in architecture

In the 1960s and 1970s several designers in Europe and in the USA attempted to use paperfolding for architectural applications. One example 'A folding house for farm workers' published in Life magazine, promoted a design by the Canadian Herbert Yates with Sanford Hirshen and Sim

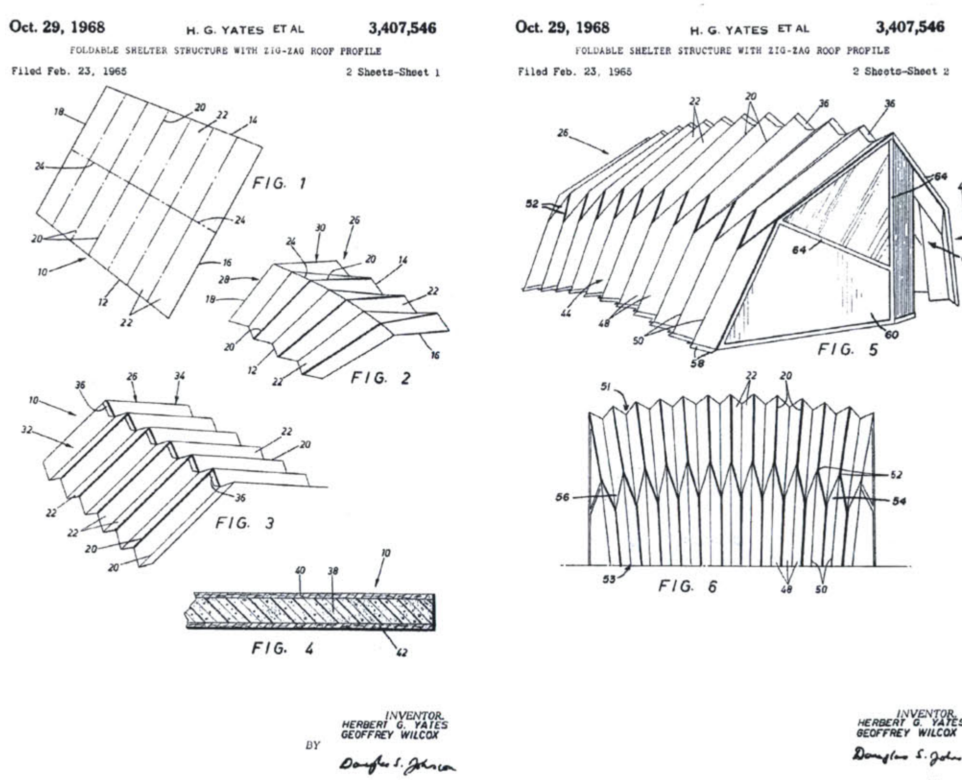


Fig 2.1.5 'Foldable Shelter Structure with Zig-Zag Roof Profile' Herbert Yates, 1965

Vander Ryn [Yat 68]. It was easy to build, waterproof and could be constructed by few people in a very short period of time. Yates used 3/8" thick composite sheets of 'kraft-paper', a waterproofed paper with polyethylene, combined into a sandwiched panel with polyurethane foam in the middle. His 'Foldable Shelter Structure with Zig-Zag Roof Profile' was at the core of his patent application of 1965 (Fig 2.1.5).

Further attempts to fold buildings occurred in Great Britain and Germany, but no constructed examples still exist today. It appears, however, that paperfolding found more interest in design pedagogy than in construction in the first half of the 20th century.

Paperfolding in education

As with paperfolding in general, finding sources in the context of pedagogy can be adventurous and a literature review provides surprising insight when looked at chronologically. Joan Sallas argues that paperfolding and textile folding techniques have influenced one another and sees the development in both areas as interdependent. Apprentices of 'napkin-breaking' familiarized themselves with paper at first in order to gain expertise in accuracy, sharpness and refinement. Folding methods and folding pedagogy became inseparable and developments in technique, style, variations of models, practicality and mobility transferred from paper to cloth and cloth to paper. Sallas quotes the above mentioned 'Tranchierausbildung' as one of the first examples of formal napkin folding pedagogy. Napkin folding was taught at the University in Padua in the 17th century and August Herrmann Francke established a napkin folding course as part of the school curriculum at the Pedagogium Regium in Halle in 1705 [Sal 10].

Publications until the middle of the 18th century featured folding patterns and verbal advice on folding. One part of the anonymous 'Aanhangzel van de volmaakte Hollandsche keukenmeid' represents the first known rigorous step-by-step instructions for folding in Europe (Fig 2.1.6) [Sal 10]. At this time napkin folding shifted from a trade executed by male to female domestic staff and the publication might have been used for this new demand for folding instructions.

Friedrich Fröbel

A very significant contribution to the development of paperfolding in Europe during the 19th century needs to be attributed to Friedrich Fröbel (1782 to 1852). He studied under Johann Heinrich Pestalozzi, the Swiss pedagogue and educational reformer who promoted the motto known as learning with 'head, hand and heart'. After his studies Fröbel served in the German army. He took care of his deceased brother's children after the war, which started his career in educating children. He established a school to train female teachers in Keilhau in 1817, expanding some of

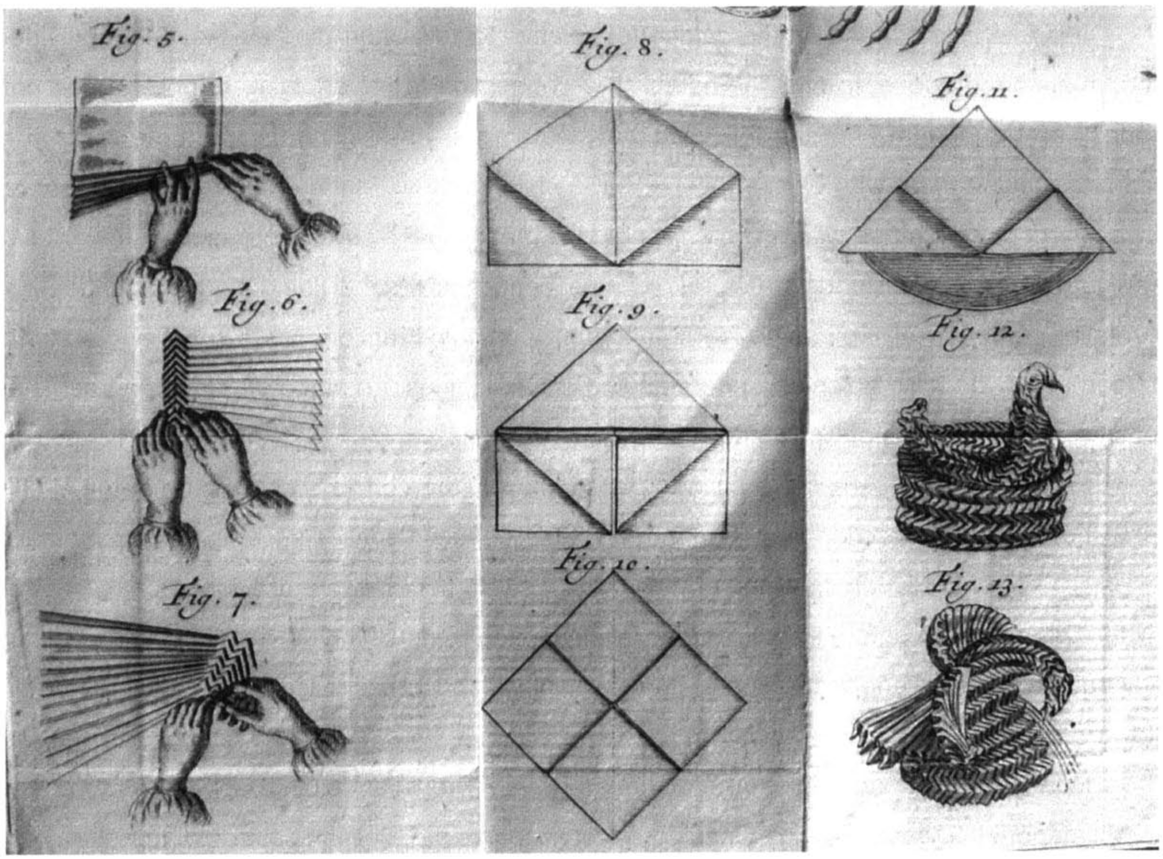


Fig 2.1.6 Figure in 'Aanhangzel van de volmaakte Hollandsche keukenmeid (1763, first edition 1746)

Pestalozzi's ideas [Bra 72], [Moo 75]. He worked on a curriculum that relied on toys or teaching tools to structure his pedagogy. His own first Kindergarten opened in Bad Blankenburg in 1837, where he fully integrated his curricular ideas. Many of the young women, trained by him, traveled the world to spread his educational theories.

Fröbel's main conceptual reference for education was 'play', which is why he invented devices or means to facilitate such activities. 'Spielmittel, which can be thought of as means or facilitators of play, encompassed 'Spielgaben', his gifts, and 'Beschäftigungs- or Bildungsmittel', which were activities and means of facilitating education. His goals consisted of stimulating a child's development in terms of feelings, anticipation, thinking and recognition, and to activate a child's physical development, fantasy and creativity. The gifts played a central role and Fröbel ranked them such that they depended on each other. He started with spheres and then continued with cubes and subdivisions of the cube. Paperfolding was not mentioned as a gift, but it was implied in one of his activities. It is unknown precisely when and how paperfolding was introduced into the curriculum; however, his followers could have easily applied some of his categories toward paperfolding. The instructors at his school and aspiring kindergarten teachers explored and subsequently published their ideas that included paperfolding.

It is useful to expand on the categories relative to sets of building blocks, the subdivided cubes mentioned above, in order to establish a connection to paperfolding. Fröbel's three categories can be thought of in the following way. Copying or abstracting things in the world of a child and making them with blocks means that a child creates 'Forms of Life'. An ornament, or fully abstract pattern made with any of Fröbel's gifts means that a child is engaged with 'Forms of Beauty'. If a child understands structural or formal concepts by subdividing a platonic shape into its smaller parts, for example, s/he learns about proportions. Fractions, as 1/2, 1/4 and so forth, are learned as the child gains an understanding of what things are made of on an abstract level and s/he creates 'Forms of Knowledge'.

Fröbel's activities enable a child to transition from the concrete or 'existing in the child's real world' toward abstraction via the introduction of geometric entities such as points, lines and planes. The activity reveals qualities of these entities via specific types of engagement and that is where paperfolding enters the pedagogy. He defines folding, cutting off, assembling and gluing as adequate types of engagement and many books with such examples were published by his followers.

How did Fröbel's disciples, the actual authors of Fröbelian folding, apply the aforementioned categories to paperfolding? One of the first accounts of Fröbelian paperfolding is the 'Manuel Pratique des Jardins d'Enfants' edited by Jean François Jacobs, published in Brussels in 1859. Jacobs listed models such as the salt-cellar, the windmill, the sailing boat, the bird, the double boat, and the gondola with seats.

The Forms of Life consisted of traditional folds known throughout Europe which continue to be popular today. Their purpose was to introduce children to paperfolding before they go on to more complex categories. Lister assesses that the Fröbelian movement did not, in general, seem to have taken the Forms of Life very seriously. He argues that they were merely a repetition of previously known folds with little exploration of new designs. This might have been one of the factors that caused the movement to stall, when paperfolding was perceived as too strict for pedagogy. Its success receded in the 20th century.

The Forms of Beauty made up the greater part of Fröbelian folding and were intended to spark a sense of creativity and artistic beauty in children. The blintz fold provided the base for everyone to experiment with symmetric folding patterns with an infinite number of possible variations (Fig 2.1.7). Museum collections of albums dating from the 19th and early 20th centuries give us an idea of how children were encouraged to play with such variations. Not all examples of the Forms of Beauty were based on squares. Triangular and circular designs exist in these paperfold-

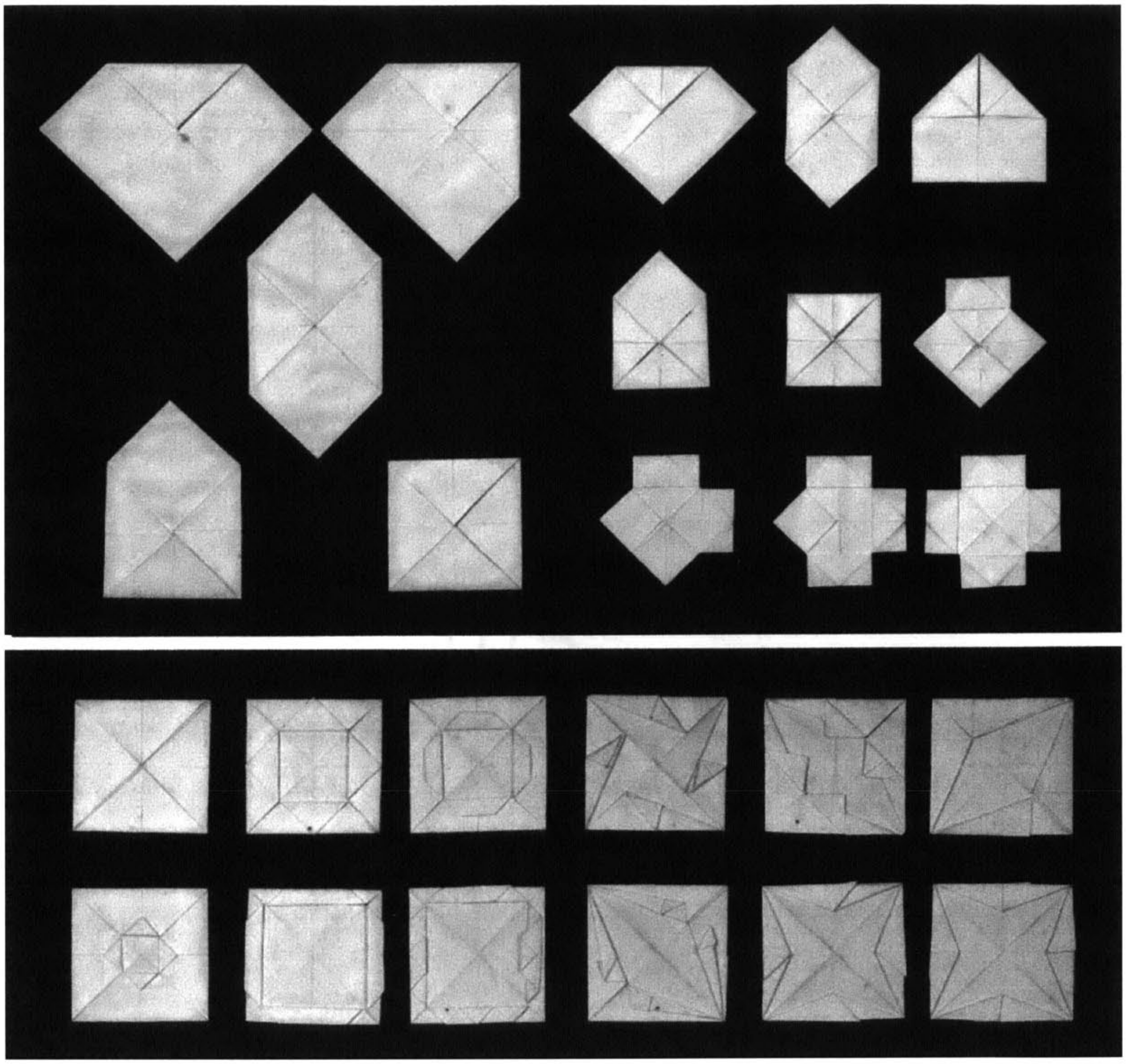


Fig 2.1.7 Examples of the blintz fold and its variations (reconstructions, Fröbel Museum)

ing albums by Kindergarten teachers (Fig 2.1.8).

Eleonore Luise Heerwart published 'Course of paper-folding' in the late 19th century, which I was unfortunately unable to attain. One of her diagrams from her 'Course of Paper Cutting', published later, attests to the rigor Fröbel's disciples used to provide folding instructions (Fig 2.1.9) [Hee 89]. Her work most probably falls in the Folds of Beauty category.

Lister argues that the Forms of Beauty quickly deteriorated and contributed to the downfall of paperfolding in the movement of Fröbel's pedagogy. Teachers did not succeed in stimulating creativity and the examples became mere repetitive exercises in copying, which defeated the original purpose. The encouraged creativity of the Forms of Beauty was not applied to the Forms of Life. Lister concludes that Fröbel's followers may not have had enough appreciation of paper-

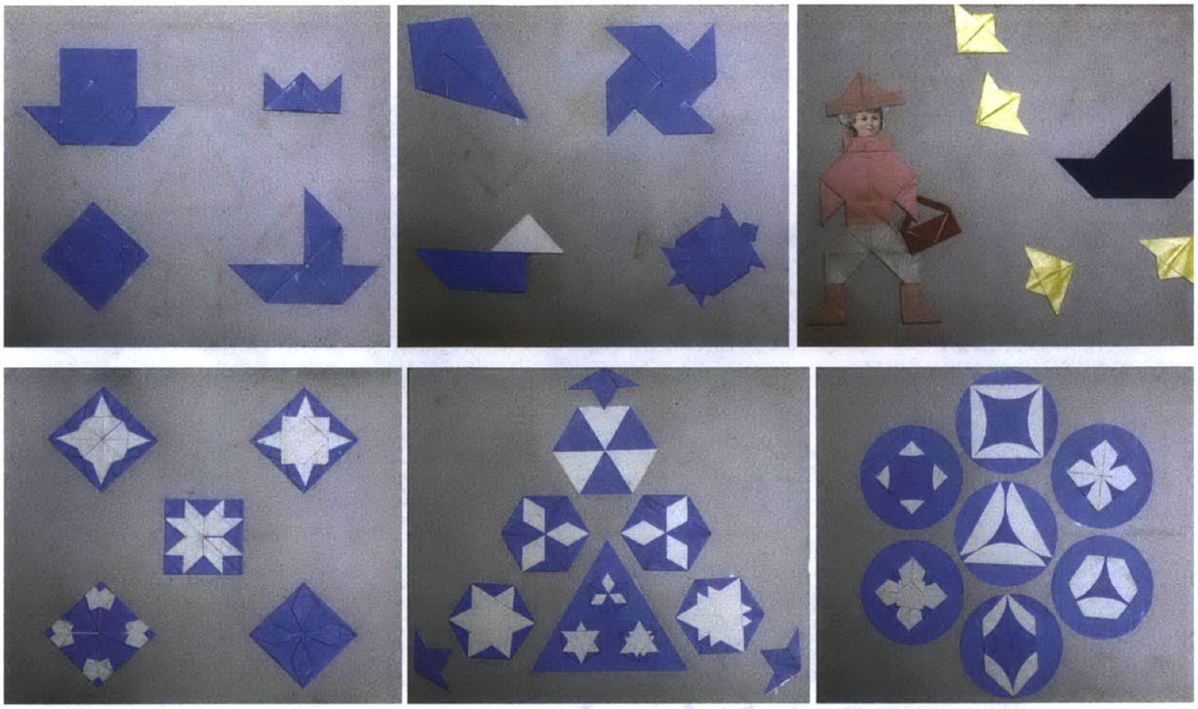


Fig 2.1.8 Fröbelian paperfolding, Forms of Life (above), Forms of Beauty (below) (CCA collection)

folding and its possibilities. [Lis d]

Fröbel became popular in Japan after WW II as part of an effort to rebuild the education system in the style of European schools. The patterns were known in Japan, but it is unclear whether they were introduced with the Fröbelian Kindergarten.

The Forms of Knowledge were restricted to mathematical folding and I could only find

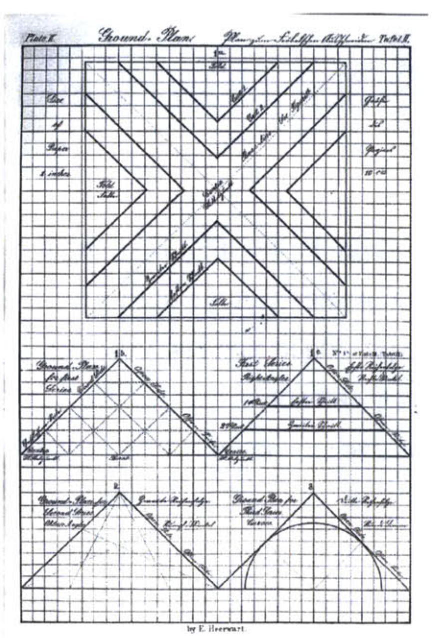


Fig 2.1.9 Example in 'Course of Paper Cutting' (1889, Eleonore Heerwart)

verbal accounts of these from Fröbel's followers. It seems that this kind of geometrical folding might have followed Fröbel's original ideas regarding his cubes, where a child would discover fundamental concepts of geometry by subdivision.

Fröbel's work is a source of inspiration for the closing chapter of the dissertation as occupations and categories in his curriculum provide a framework that introduces intentions or goals for designers, while operating within a specific geometric category.

T. Sundara Row

Folded edges can represent lines. When creases cross, angles become apparent between them. When lines intersect, points or vertices appear. Pedagogues interested in exploiting haptic qualities of learning can teach such mathematical concepts in a playful way via the use of paperfolding. Dionysius Lardner published many works in the field engineering and applied mathematics, and may have been the first to propose the use of paperfolding in his geometry textbook of 1840 [Ken 87]. Another early account of this idea is T. Sundara Row's 'Geometric Exercises in Paperfolding', published in 1893, with over 200 examples of paperfoldings and their mathematical relationships. The translated edition by Wooser Woodruff Beman and David Eugene Smith as well as their 'New Plane and solid Geometry', another book on geometry, are still held in high regard today.

Gershon Legman, American cultural critic and folklorist, mentioned that Row's book was

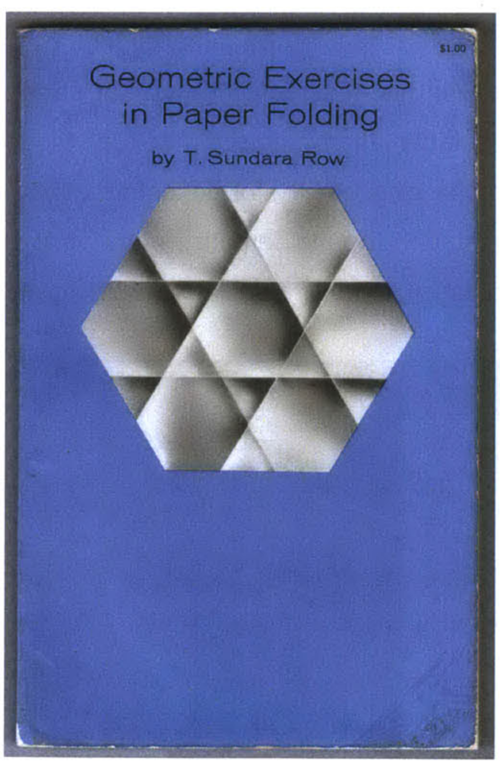


Fig 2.1.10 Geometric Exercises in Paper Folding by T. Sundara Row, 1966 edition

first published by The Open Court Publishing Company of La Salle, Illinois in 1901 in his origami bibliography. Lister and Legman differ on whether the date of the second edition is 1904 or 1905. Dover publications Inc. of New York issues a paperback edition in 1966 and still prints it today (Fig 2.1.10). It is thereby most probably one of the oldest books on paperfolding in a Western language that still remains in print.

It is unclear who exactly T. Sundara Row was. The introduction of the book ends with 'Madras, India, 1893', possibly the place and date of the original publication. 'Row' was spelled 'Rau' with an accent on the a in other editions and it would seem that Sundara Row was Indian. It remains unclear in which language the original was written. The editors' preface to the English translation mentions a reference in Klein's 'Vorlesungen über ausgewählte Fragen der Elementargeometrie' (lectures on selected questions of elementary geometry). The connection between Felix Klein, known for his Klein Bottle, and Sundara Row, however, appears to be unclear. [Lis e]

Row's introduction also mentions Fröbel's 'gift number VIII' as source of inspiration and refers to it as an available toy kit or product [Row 41]. Fröbel's gifts end with number seven, but his followers invent several more and eight is in fact related to lines as edges of surfaces. Row elaborated on the advantages of paperfolding as an appropriate preparation for a pupil to appreciate science. Row's initial collection of diagrams was replaced and expanded with many photographed models of the discussed examples by Beman and Smith (Fig 2.1.11).

The development in the USA continues with D.A. Johnson's 'Paper Folding for the Mathematics Class' in 1971, published by the National Council of Teachers of Mathematics [Joh 57]. Interest persists among educators and A. Olson follows with his 'Mathematics through paper folding' in 1975, published by the same council [Ols 75]. In Japan, the biologist Dr. Kazuo Haga, introduced 'Origamics' in the 1980s while being an instructor of geometry for school teachers. He worked with Koji Fushimi, a physicist, who was interested in the mathematics of the crane constructions (orizuru) and their collaboration led to a publication, recently translated into English

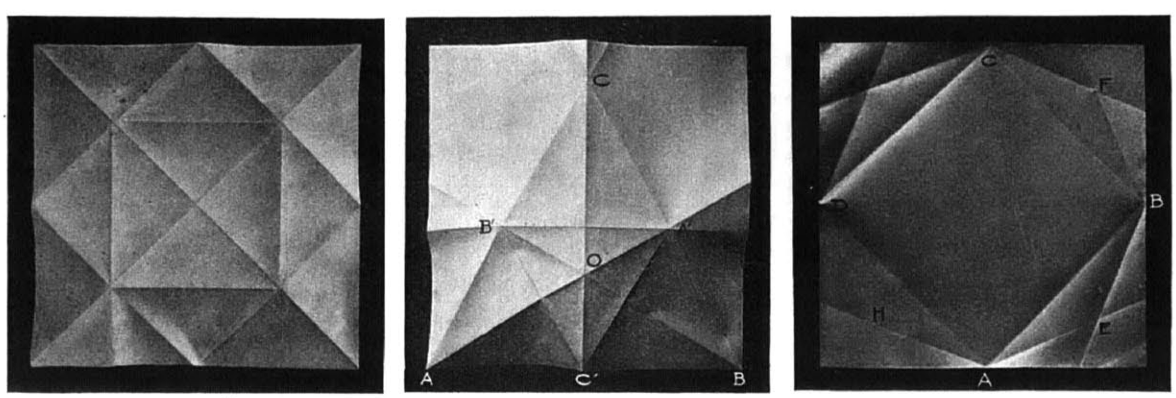


Fig 2.1.11 Photographed models of exercises by Sundara Row

[Hag 08].

Among probably hundreds of contemporary origami instruction books worth mentioning, 'Genuine Origami' by Jun Maekawa stands out as it describes 43 mathematically based models [Mae 08]. Maekawa fashioned the publication in the style of a text book and aimed at explaining some of the mathematical fundamentals behind the models of modular and polyhedral origami.

OSME

Paperfolding found its way into the pedagogy of aesthetics and mathematical constructs, but also ventured into science and engineering, which can be observed in the development of the conference on Origami, Science, Mathematics and Education (OSME).

The initiating conference was held in Ferrara, Italy in 1989 with the name 'The First Meeting of Origami, Science, and Technology'. Organized by Humiaki Huzita, it laid the foundation for people interested in paperfolding within the mentioned areas. The widely cast net might have been motivated by the uncertainty as to which concerning disciplines might have something to say. For the next gathering, the name was changed to the 'Second International Meeting of Origami, Science and Scientific Origami'. Held in Otsu in 1994 it omitted the word 'technology' in favor of 'scientific origami', a term that has not become popular in comparison to mathematical origami, which has been established as a standard in the scientific folding community. At the third OSME conference in 2000, the word education appeared and 'scientific origami' was altered to 'mathematics' [Hul 02]. Thomas Hull, an educator in mathematics himself, edited the proceedings and might have been instrumental in bringing pedagogy into the picture. Hull elaborates that origami can be used to explain concepts and solve problems in mathematics beyond geometry. In his 'Project Origami' he covers topics such as calculus, abstract algebra, discrete mathematics as well as topology [Hul 06]. The fourth conference with its proceedings edited by Robert Lang keep the name OSME. Similarly the fifth conference adheres to the name [Lan 09], which attests to the settling in of the contributing disciplines.

Paperfolding versus Origami

Lister and Paul Jackson, a prolific British folding aficionado, differ on the definitions of origami and paperfolding, but both acknowledge there to be a difference and I will elaborate on the origins and possible definitions of the words themselves.

The word 'origami' might have existed in Japan as early as the Western middle ages and Hatori Koshiro suggests the late 12th century. Its use seemed to have originated for rectangular horizontal sheets of paper for lists and letters called 'tategami', when folded the long way, and 'kirigami', when folded the short way. Certificates of authentic manufacture in the Edo era and formal lists for engagement gifts still use this format today. Curiously the Greek word 'diploma' appears to have similar origins as a folded certificate. In the eighteenth and nineteenth centuries the words 'orikata' and 'orisue' found use to describe ceremonial and recreational paperfolding. According to Lister, traditional Japanese schools used the term origami in contrast to kindergartens with a Fröbelian approach, which used 'tatemigami'. This provides an interesting example of a distinction between the two traditions in Japan. 'Origami' established itself as the term for recreational paperfolding since the end of the 19th century in the East. Lillian Oppenheimer, attributed for popularizing origami in the West, used origami in lieu of the Chinese word ' zhe-zhi ' for the formation of her Origami Center in the USA in 1958 [Lis f].

Several other words exist in Europe, 'papiroflexia' being the usual term for paperfolding in Spanish speaking countries, for example. It is the equivalent to 'Papierfalten' in German or 'paperfolding' in English, but it is a recent word, invented by Dr. Vicente Solorzano Sagredo of Argentina probably around 1910. Before that the Spanish would talk of 'making pajaritas', a traditional folding of a little bird, and Vicente Palacios of Barcelona explains that this expression not only designated folding an actual 'pajarita' but also any other crease pattern.

The mentioned words are not very old and they may have originated as a result of the use of paperfolding in Fröbel's Kindergarten. The first few decades after his death in 1852, when his students developed their methods, serve as a reasonable guess. Lister argues that 'paper folding' was the natural translation of Fröbelian work into English. Before Fröbel there was no concept of paperfolding as an independent activity, it was simply folding paper. Without the concept, there was no need for a special word. The necessity for a word could mark the origin for a filed of knowledge as an occupation attained different kinds of values that resided in disciplines such as education. Neither 'paper-folding' nor 'paperfolding' appeared in the first edition of the Oxford English Dictionary of 1928. The words 'paper' and 'folding' are long-standing English words and their combination into 'paper folding' and later to the hyphenated 'paper-folding' is a natural process in a language. The particular date when 'paper-folding' became an established word is indeterminable, but Lister believes around 1880 to be a good estimate. Today, 'paper-folding' is evolving into 'paperfolding' without the hyphen, which is the form I use in this dissertation.

Paul Jackson raises questions regarding the definition of 'origami', 'folding' or 'paperfolding' on the British Origami Society's website and David Lister responds in one of his own postings. He asks, if it is correct to use the word 'origami' for paperfolding outside of Japan, for example, for Chinese paperfolding and refutes that idea. Is it then legitimate to call Western paperfolding 'origami'?

'Origami' has of course been absorbed into English and other languages and to try to revert to 'paperfolding' would not only be impossible, but would be perverse according to Lister. He points out that when he is writing on more serious aspects of paperfolding, he deliberately uses the word 'paperfolding' unless the paperfolding is Japanese. He does not think that the term origami applies to pleated paperfolding of the kind used in bellows, accordions, cameras, umbrellas and lamp shades of the 1960s for example.

Paul Jackson provides the definition of 'an easily recognizable representation in folded paper of an easily recognizable subject (elephant, star, etc.). This definition is specific to abstracting subjects or things and would exclude non-figurative tilings for example. Lister thinks that a definition should relate to the technique rather than the product and mentions the creation of reverse folds, the further folding of paper that has already been folded and the superimposition of creases.

The British Origami Society included a definition of origami when they establish themselves in 1967:

- *The Society defines origami as the folding of paper of any regular shape to form two or three dimensional models of living creatures, inanimate objects and abstract forms.*
- *While the Society holds that Origami in its purest form does not admit the cutting of paper, the Society does not exclude cutting provided that it is limited in extent, adds significantly to the value of the model and provided that the model retains the main characteristics of uncut Origami.*
- *The Usual medium of Origami is paper, but the Society recognizes that the techniques of folding may be applied to other materials.*
- *The Society recognizes techniques of manipulating and cutting paper other than Origami and seeks to foster the interchange of ideas between the pursuit of origami and other paper techniques.*
- *The Society considered changes to the definition such as 'the art and science of folding paper' or 'the art and science of folding', but the original was kept as historical record.*

Lister points to several advantages of this set of definitions as it includes 'cutting' and 'not cutting'. The scope is not restricted without diluting the essential meaning and integrity of 'mainstream origami'. It recognizes that folding is not confined to paper and acknowledges other paper techniques such as the ones used in paper sculpture.

Lister refutes Jackson's comment that origami is not paperfolding. Origami is paperfolding, but it is one particular kind that deserves separate recognition as something in its own right and with its own identity. [Lis g]

I believe the distinction between the terms helps to delineate the boundaries of a possible kind of knowledge. Following Kubler's way of writing a history, I propose to define paperfolding via its geographic occurrences and specific incidents in time. The mentioned events in the European history of paperfolding serve as examples here.

This will become especially relevant when I discuss curved crease paperfolding, the specific subset of paperfolding I study in Huffman's work. I agree with most of Lister's comments and would define paperfolding as a form of knowledge that follows all of the British Origami Society's qualifications with a few added characteristics. It does not focus on figurative representation, it is predominantly European and follows early attempts in furthering the field in a mostly Western tradition. David Huffman never used the term origami himself and for the purpose of this dissertation I adhere to the use the term 'paperfolding' to describe his work.

2.2 A history of curved-crease paperfolding

When we consider historical accounts of curved creases, the trajectories of art, mathematics and education cross again, however, only few examples can be found. This chapter describes developments in chronological order and does not differentiate between disciplines as the previous chapter did. This chronology presents the sources I was able to find and I hope to not have omitted any significant contributions to curved crease paperfolding. The chapter ends in the 1970s as I discuss the work by contemporary folding artists in the final chapter on design approaches.

Napkin Folding

Joan Sallas has discovered very early accounts of curved crease paperfolding and states that curved creases most probably existed several centuries earlier than his sources suggest. Starting in the 17th century the previously mentioned German publications elaborated on techniques for curved crease paperfolding that focused on table decoration. The earliest example Sallas quotes is the 'Trincir-Buch' by Georg Philipp Harsdörffer from 1652, which describes 'scale folds' that are meant to represent fish scales. The publication also includes a comment that 'one can fold round folds' [Sal 10, p59].

Andreas Klett mentions that one can obtain beautiful shapes by using curved creases or as he says round folding in his 'Trenchir- und Plicatur-Buechlein from 1677 (Fig 2.2.1 left). The pleated models appear to be abstract decorative designs with little to no figurative aspiration. The tradition of napkin folding continued for many decades, but appeared to decline in the 18th century [Sal 10]. Some examples in a publication Sallas attributes to the name Glorez, display less symmetrical patterns and one of them has no symmetry at all (Fig 2.2.1 right).

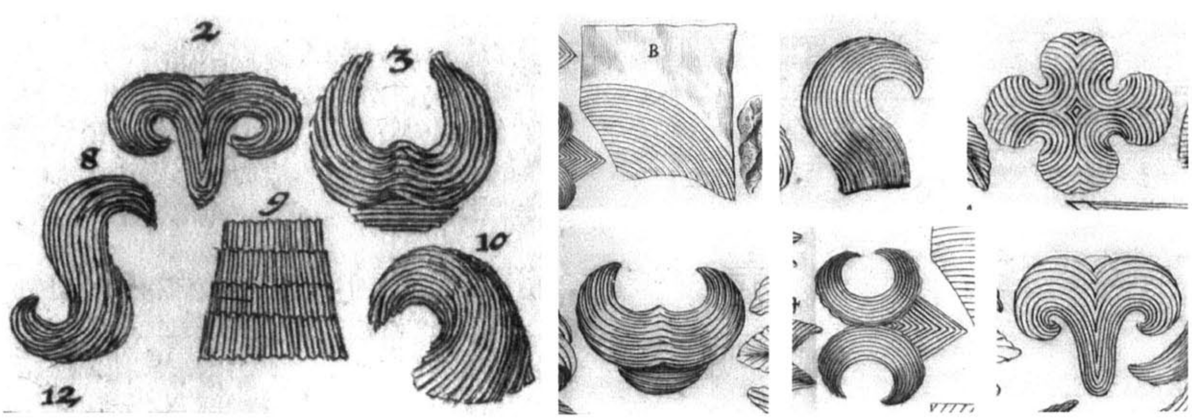


Fig 2.2.1 Curved crease napkin folds by Andreas Klett 1677/1724 and details from Glorez 1699/1701

'The Bauhaus model'

Several centuries later Josef Albers taught at the first Bauhaus in 1927 and 1928 and explored curved crease paperfolding in the context of architectural pedagogy. A student of his makes a model that is made by pleating concentric circles, the technique of folding paper in alternating directions of mountain and valley creases (Fig 2.2.2 left). The creases automatically fold into the depicted configuration [Win 78]. Another example exists from 1937 at Black Mountain College (Fig 2.2.2 center) [Adl 04]. The model becomes a subject of investigation for folding experts, and Irene Schawinsky, the wife of Alexander "Xanti" Schawinsky, made a variation of the model (Fig 2.2.2 right). She was a Bauhaus student and later taught at Black Mountain College during the time Albers worked there. Her version of the model features a large hole in the center [Pha 44].

Thoki Yenn, Danish paper sculptor and founder of the short lived Dansk Origami Center, published his version of the model in the 1980s, which he called 'Before the Big Bang' [Yen 01]. Kunihiko Kasahara learned of the model from Yenn and made further variations, which he published in 'Extreme Origami' in 2003 [Kas 03]. The model finds interest for folding experts, who are mentioned in the final chapter of the dissertation.

Students made the model in Josef Albers' 'Vorkurs', a foundations class in design. He decided to teach design via the use of paper models, because it was an abstract exercise that allowed students to focus only on design and paper, not on pragmatic or functional requirements, for instance.

In a short article, republished by Londonberg in 'Papier und Form', Albers suggests to allow students to try out designs without any a priori knowledge of architecture or established design methodologies. He calls this 'non-expert experimentation'. The material itself is the only constraint for designing. This was very much aligned with the then newly established Bauhaus curriculum that focused on material logics. Albers points out that working with materials and exploiting their properties for a design leads to a fundamental understanding of an efficiency of

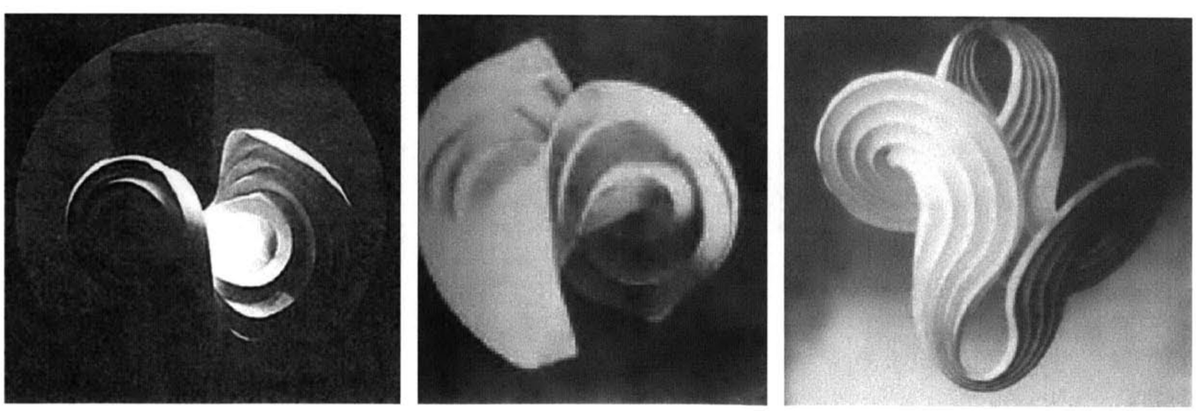


Fig 2.2.2 'The Bauhaus model',1927 and 1937, a later version by Irene Schawinsky

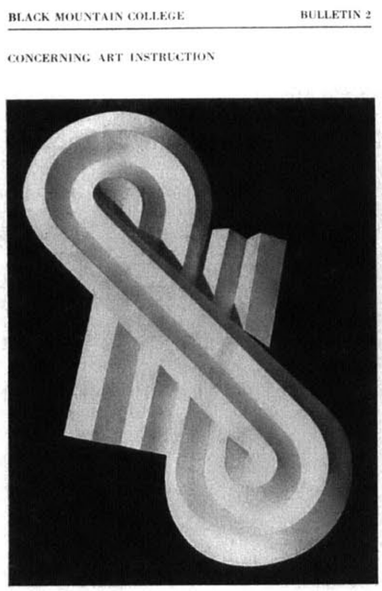


Fig 2.2.3 Black Mountain College Bulletin 2, 1944

means [Lon 72]. A young designer can learn how to utilize a material to its maximum potential, which leads to light design proposals with little waste [Bec 70]. Designing curved crease models also provides students with unexpected revelations, which has additional pedagogical value that I elaborate on in the final chapter.

Further evidence of curved crease paperfolding in pedagogy can be found at Black Mountain College. Bulletin 2 'Concerning Art Instruction' from 1944 shows a pleated s-figure with curved creases most probably from Albers's foundation class. The decision to use it on the cover displays an appreciation for paper models (Fig 2.2.3). Albers continued to teach paperfolding and was photographed with a large version of the model at the Hochschule für Gestaltung in Ulm (Fig 2.2.4).

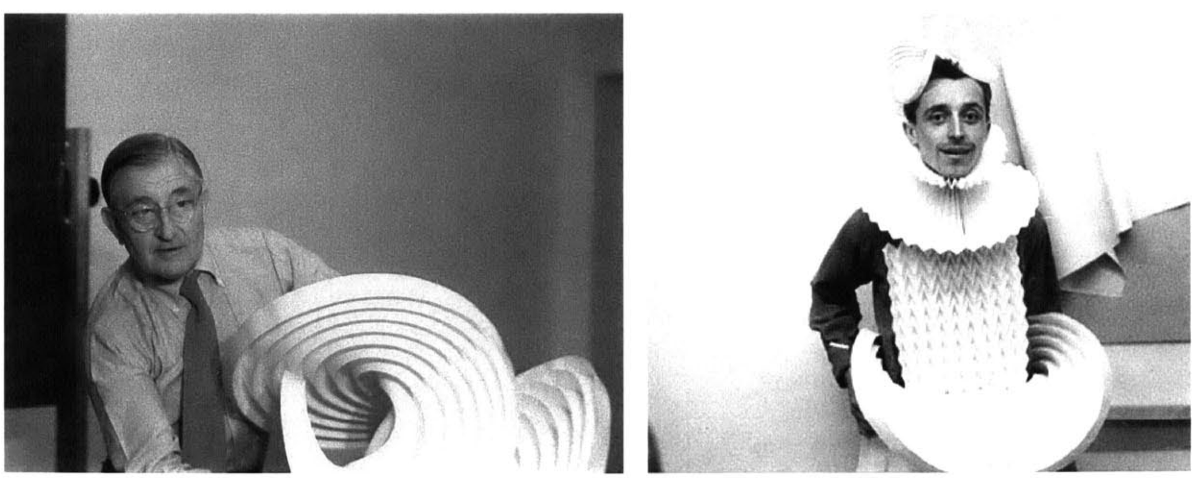


Fig 2.2.4 Josef Albers at the College for Design in Ulm (1954, Hans Conrad), Student of Josef Albers, HfG-Archiv Ulm, Naske permanent loan, (1955, unknown photographer)

The image on the right is from the same time period and depicts a student holding a large version of the model in his hands.

Less known for his folding talent, but certainly recognized for his art books Kurt Londenberg (1914-1995) published 'Papier und Form' in 1972, followed by several editions later on (Fig 2.2.5 left). The book features sculptural works of paper and presents paperfolding in various contexts. Among them is a model that appears in the 'architectural folding' chapter (Fig 2.2.5 right). Many of the photographed models were made specifically for the book and he saw this publication also an educational contribution [Lon 72]. Londenberg attributed great significance to Bauhaus educator and artist, Josef Albers, and republished the above summarized article on working with paper.

Hiroshi Ogawa [Oga 71] worked in a similar fashion as Londonberg as he also made the paper models for his publication (Fig 2.2.6). Ogawa focused on 'paper sculpture' and wanted to convey what could be achieved in terms of artistic expression. He appears to have had a more general audience in mind.

Both authors' works display artistic qualities in terms of the depicted objects themselves, the expressive nature of the photographs and also as art books. They both do not follow the fold-no-cut rule. They refrain from elaborating on personal artistic motivations, but want to demonstrate the design potential of the material. The 2 books served slightly different purposes in different countries. Londonberg thought of his book as collection of works in paper in many different design disciplines and industries. He foregrounded connections to pedagogy, but also presented paper as industrial material and pointed to large scale implications. Londonberg used the term' Papierarbeit' meaning 'work of or with paper'. Ogawa on the other hand was interested in conveying techniques and included crease patterns at the end of the book. Many of the works described

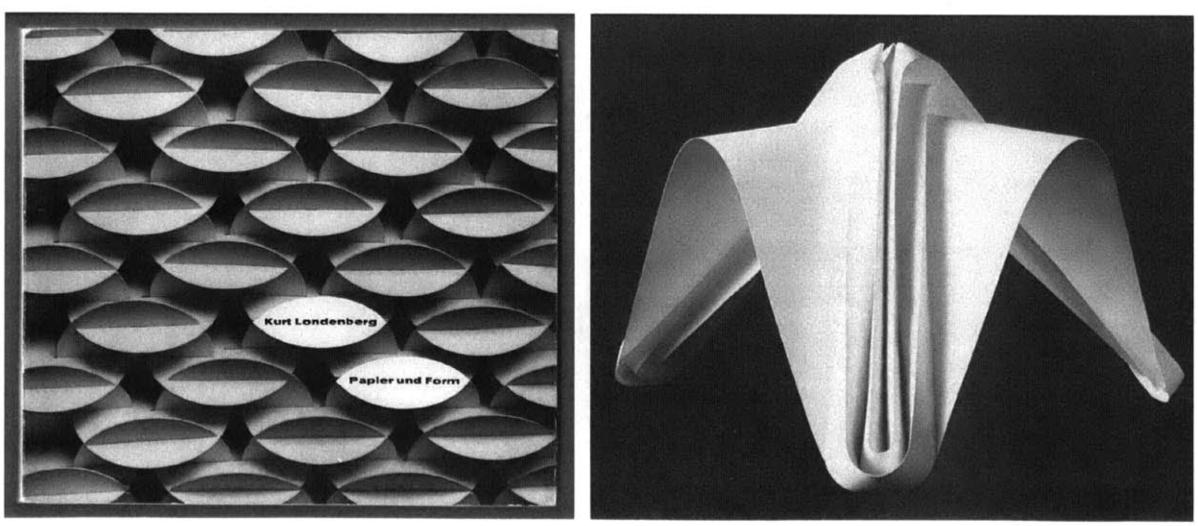


Fig 2.2.5 Cover and a design in 'Papier und Form' (1972, Kurt Londenberg)

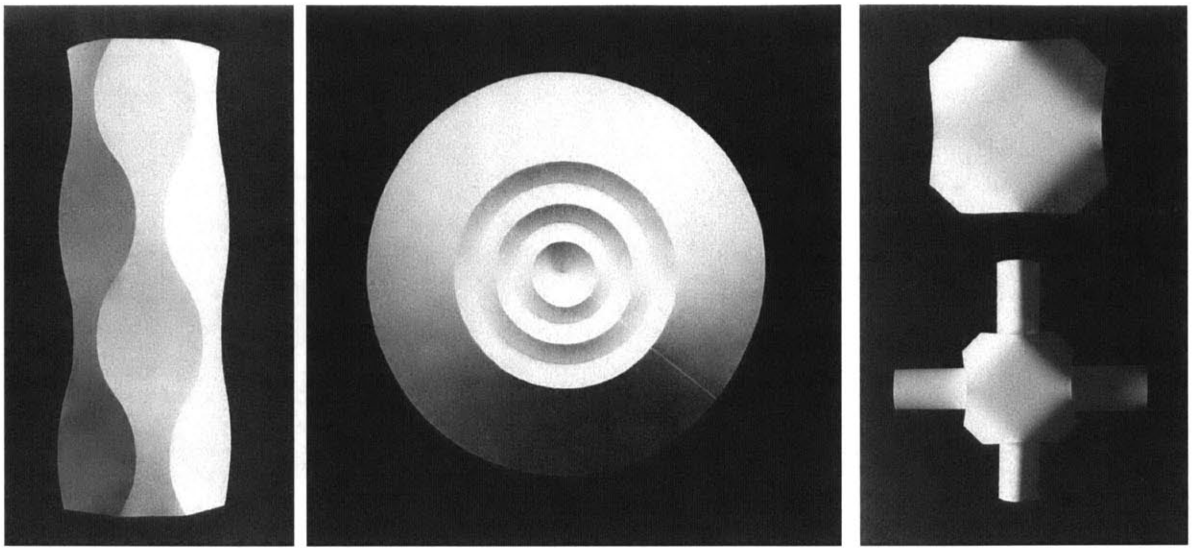


Fig 2.2.6 Designs in 'Forms of paper' (1971, Hiroshi Ogawa)

in both books however are good examples of what I like to think of as paperfolding.

David Huffman owned the 1971 edition of Hiroshi Ogawa's 'Forms of Paper', but it is unclear when exactly he acquired the book. Huffman appeared to focus on tilings with straight creases in the 1960s and it is hard to estimate when exactly he discovered curved creases for himself. The 3 examples in the figure are comparable to some of Huffman's investigations (Fig 2.2.6).

David Huffman and Ronald Resch

The most expansive work from the 1970s that utilized curved creases has to be attributed to the artist Ronald Resch and the computer scientist David Albert Huffman. Two examples of their work use a variety of curves (Fig 2.2.7). They discussed paperfolding in 1973, when Huffman visited Resch during a sabbatical leave at the University of Utah. By that time, Resch had completed an impressive body of work that included curved crease paperfolding. Huffman remained true to his roots and took a more analytical approach that included rulings in the years to come, while Resch was more interested in applied techniques for sculptures and architectural structures. Both published little and had a strong connection to computational processes. Only Resch used computers to realize some of his sculptural work [Res 74]. Resch and Huffman generally followed the 'fold no cuts' rule of folding purists. They both described their work as 'folding' or 'foldings' and did not use the term origami. Huffman's cyclic tiling was made by hand with a sheet of vinyl similar to the ones Resch used at that time (Fig 2.2.7). Resch's sculpture on the left was constructed as part of an academic paper that conjectured that any space curve can be a curved crease. I elucidate the work in the chapter on mathematical developments.

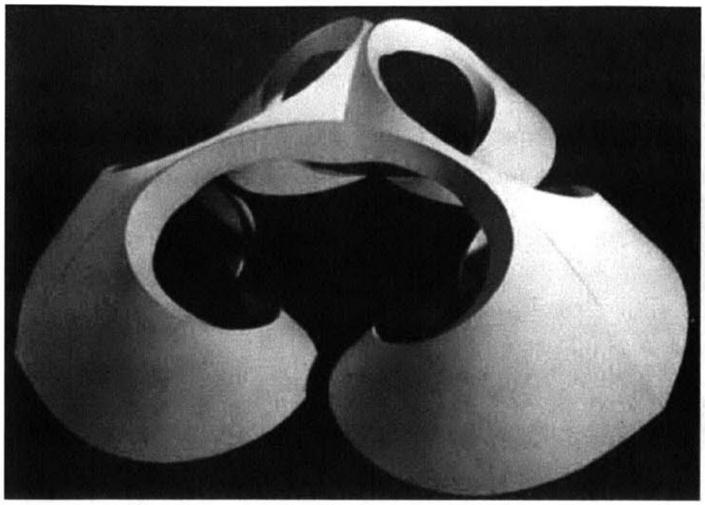
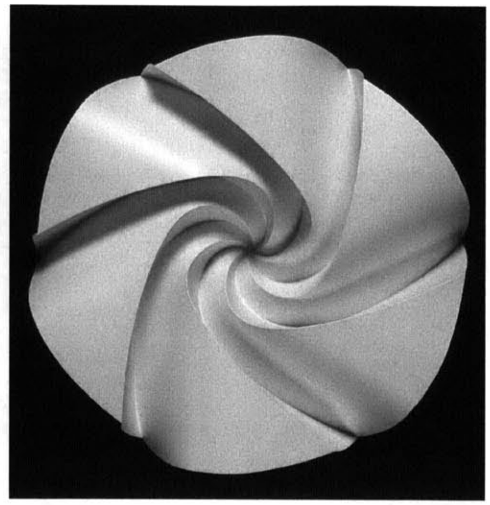
 

Fig 2.2.7 'Space Curve' (Ron Resch), Vinyl model (David Huffman)

Resch, being concerned with fabrication methods for his expressive art, created his own work flow and used a large plotter to pre-crease sheets of vinyl [Sch 09]. We can see Resch operating a plotter manufactured by Gerber that the University of Utah owned (Fig 2.2.8 left). The image on the right depicts a slightly better image of a similar computer controlled plotter the company sold in the early 1970s. Resch used ball pens attached to a Dremmel tool and sheets of vinyl to create his sculptures at the time.

Product design

Developable surfaces combined with curved creases provide a geometric resource for designers. The definition of the surfaces and behavior of the creases allow for the manipulation of a flat surface that can assume fairly complex shapes. Most materials come in the form of sheets, so called 'sheet goods', and a designer can shape these sheets without the need for a mold.

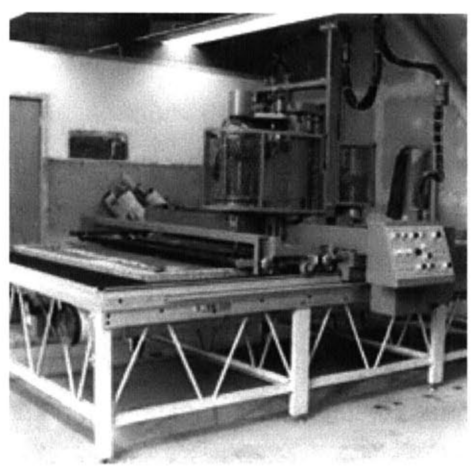
 

Fig 2.2.8 Ron Resch operating a Gerber plotter (circa 1969), Gerber plotter model (circa 1970)

The lamps by the Le Klint serve as an excellent example to demonstrate the potential of curved creases in design. The designs consist of one material and a method for folding the company has perfected over many years. Le Klint sells lamps that have been folded into pleats with straight lines, starting in 1943 [Kli 43]. The designs are still produced from long sheets of lamp shade foil by hand today and are mostly based on cylindrical configurations [Jac 08]. Architect Poul Christiansen, born in 1947 and trained at the Royal Danish Academy of Fine Arts' School of Architecture, designed the lamps (Fig 2.2.9). He worked for Ib & Jørgen Rasmussen from 1977 to 1986 and founded 'Komplot Design' together with Boris Berlin in 1987. In 1969 Christiansen discovered that folding with mathematical curves gave the lamp shades beautiful and unique sculptural shapes. His most famous creation is the SinusLine series which he developed by combining sine curves such that they fold into a spherical shape. In an email Christiansen [Chr 11] reported that there were no classes on paperfolding in the late 1960s. 'I found out about this all by myself, and it was really a wonderful feeling to discover all these 3d shapes, when folding along curves.' he said. He first tried to implement the shapes in architectural projects, inspired by similar constructions of Utzon, Nervi, Candela, and other architects from that time. He realized that a more direct way to use his findings was to make lamp-designs for the Le Klint company, a place he passed by every day on his way to the architecture school in Copenhagen.

As we can see the historical evidence of this subset of geometry surfaces sporadically and in different places. I discuss contemporary examples of curved creases by designers and artists in the final chapter that proposes several design approaches.

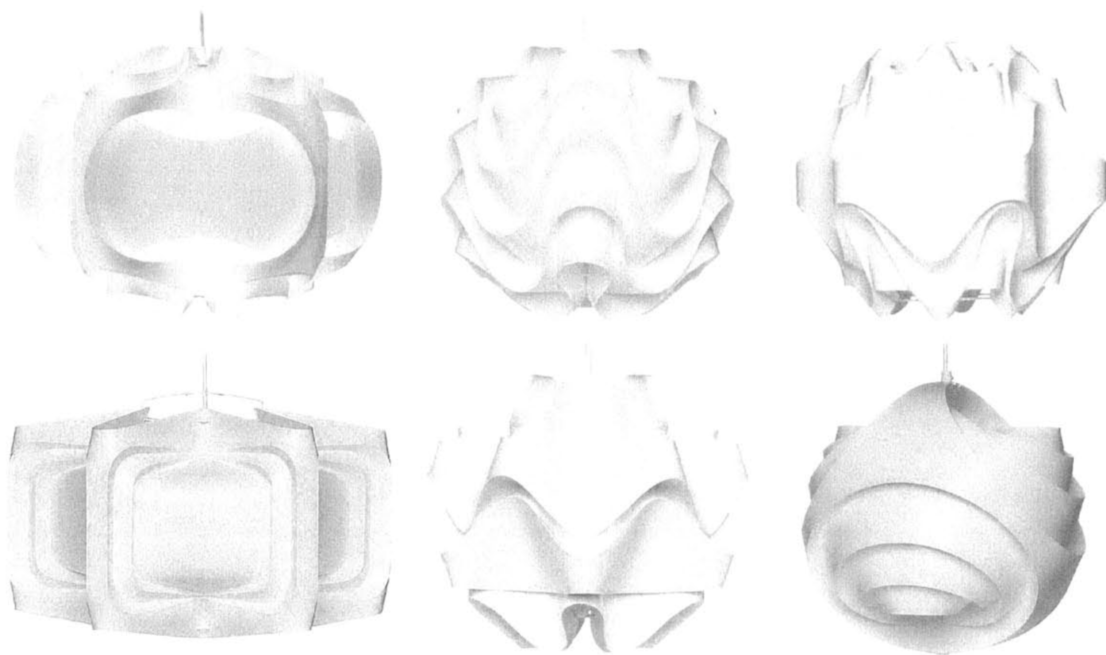


Fig 2.2.9 Lamp designs #171 to 178 by Poul Christiansen, 1969 to early 2000's

2.3 Mathematical and computational approaches to curved creases

In order to make curved folding teachable and applicable to design, it is necessary to understand the geometric nature of curved creases. Curved creases are poorly understood even today. The problem has been investigated by mathematicians, computer scientist, engineers and artists with fairly different approaches. Some investigations have resulted in the development of tools and simulation software, which I address in the individual sections.

This section presents the work in the categories of a previously published paper by Erik and Martin Demaine, Tomohiro Tachi and myself [DDKT 11]. I begin with general definitions of developable surfaces and subsequently expand on the categories of 'Constructive geometric approach', 'Differential geometric analysis', 'Inverse calculation of a crease' and 'Discrete geometric approach' beyond the previously published paper.

Developable surfaces

Paper, mathematically speaking, can only assume the shape of a developable surface between creases. A developable surface has to follow certain constraints. The surface can be mapped onto a plane without distortion of curves, which makes it a special kind of ruled surface with a Gaussian curvature equal to 0. The tangent planes to a developable surface at every point along a ruling are coplanar. One could imagine holding a ruler up to that surface and it would touch along a rule line, but these lines are not arbitrary. Together these rule lines form the ruling.

There are four main types of developable surfaces (Fig 2.3.1) [Law 11]: The first type is the planar surface, which has an ambiguous ruling. The second type are surfaces which can be described as a generalized cylinder, where all generating lines are parallel, swept by a set of mutually parallel lines. The third type are surfaces which can be described as a generalized cone where all generating lines intersect in at a common vertex, the apex, of the surface. The fourth type are tangent surfaces. They are surfaces whose rule lines are tangents of a general space

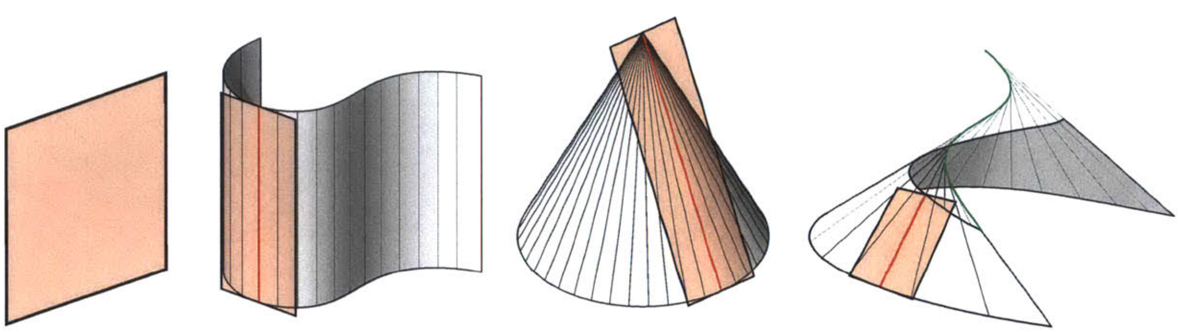


Fig 2.3.1 Developable surfaces: plane, general cylinder, general cone, tangent surface

curve, which is called the edge of regression [FT 99].

Paper surfaces can consist of 2 kinds of combinations of developable surfaces. They can be joined smoothly at rule lines or can be joined at creases.

Constructive geometric approach

One of the simplest design methods for curved crease folding is to use reflection. We start with a single developable surface, for example a general cylinder, and cut it with a plane. We subsequently reflect or mirror the surface through the plane (Fig 2.3.2). We obtain a curved crease that, by definition, lies in a plane.

This reflection is a special case of curved folding, where the rulings don't change direction in the flat state and the crease lies on a single osculating plane [Huf 76]. All developable surfaces are available for this kind of approach and this simple yet effective method has been used by several artists. The taxonomy in chapter 4 begins with examples by Huffman that are based on reflected cylinders and cones.

The crease pattern for reflections of cylinders consists of parallel rulings, and reflections of cones result in rulings that converge in a single point, the apex of the cone. The predictability of the location of the rulings lends itself to developing discrete solvers such as Jun Mitani's ORI-REF software [Mit 11], discussed in the final chapter on design approaches.

Differential geometric analysis

We can obtain fundamental results via a differential geometric approach, i.e., understanding the local behavior of the paper surface. Investigating Ron Resch's work gives us early clues into this approach.

Resch worked with computer scientist, Ephraim Cohen, who wrote many programs for him between 1971 and 1972. Cohen reports in an e-mail conversation [Coh 14] that the Computer Science Department at the University of Utah owned 2 computers at that time. The computers, PDP-10's, served different purposes. One of the PDP's was used for time-sharing and had a

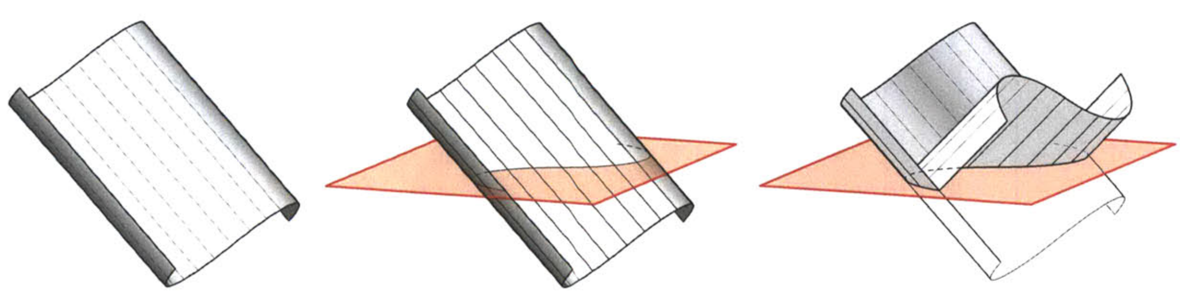


Fig 2.3.2 Reflection of a general cylinder through a plane

number of workstation-like posts, some equipped with early monitors and mice. The other was a single-user machine and one could sign up for an hour at a time. The machine was equipped with an interfaced CRT (cathode ray tube monitor) and a 4″ by 5″ camera or 16mm film camera to take pictures of what was displayed on the screen. Cohen used mostly SAIL, an algol-like language, and only rarely reverted to Fortran or Assembler.

Resch and Cohen investigated curved creases together. Cohen conjectured that every space curve could be used to construct 3 distinct pairs of surfaces that have a curved crease on the curve in his manuscript 'Attaching a developable surface to an arbitrary space curve' [Coh 75]. Resch and Cohen most probably wrote a second version of the manuscript together [RC]. Huffman keeps a copy of both manuscripts among his notes. According to Cohen the work was never published. Cohen implemented the idea in his software 'Dev', which computes a pair of developable surfaces from a space curve. His software was able to compute different folding angles, but he used constant angle configurations with 90° for the published renderings of the 'Space curve'

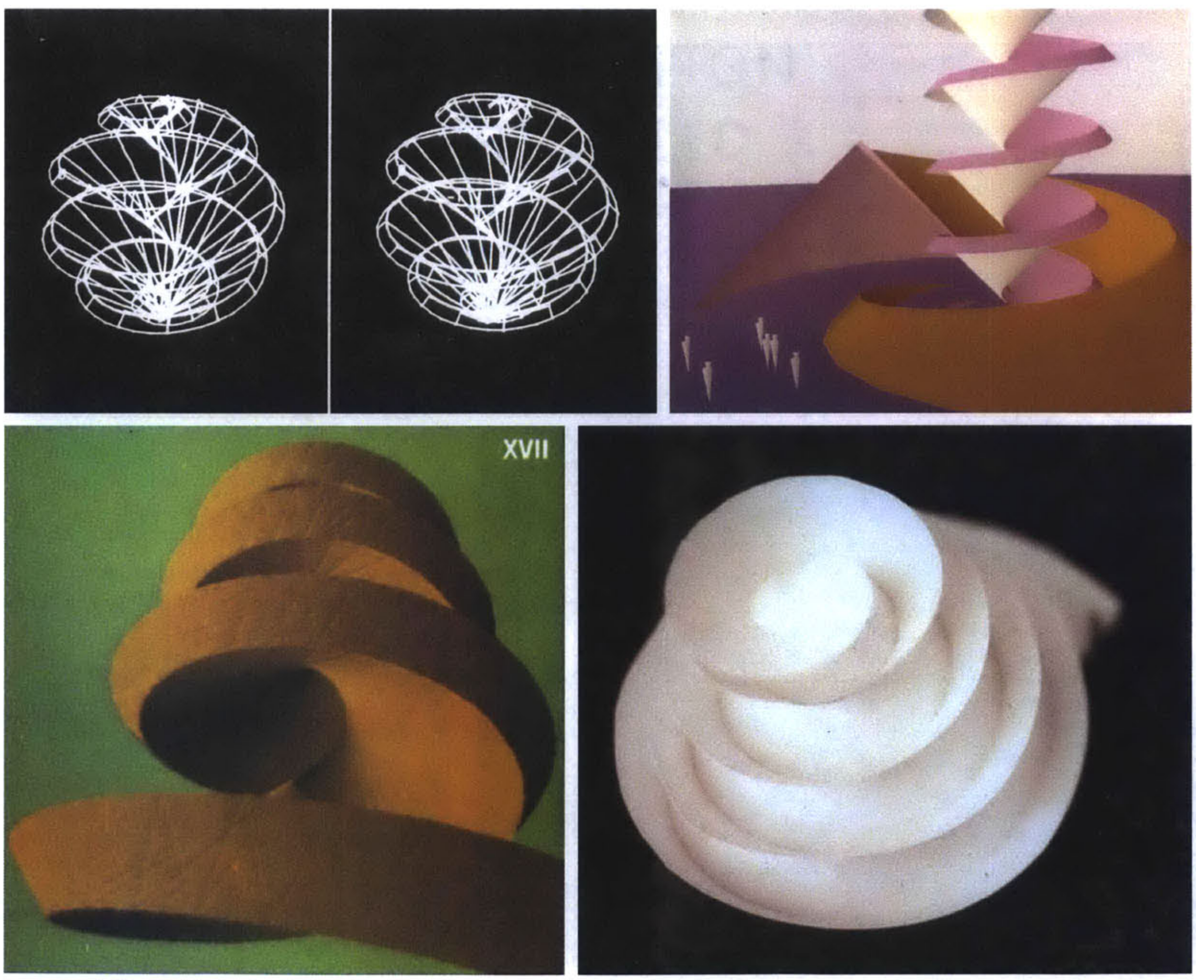


Fig 2.3.3 Variation of 'Space curve' (Ron Resch, renderings and photograph by Ephraim Cohen)

(Fig 2.3.3 and 2.3.4).

An iteration of the design with an upward spiraling curve was visualized in the form of early computer renderings, the kind of image the University of Utah with its efforts in computer graphics was known for (Fig 2.3.3). The screen shots were taken with the previously mentioned photo camera that was permanently mounted to one of the PDP's. In the upper left images we can observe how Cohen was able to construct the 3d configuration digitally.
Resch's physical paper version of the design is depicted on the lower right [Res 74].

A second variation of the design consisted of a tiling with 3 parts (Fig 2.3.4). The similar arrangement of images shows screen shots of renderings and Resch's paper model with less material in the center area.

Huffman contributed in several ways to our current understanding of curved creases. He was interested in finding representations of creases in ways that can be useful for computational processes. His 1978 paper 'Surface Curvature and Applications of the Dual Representation' [Huf 78] introduced a dual representation that was based on mapping the normal vector of a

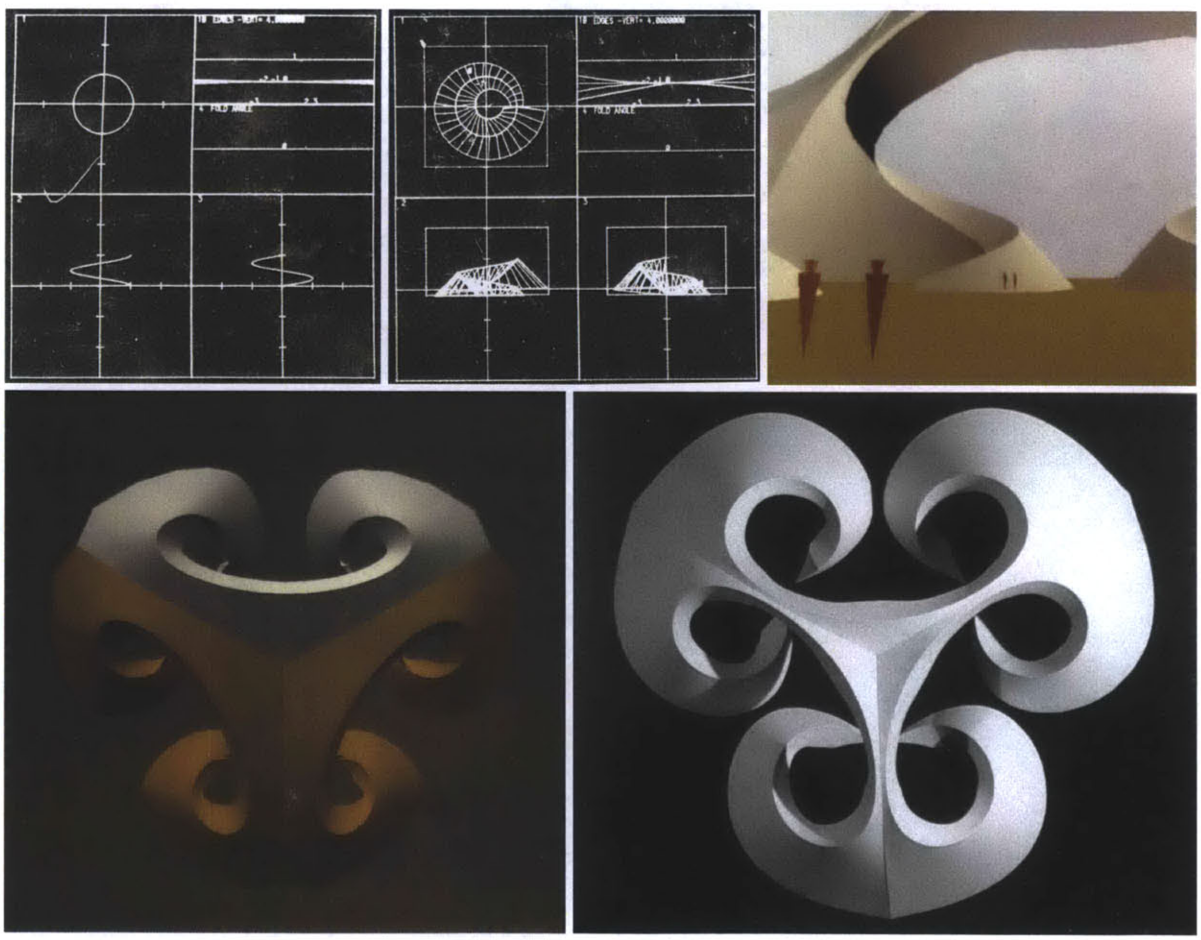


Fig 2.3.4 'Space curve' renderings (1971-72, Ephraim Cohen), Paper model (1971-72, Ron Resch)

plane on the Gaussian sphere. Given a vertex with 4 edges as shown in the top-left drawing (Fig 2.3.5) we can draw the normal vectors to the planes that are described by the edges and intersect them with a sphere. The resulting points on the sphere are labeled the same way as the planes in the original configuration. A trace on the sphere can be drawn between adjacent planes in the dual picture as shown in the bottom left of the same figure. For infinitesimal folding, this spherical diagram is approximately planar, so it can be drawn in the plane as shown on the far right. Huffman elaborates on proportions and relationships that can be studied in this dual image. He creates the flat foldable crease pattern to the left of the dual.

Huffman's most studied earlier paper 'Curvature and Creases: A Primer on Paper' from 1976 [Huf 76] represents the core of what we know about curved creases today, which reveals the behavior of a crease at a vertex. Huffman uses the dual representation already in this paper and elaborates on observations for special cases. He draws a crease that lies in a single plane parallel to the projection plane of the drawing (Fig 2.3.6). The dual representation, shown below, indicates that the 2 regions of the paper on the left and right side of the crease are concave and convex. The tangents in the dual representation of the bold point 3, denoted as L_3 and R_3, are parallel to each other. Huffman compares this special case to a general case in the 2 diagrams on the right, where the rulings on the left and right side of the crease are not on the same line in 2d.

Dmitry Fuchs and Serge Tabachnikov [FT 99] [FT 07] build on Huffman's work and contributed to the understanding of curved creases. They assess that it is possible to fold an arbitrary 2d curve on paper into a 3d crease with higher curvature. If the 2d curve is strictly convex and closes onto itself (e.g., a circle) then the folded 3d crease is not in a plane. They also elaborate on the behavior of rulings along folded creases. They describe which configuration a single general curve in 2d can assume when folded in 3d.

Erik Demaine, Martin Demaine, Vi Hart, Gregory Price and Tomohiro Tachi [DDHPT 09]

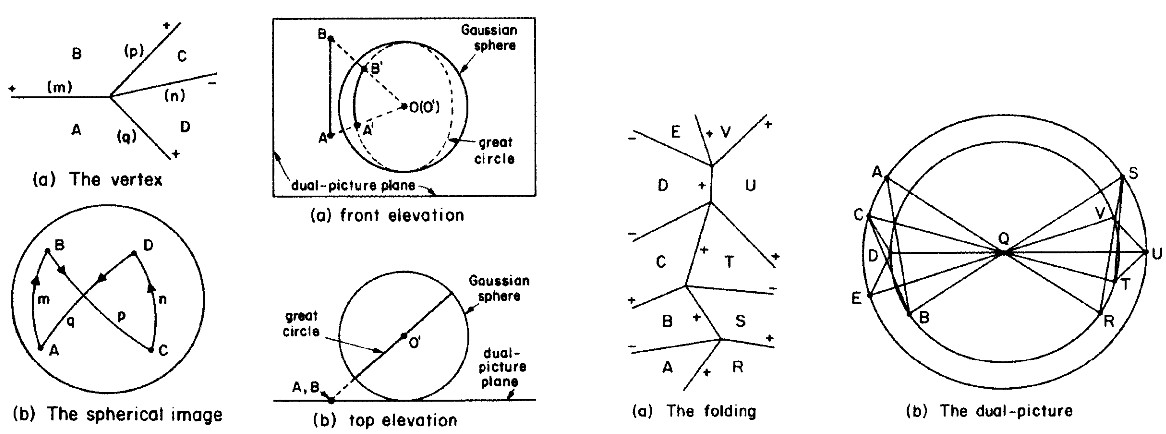
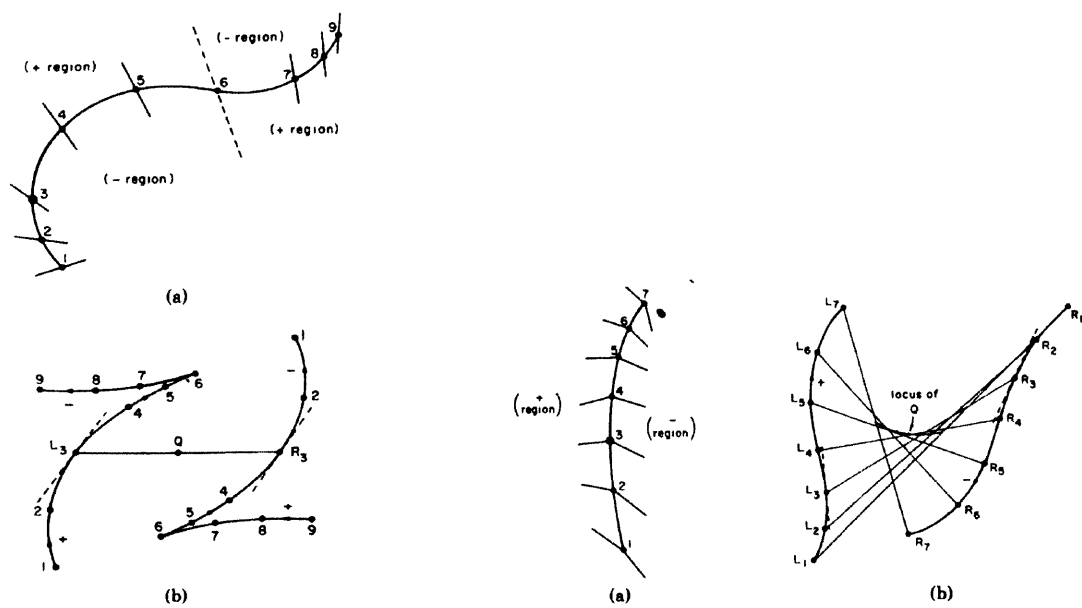


Fig 2.3.5 Mapping of trace on Gauss sphere, crease pattern and corresponding dual (D. Huffman)



Representation of a convex crease contained in a single osculating plane. (a) Convex crease with associated generating lines and (b) corresponding trace.

Representation of a convex crease with a changing osculating plane. (a) Convex crease with associated generating lines and (b) corresponding trace.

Fig 2.3.6 Crease in plane and dual, General case and dual (D. Huffman)

describe how paper behaves between creases and, in particular, formalize the existence of rule lines. They mathematically explain why only curved creases can produce interesting curved surfaces. In other words, a surface surrounded by straight creases (folded with an angle α, $0 < \alpha < 180°$) cannot bend and must stay polyhedral.

Even though these local analyses form the base of other geometric design approaches, these general results themselves often stop after the first crease, while multiple creases are typically used in practical designs.

Inverse calculation of a crease

If a specific 2d curve is used as the crease template and one of the surfaces is bent, what will the surface on the other side fold into? Robert Geretschläger [Ger 09] sets out to understand curved creases by predefining the geometry of a piece of paper in its curved state. He then assumes the 3d path of a crease and calculates the position of the part of the paper on the other side of the crease (Fig 2.3.7).

This type of inverse approach is useful to construct reusable modules for tessellated designs. Jeannine Mosely analyzes the curved creases of her own 'cube shape', a tiled volumetric model that consists of 4 semicircles [Mos 02]. In her tessellation work a curve is numerically calculated such that cones and cylinders symmetrically align in a plane.

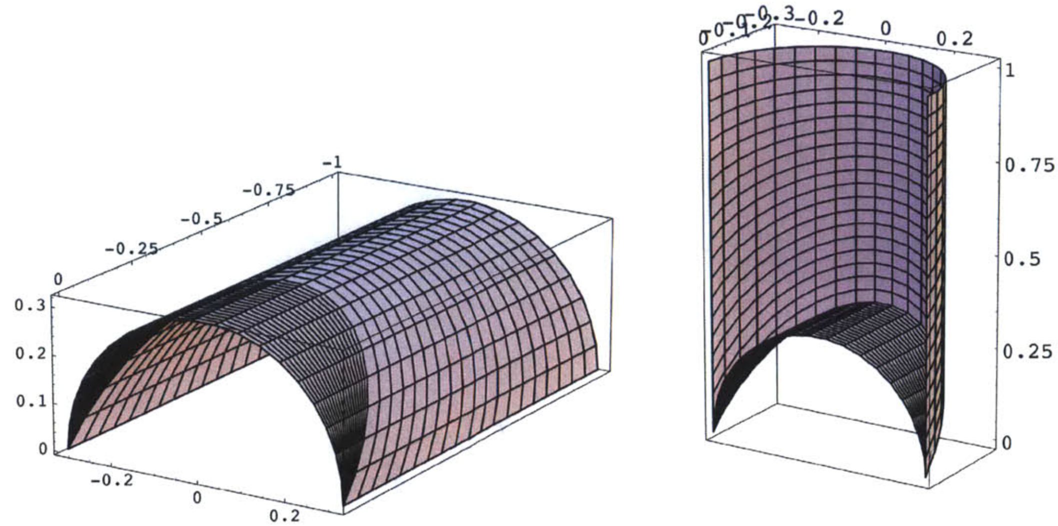


Fig 2.3.7 Calculated curved crease (R. Geretschläger)

Discrete geometric approach

Discrete representations of curved creases are easier to work with computationally and therefore play an important role for simulation. In order to deal with fully generalized curved folding without predetermined assumptions about the form of the surfaces, we need to globally solve geometric problems via a discretized approach.

Early work in this domain was undertaken by Resch and Cohen and resulted in the computer-controlled simulation of artistic sculptures, called 'birds' [Res 73]. Cohen's program 'Rule' approximated the ruled surface between two space curves as thin triangles, joined and folded along their long edges [Coh 14]. The digitally simulated models are subsequently folded in paper and vinyl (Fig 2.3.8). Resch includes the designs in his exhibition 'Ron Resch and the Computer' in 1972 at the Utah Museum of Art, University of Utah, Salt Lake City. The work constitutes very early examples of computer-generated paper folding.

Haresh Lalvani, an artist and geometer who trained with Buckminster Fuller, uses sets of parameters or 'genes' to modify families of polyhedra in his 'Morphological Genome Project'. The polyhedra shown with red top surfaces (Fig 2.3.9 left) are converted to developable cylindrical shapes (Fig 2.3.9 right) and are grouped into clusters of similar types [Gan 03]. The practical application of this work consists of metal column covers for interiors manufactured by Milgo/Bufkin in Brooklyn. The company, owned by Bruce Gitlin, has realized other artists' works for many decades and the designs developed with Lalvani are part of the permanent collection of the Museum of Modern Art in New York.

Yannick Kergosien, Hironoba Gotoda and Tosiyasu Kunii [KGK 94] take an engineering

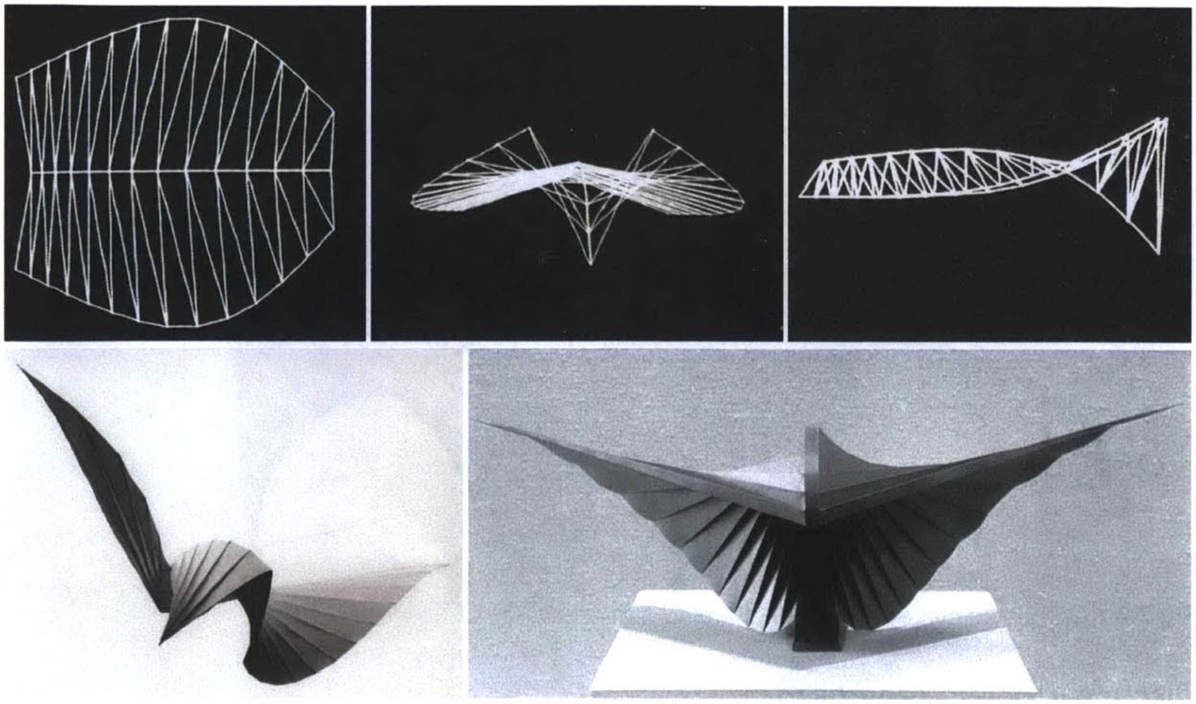


Fig 2.3.8 Computer generated birds (early 1970s, Ron Resch and Ephraim Cohen)

approach to investigating simulations of paper. Starting from a generic surface they are able to fit a developable surface. If the boundary curve creates crossing rule lines their algorithm finds a curved crease within the boundary (Fig 2.3.10). The work is useful for computer graphics, but unfortunately does not explain the mathematics behind the relationship of the crease in its flat and

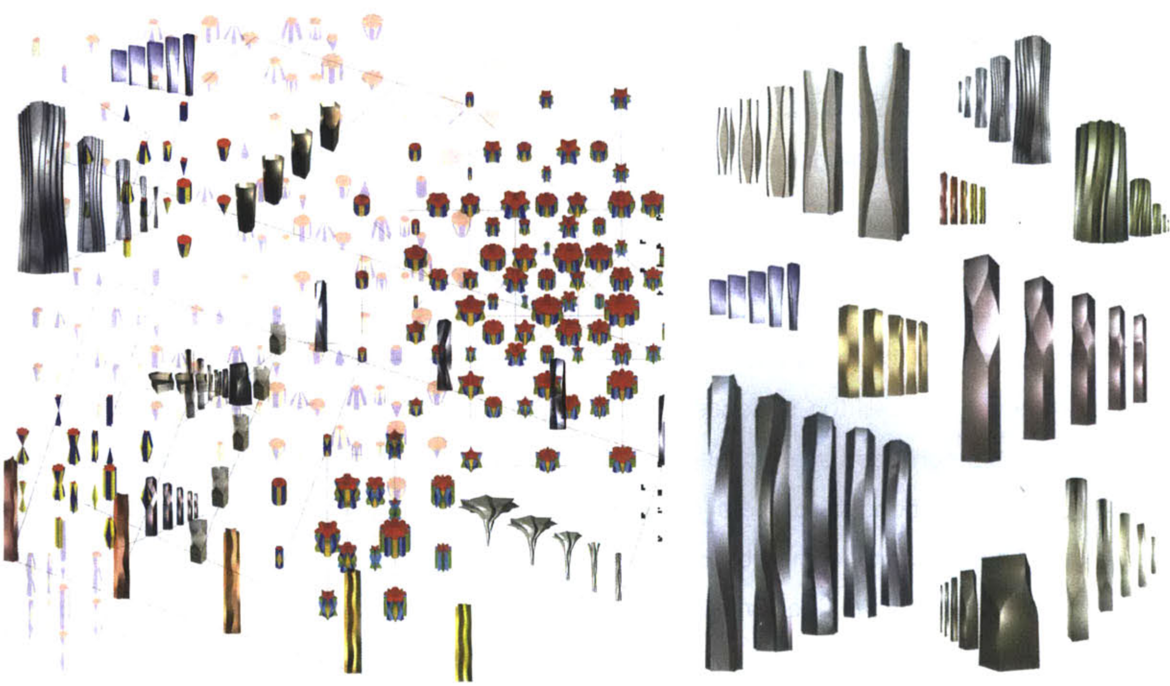


Fig 2.3.9 Morphoverse with solutions and column covers by Haresh Lalvani

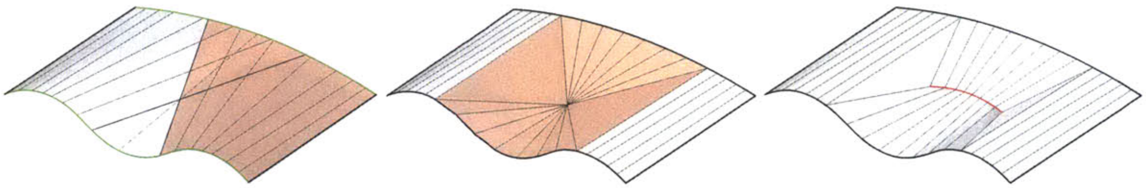


Fig 2.3.10 Patch with invalid rulings, area replaced by cone, new curved crease (Kergosien et al.)

corresponding folded state.

Martin Kilian, Simon Flöry, Zhonggui Chen, Niloy Mitra, Alla Sheffer and Helmut Pottmann [KFCMSP 08] model curved folding using planar quadrangle meshes (PQ-meshes) and deploy an optimization-based method. A case study they investigated is a car design by Gregory Epps made of a single piece of paper. The 3d scanned physical paper model undergoes an elaborate process of analysis, rule line searching, plane fitting and edge optimization. The outcome is the description of a piecewise-smooth developable surface (Fig 2.3.11). The reconstruction of Epps's car shows tangent surfaces in blue, cones in red, cylinders in green and planes in yellow. The proposed process can post-rationalize any 3d scanned paper model, which is useful for fabrication for instance, but does not describe the folding process or generate novel forms, which still presents a main challenge today.

Erik and Martin Demaine and Jenna Fizel develop software to simulate the physics of pleated paper folding [Dem 10]. Their discrete simulations reproduce the observed physical behavior of the circular hyperbolic paraboloid, which is the Bauhaus model mentioned earlier. A recent numerical approach by Marcelo Dias and Christian Santangelo [DS 12] suggest the Bauhaus model might exist.

Tomohiro Tachi, architect and geometer has developed Freeform Origami, software that can simulate the folding of a crease pattern comprised of line segments [Tac 13]. Constraint settings in the software make sure that the simulation is based on what Tachi calls 'rigid folding' [Tac 09], which means that no planes stretch or warp during simulation.

The simulator can be used with an approximated curved crease pattern when the rulings

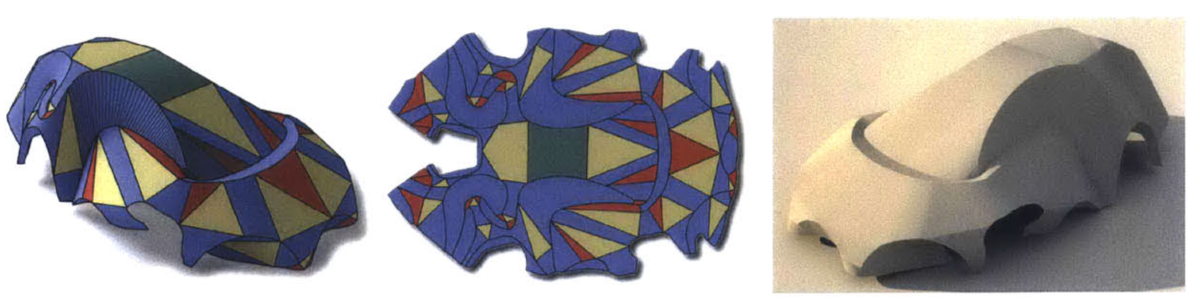


Fig 2.3.11 Post rationalized model, crease pattern, rendered model (Kilian et al.)

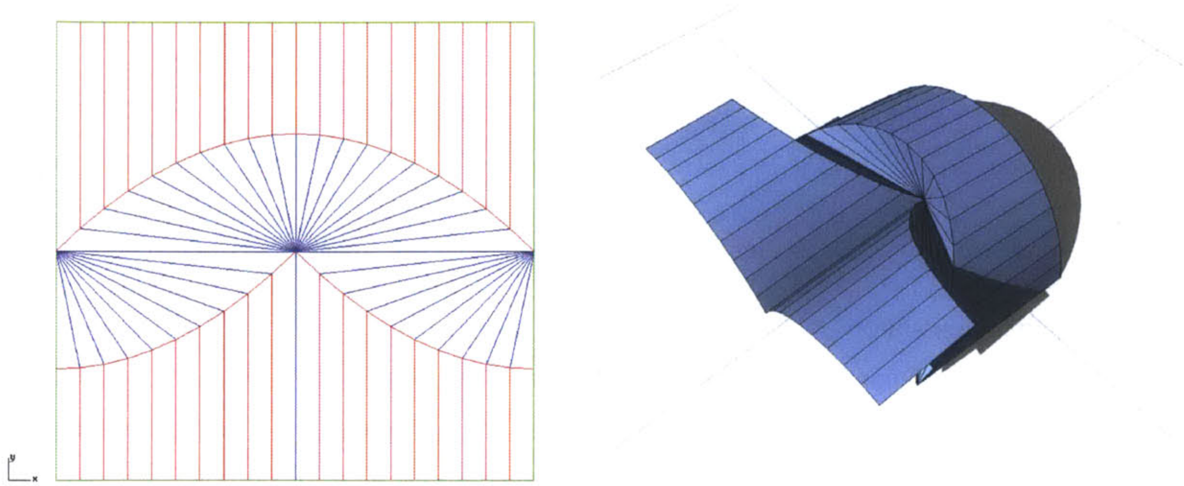


Fig 2.3.12 Discrete representation of a design by Huffman, (Duks Koschitz and Tomohiro Tachi)

are known in 2d. This is especially useful when simulating designs by Huffman as he determines the location of the rule lines, which are based on the refraction properties of conics. If the rulings are connected by polylines that approximate the curves of the crease pattern we obtain a possible discrete representation, which we can simulate.

A part of the 'Arches' design by Huffman was used as first example for this approach (Fig 2.3.12). The crease pattern consists of a discrete version with line segments of his continuous crease pattern with parallel rulings and rulings that meet in a vertex. The simulated model consists of conical and cylindrical approximations in the folded state. I further elaborate on this method as a design approach in chapter 5.

Other accomplishments by Tevfik Akgun (analysis of sculptural work by Ilhan Koman) [Akg 06], Rob Burgoon (discrete shells origami) [BGW 06], S.D. Guest and S. Pellegrino (deployable structures) [GP 92], Biruta Kresling (structural buckling analysis of bamboo) [Kre 02], Fady Massarwi (polygonal folding from generalized cylinders) [MGE 07], Shin-ya Miyazaki (virtual manipulation system for origami) [MYYT 96], Jeannine Mosely (Curved Origami) [Mos 08] and Surface Transitions in Curved Origami) [Mos 09] and Saadya Sternberg (Curves and Flats) [Ste 09] have also contributed to curved crease investigations.

3. David Albert Huffman (DAH)

DAH, the person and his ways of working

In order to better understand Huffman's intentions and his approaches to dealing with curved creases, it is useful to study the many objects and visual artifacts he produced over the course of his career. He made models of geometric phenomena, designed sculptural objects, and created puzzles in the context of what one might call a hobby. He photographed his work as final product and in some cases also documented the making of the work. His ways of working suggest a connection to MIT's culture of thinking in conceptual models and making physical models. His early academic investigations related to machine vision and later work was concerned with describing the behavior of straight and curved creases. Both motivated the making of visual artifacts such as diagrams, drawings, sketch models and many paperfoldings.

The following biography is by no means complete and merely strives to highlight moments in Huffman's life which provide clues to how he might have thought of design related issues. I argue that he saw the world with an analytic mind and considered design as a problem that might have a solution. Certain events, documented with archival material in this chapter, illustrate his bias toward problem solving. Others, show that he sometimes privileged aesthetic decisions over reasoning.

3.1 DAH, a gifted computer scientist

David Huffman Copp, born on August 9th 1925 in Alliance Ohio, obtained his new name, David Albert Huffman, in 1927 when his mother petitioned for a name change in the course of her divorce. During his first years he was slow to speak, which appeared to be related to early family incidents, but a series of tests showed that the prodigy was soon on his way. By the age of 18 he finished a Bachelor's Degree in Electrical Engineering at Ohio State University (OSU). In an interview for the Scientific American Huffman described his affinity to mathematics as possibly related to his early childhood. 'I like things neat,' he said. 'I like to wrap things up and get definitive answers, possibly because of the uncertainties of my early life.' [Sti 91]

He joined the U.S. Navy and served as 'Radar Maintenance Officer, Sonar Officer and Radar Counter Measure Officer' on the USS Duncan, a destroyer that cleared mines in Asian waters after World War II. Upon his return he taught as an instructor at OSU in the Electrical Engineering Department and supervised a classified project on guided missile control with the Air Force Air Material Command, which seems to be his first military involvement as an academic. He earned his Master's degree in Electrical Engineering at OSU and got married to Betty Jane Ayres. The same year he was admitted to the Graduate School in Electrical Engineering at the Massachusetts Institute of Technology (MIT) in Cambridge.

In 1951 Professor Robert Fano gave his graduate students the option of working on a term paper instead of the final exam, but did not tell them that neither he nor Claude Shannon, the creator of information theory, knew the solution to the proposed topics. One of the posed problems was how to represent numbers, letters or symbols efficiently with binary code. Huffman decided to work on this information compression problem and almost gave up by the end of the term as he could not prove that many of his approaches were the most efficient. Right after discarding his work he realized that he might have found a solution, which came to be known as the Huffman Coding Procedure, a fundamental idea of loss-less compression in computer science. His approach assigned the shortest code for the most used characters and the longest for rarely used ones. He relied on a coding tree in which every leaf was the probability of each character and was thereby able to reduce code symbols significantly without losing any information.

In 1952, Huffman published 'A Method for the Construction of Minimum-Redundancy Codes' in the proceedings of I.R.E. 40, No. 9. He also attempted to pass his doctoral examinations, yet without success. He later described this experience as a cultural shock and remembered how terrible it was to think of himself as a failure. A year later he defended his Doctor of Science in Electrical Engineering with 'The Synthesis of Sequential Switching Circuits' advised by

Samuel Caldwell. This early methodology for devising asynchronous sequential switching earned him the Louis E. Levy Medal from the Franklin Institute a few years later. It also allowed him to join the faculty ranks at MIT until 1967, while also holding a visiting position at the University of California, Berkeley in 1961 and Stanford University, Palo Alto in 1964.

During his time at MIT Huffman was again involved in several classified and unclassified military projects as technical aide, developer and consultant. The work for the Navy's Institute for Defense Analyses appears to be related to his compression work for communication technology. He invented a Barrier-grid iconoscope, an infrared television designed for use in darkness and fog, for the Air Force Air Material Command, at Wright Patterson AFB. In August of 1960 Huffman received a letter from the Operations Evaluation Group, Navy Department, confirming his position as a member of the consultant panel. His compensation was set at $100 per day plus expenses. A diary entry for 1962 reads: 'Cuban crisis around Oct. 22-28 / Air Force meeting Oct 25-26'. It is likely that he participates.

Other groups he works for are the Air Navigation Development Board Psychological planning group, the A. and M. Physical Science Lab, New Mexico State College, the President's Scientific Advisory Board and the US Air Force Operations Evaluation Group. One particular involvement with the National Security Agency (NSA) is worth investigating further as it shows Huffman's attitude towards his military work. On July 12th 1955 he completed his application for the NSA and served as consultant during a few meetings in Washington DC in July. Several letters in 1957 indicate that the NSA hosted officials from Electrical Engineering departments of 72 colleges and universities. The letters confirm that he knew of the Bell Labs Baker committee and that he was invited to attend the Panel of the Science Advisory Committee, Office of Defense Mobilization, Office of the President, which he subsequently joined as consultant. On July 18th, 1957 a handful of the nation's top scientists strategized on the Agency's future technological survival.

Huffman planned on going to Russia for a conference and faced difficulties regarding traveling abroad. As a result he wanted to resign from the NSA FOCUS Advisory Committee on September 17th, 1962 in order to be able to travel internationally. The final acceptance letter of his resignation dated September 28th 1962 states that he will need to be debriefed and that his authorization for access to sensitive cryptologic information will be terminated. It took him several attempts to obtain permission to resign, which suggests that Huffman valued his work as scientist above any patriotic motivations. In an e-mail interview his daughter describes his political inclinations 'I can't really say how he was feeling politically during these years. Later on he was teaching at an 'alternative/liberal' school, UCSC.' She continues 'He was generally a Democrat and in favor of civil rights. [He] understood the need to defend oneself, as a country, but was not adamant or militaristic about it. He had a strong sense of right and wrong.'

3.2 DAH, a visual biography

In order to elucidate Huffman's thoughts on design and creativity this section presents the previously mentioned models, drawings photographs and other artifacts to show how his sense of aesthetics developed over time.

Edges and Lines

Huffman keeps a log of a final technical report for a project at Stanford Research Institute (SRI) for the Air Force in 1972, but not the report itself. It was likely related to Huffman's achievements in machine vision as that was the focus of his work at SRI. It is unclear when exactly Huffman started to gain interest in the logical analysis of scenes with ambiguous or impossible objects that are represented by lines. However, by the mid 1960s Adolfo Guzman-Arenas, who occupied the office next to Huffman's on the 8th floor of 545 Technology Square, conducted work in that field. In an e-mail conversation, Guzman mentions [Guz 14]: 'Prof. Huffman was in my Ph.D. thesis committee (1967-68), together with Profs. Minsky, Papert, and Licklider. So he knew my thesis work. I think he was puzzled by the idea that the 'propagation of the restrictions' (discovered by me) imposed by the vertices of polyhedra must have some reason, which needed to be discovered.'

Max Clowes, British researcher in Artificial Intelligence, was in London in 1970 and was also working on the interpretation of pictures by computers. Guzman spent the summer of that year at the University of Edinburgh, when Donald Michie organized his Sixth Machine Intelligence Workshop. Guzman explains 'I attended it. So did Prof. Huffman, who then presented his work 'Impossible objects as non-sense sentences'.' Prior to the conference Clowes had wanted to formalize the propagation of labels with Guzman, but they did not engage in the effort. Huffman did, refined the work and published it.

The scenario we need to imagine consists of a machine trying to make sense of what it sees. Starting with a digitized image or video feed of a scene a machine can detect edges of objects and can then, using Huffman's labeling and reasoning (Fig 3.2.1), infer, if the intersecting edges describe a polyhedral object or if these edges result in what he called an impossible object. He described his proof as an image grammar [Sti 91]. 'I wanted to create a sieve so grammatical pictures would go through and ungrammatical images would be seen as unrealizable,' he said.

Huffman's interest in the work started at a time when he was already contemplating leaving MIT. After several trips to the West that fueled a fascination with the outdoors of the Grand Canyon, Yellowstone and alike, Huffman became a visiting professor at Stanford University and worked at the Stanford Research Institute (SRI) in Menlo Park. The Artificial Intelligence Center

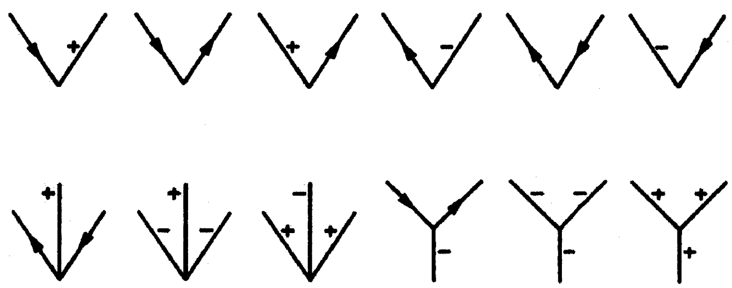


Figure I. Catalog of labelled-line configurations possible at nodes in pictures of trihedral objects.

Fig 3.2.1 Huffman's labeling convention for trihedral objects

at SRI was focused on an autonomous robot called Shakey and the team used his notation for polyhedral object recognition. In 'Artificial Intelligence: A New Synthesis' by Nils Nilsson [Nil 98] we can find a depiction of a typical scene Shakey would have encountered and the illustrations explain how Huffman's notation is used.

While this work is centered around turning an image, or rather a grouping of lines into symbols that a machine can use, it may nevertheless be the first time Huffman is dealing with visual imagery and geometry. The problems in machine vision challenged Huffman to think about line representations of objects, how a machine can infer whether random lines could be the edges of an obstacle, and how notational systems could help us understand the nature of 3d objects. He made his own lecture slides of such impossible objects a few years later in 1974 (Fig 3.2.2).

In another version of the work Huffman made wire models of two objects that look like a polyhedron, but only when looked at from a specific viewing angle (Fig 3.2.3). Some of the wires traverse through the individual volumes, which means they are closer to his impossible objects. The top images show the models and the bottom images, taken by Huffman himself reveal the described optical illusion.

The Huffman family moved to Santa Cruz in 1967, where he obtained a position at the University of California at Santa Cruz (UCSC) as the first head of its new department of computer science. He played a major role in the department's academic programs, the hiring of its faculty and served as chair from 1970 to 1973. After his first year at UCSC Huffman traveled to the Department of Computer Science at the University of Illinois at Urbana-Champaign and met Ed Jordan, Donald Gillies and Ron Resch at a dinner in October of 1968. This encounter was followed by further meetings with Resch in later years.

Back at UCSC Huffman began to work on a paper that describes the potential of NOT elements, elementary building blocks of a digital circuit. 'How to Say 'No' Once and Really Mean It,' is the humorous title of a paper Huffman presented at University of California Berkeley that proves

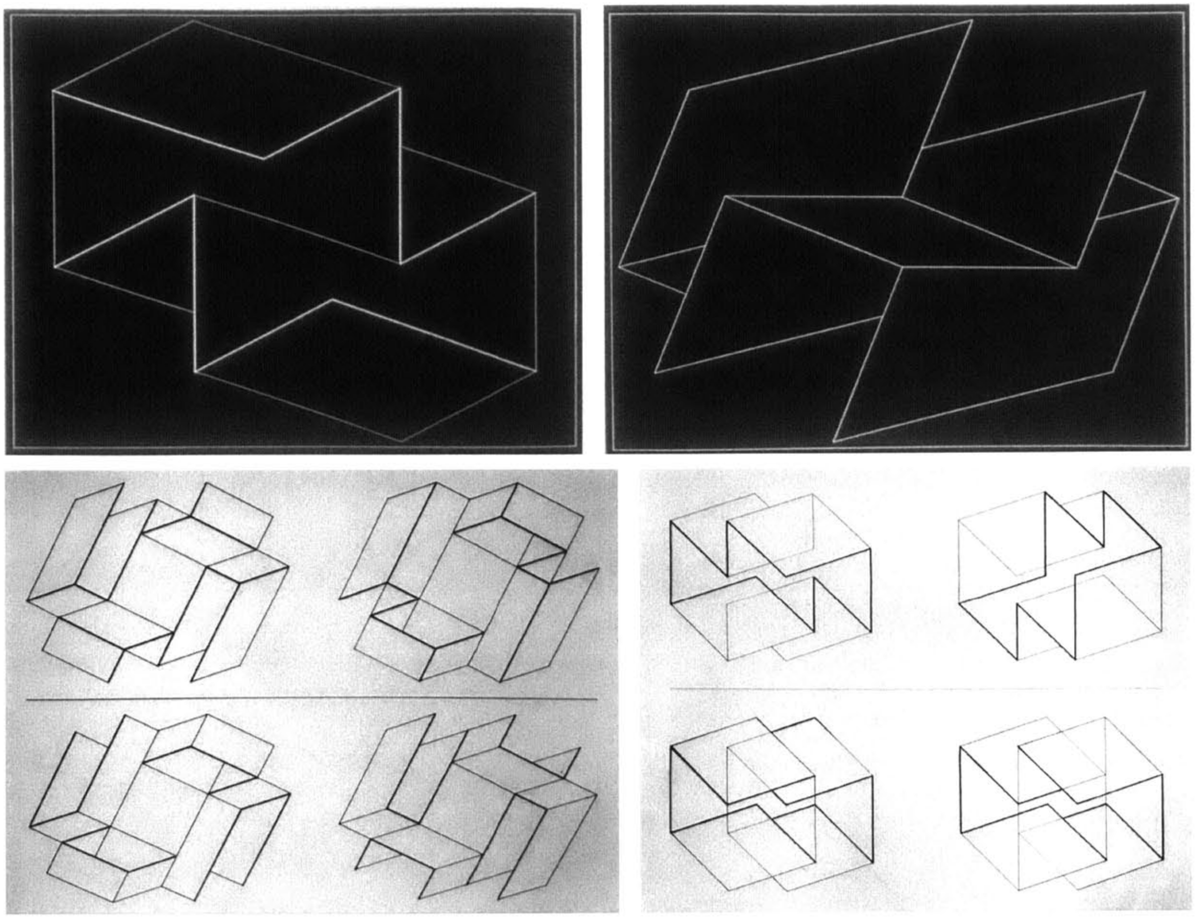


Fig 3.2.2 Lecture slides of impossible objects (1974, DAH [DAH])

the fundamental concept that a hypothetical digital computer can be designed using only NOT gates. The Boolean element NOT takes a zero or one and converts it into its binary opposite. Huffman reported [Sti 91]: 'It was totally impractical, but it was a kind of a mind exercise that showed how it could be done. I enjoy pushing things to their theoretical limits.' The choice of the title shows us that he was willing to play with humor even when faced with fundamental scientific concepts and we will see further examples that display a similarly playful attitude. His models for '4-lobed, cloverleaf design' in the taxonomy for example (Fig 4.2.19) are made of the cartoon section of a newspaper.

It is not surprising that Huffman took interest in the idea of tensegrity, a structural system investigated by Kenneth Snelson and Buckminster Fuller, where compression members don't touch each other. Fuller coined the term in the 1960s as a neologism of 'tensional integrity'. Huffman's tensegrity models consist of lines and edges similar to his previously mentioned wire models, but he needs to use compression and tension members, which expanded his material repertoire to wood dowels and fishing lines in Nylon (Fig 3.2.4 left). Huffman appreciated the aes-

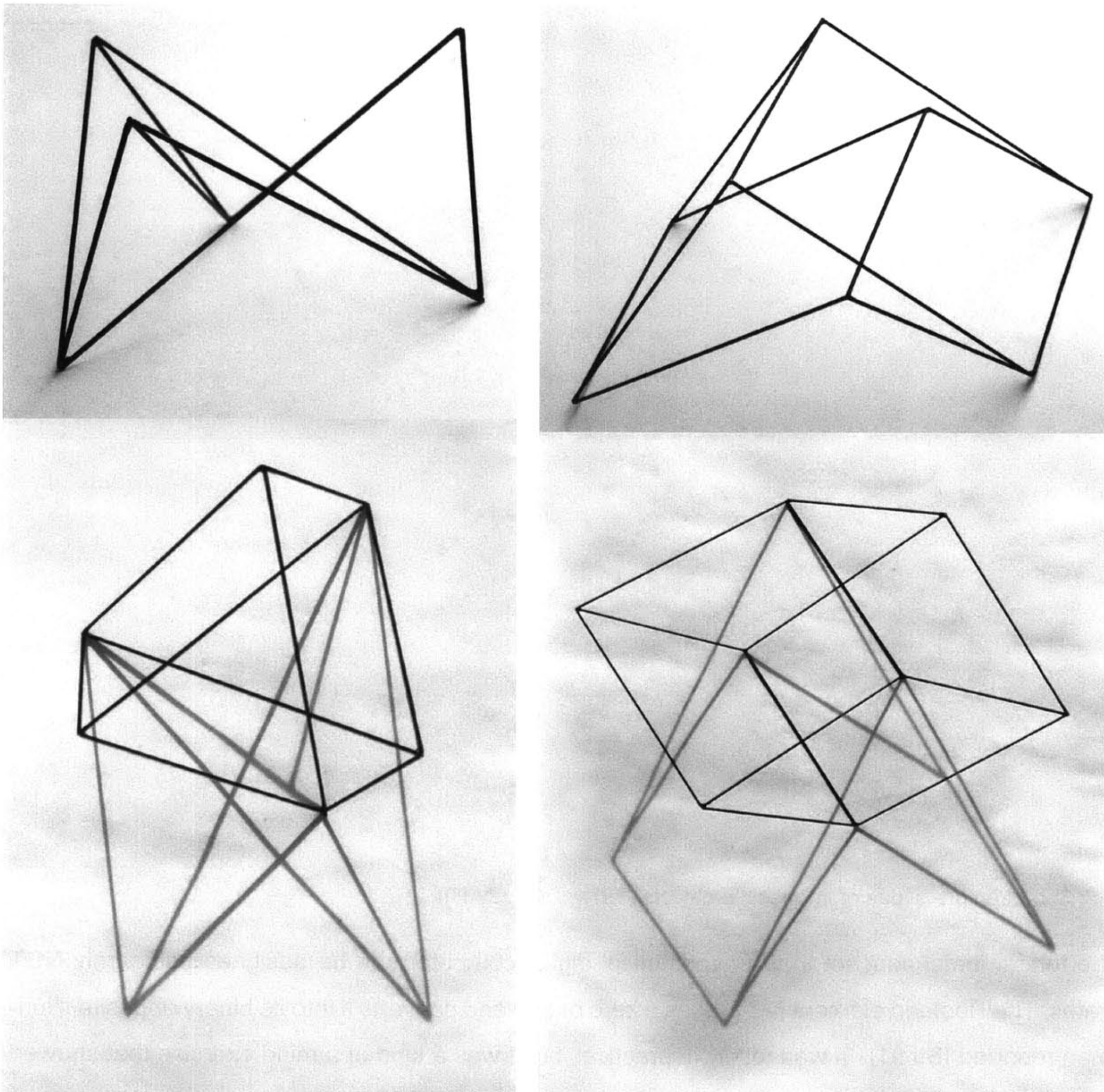


Fig 3.2.3 Wire models of impossible objects, top (1974, DAH [EAH]), bottom (1974, DAH [DAH])

thetic potential of the system and photographed a painted model similarly to the impossible wire models (Fig 3.2.4 right). This time however there was no optical illusion to be displayed.

Paperfolding and curved geometry

It is unclear exactly when Huffman started to ‘exchange paper writing for paperfolding’ as Gary Stix eloquently put it in his Scientific American article [Sti 91]. The first years in Santa Cruz might be a reasonable guess. Huffman’s fascination with folding appears to have originated with models made of graph paper that fold flat. Huffman produced a significant amount of these tiled models, which in 2013 have become a topic of further investigation [DDDR 13].

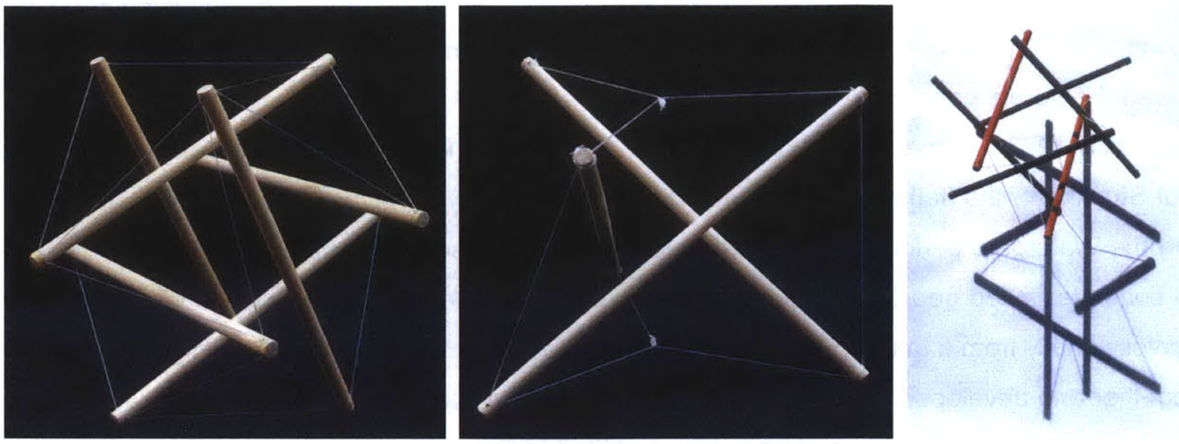


Fig 3.2.4 Tensegrity models (1974, DAH [EAH]), (1974, DAH [EAH]), (1974, DAH [DAH])

Very little dated evidence exists from 1970 to 1974, which seem to be formative years for Huffman. We can study the relationship to Richard Riesenfeld and Ron Resch, both at the University of Utah during that time, and will also consider the evolution of his academic papers and talks on paperfolding.

I mention a few wire models as they relate to surfaces and were probably made in the same time period. Huffman's slides of wire models with ruled surfaces and minimal surfaces made of highly expandable film indicate '1974' as a date (Fig 3.2.5). Huffman's daughter, however, remembers them to be from the very early 1970s, when the toy company Wham-O came out with an elastic film product. It is likely to be the material he used.

Dating the models of his Reuleaux tetrahedra made in resin and a model made of four partial cones is similarly difficult (Fig 3.2.6 left). It seems more probable that he documented his models in 1974 rather than made all of them in the same year. It is unclear when Huffman's interest in curved surfaces emerged, but the shown examples might manifest the beginnings. A well crafted wooden sculpture, also undated, might look like it could be made with curved creases, but

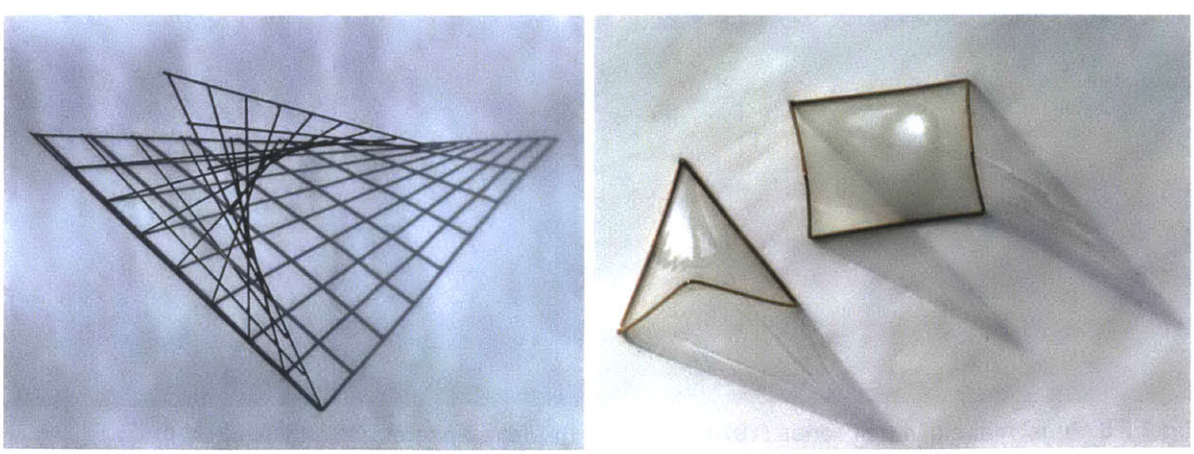


Fig 3.2.5 Wire model of ruled surface (1974, DAH [DAH]), soap bubble models (1974, DAH [DAH])

only the front half of the object can be folded in paper (Fig 3.2.6 right). The two joined surfaces along the left curved edge in the image do not alternate from concave to convex, which is not allowed if it were a paper model.

In 1973 Huffman traveled to Utah for a sabbatical leave at Tom Stockham's invitation, but Stockham, the 'father of digital recording', became very preoccupied as a tape expert during the Watergate period. Huffman befriended Richard Riesenfeld, a young Assistant Professor in computer aided geometric design, who was eager to learn all that he could scientifically and professionally from a man of Huffman's stature and achievement. They spent a good deal of time together and developed a warm friendship [Rie 14]. 'If I remember correctly, he was undergoing some stress in his marriage and he felt a bit lonely himself, I believe.' In the same e-mail conversation Riesenfled reports 'I was also alone at the time, so we enjoyed lots of 'face time', as they now say.'

Riesenfeld was working on a paper that analyzed a geometric procedural algorithm, which

Fig 3.2.6 Model made of 4 half cones (1974, DAH [DAH]), 'Tetra Spheres' (1974, DAH [EAH]), Wooden model (undated, DAH [EAH])

would benefit from original ideas, the kind Huffman would likely be able to provide [Rie 14]. 'Cleverly applied original thinking was a hallmark and theme of his research, so talking to him gave a young professor the strength and confidence to chart a somewhat different path. I felt like that more fortunate individual in the world to have this kind of connection.' he reports.

The work of Ron Resch, a gifted artist with an interest in computer science, might have been the draw for Huffman to visit Utah and there existed the basis for a marvelous collaboration between them, but no joint efforts transpired. Resch was proprietary about the area of paperfolding and did not see the power of trying to work together with Huffman. Riesenfled who lived in the same apartment with Resch provides the following insight [Rie 14]: 'Ron had difficulty in his relationships; generally he was a 'solo act', as I viewed him. Although I genuinely admired Ron's creativity and art, I could not collaborate with him because I am not an artist.' Their relationship was cordial, but pretty formal and at arm's length. Riesenfled concludes: 'There may have been a couple of tries, but nothing even got started.'

Ephraim Cohen [Coh 14], one of the previously mentioned computer scientists Resch hired with funding from an NSF grant, remembers Huffman's excitement about flat foldable models in 1973. Huffman stored several papers by Resch among his own notes including a reprint of the AFIPS Conference Proceedings, volume 42 from 1973 and the previously describes manuscript by Cohen on space curves.

The fascination with curved creases and sculptural models that can be exhibited most probably originates with Huffman's interest in Resch's art. The work depicted in a 1972 photograph of Resch's office includes a working model of 'cones kissing' on the bottom left, several 'space curve' designs and a hardly recognizable tiling with cones at the top right (Fig 3.2.7). It is very likely that Huffman saw all of Resch's work during his visit including framed tilings and precreased vinyl sheets (Fig 3.2.7). The work for 'Ron Resch and the Computer', his 1972 exhibition, might also have been back in his office. Many years later in a phone conversation with Erik Demaine in 2004 [Dem 04] Resch felt that Huffman should have attributed his accomplishments.

Huffman returned to UCSC from Utah. The year 1974 marks an incredibly prolific year as his archive contains a wealth of photographs of models, objects and geometric investigations. As the only available dates exist on Huffman's slides it is however unclear, if all the work originated in that year.

Regarding Huffman's mathematical work on curved creases we can trace his academic endeavors in terms of his lectures. The link between polyhedral objects in scene analysis and paperfolding was obvious to Huffman. His grant applications and lectures help elucidate the connection between the 2 fields. He presented a precursor to his primer on paper when he returned

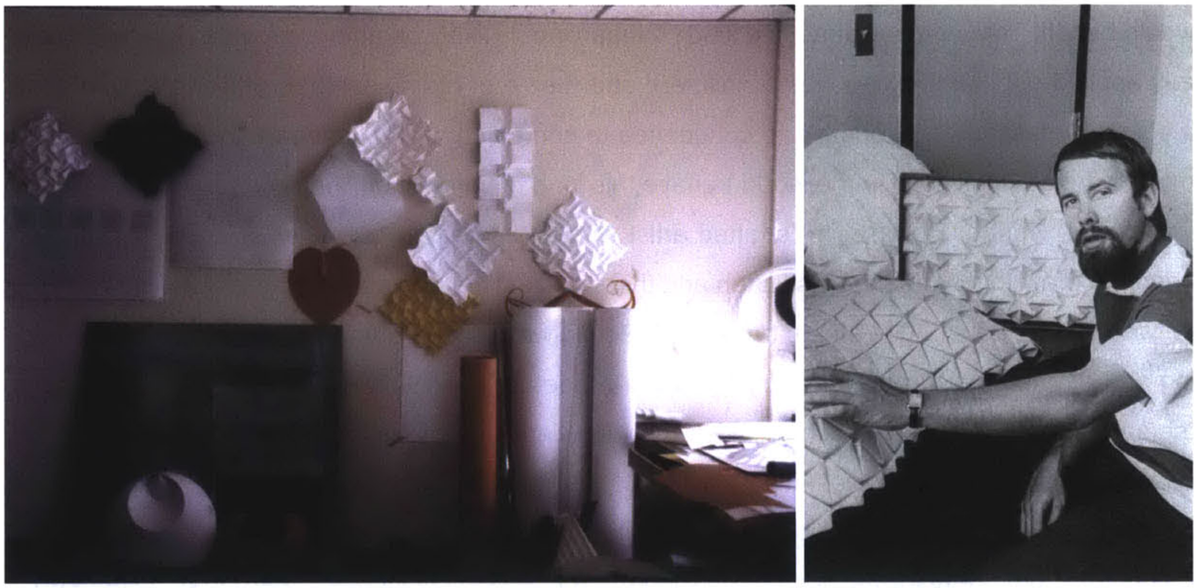


Fig 3.2.7 Ron Resch and his SLC Paper Folding Lab, (left Nov. 1972; right, undated)

to the University of Utah in May of 1974 and delivered a lecture called 'Curvature, convexity, concavity, creases, and crinkles'. Ron Resch celebrated his 'Flexible structures' exhibition on May 23rd, 1974, and showed the 'Space curve' model and his 'Tetra ball screen'. It appears likely that Huffman witnessed the work during his trip. In April 1975 Huffman lectured again at the University of Utah, this time on 'Curvature, convexity, concavity on zero-curvature surfaces'. In May of 1976 he finally presented his primer at the UCLA Extension Conference on Computer Graphics, Pattern Recognition and Data Structure called 'Curvature and Creases: A Primer on Paper'.

Huffman's NSF applications also provide background information on how curved creases and previous work relate. His paper 'Realizable Configuration of Lines in Pictures of Polyhedra' [Huf 77] quotes an NSF Grant with No GJ-28451. The final technical letter report for this grant with a starting date of July 1st, 1971, the time when he published the impossible objects work, and a completion date of June 30th, 1975 is called 'Logical Problems of Visual Perception'. Curiously, it includes Huffman's entire primer on paper as reference at the end. This occurred prior to its publication and Huffman did not elaborate on the nature of the curved crease research in the grant report.

Riesenfeld advised Huffman on an NSF grant application in a letter from January of 1975. Other documents show further desires to expand the work via NSF grant applications, which tells us that he saw curved creases to be an extension of his work on scene analysis. In March 1976 he wrote another application that was explicitly about curved creases in the context of machine vision, which he titled 'Proposal to study curvature and convexity in scene analysis problems'. He added the primer on paper in the proposal and I believe did not succeed in getting the grant.

In 1987, 12 years later, Huffman prepared a document that seems to conclude all his NSF efforts. Award No. MCS-7803822 with the project title 'The Mathematical and Physical Properties of Flexible Surfaces' with an active period from July 1st 1978 to December 31st 1987 mentions a cumulative award amount of $73,258. The document provides a summary of the relationships between his scene analysis work and curved creases:

'Lessons learned from polyhedral foldings were applied to the case of surfaces containing curves creases. For example, in a special case in which an equivalent of the PI-condition is met, the 'generating' lines that are embedded in the uncreased portions of the surface are acted upon by curved creases in a way that is closely analogous to the way that rays of light are acted upon by lenses.'

This quote represents the essence of the design method Huffman used for the majority of his models in the taxonomy in chapter 4. His use of ray reflection and refraction of lenses in optics forms the basis of how Huffman tries to predict where the rulings are located in 2d.

Exhibiting and the value of art

1978 delineates another important year for Huffman in terms of public exposure. He took part in 'White Paper 1', a group show at USCS at the Eloise Pickard Smith Gallery, Cowell College on Oct 22nd. The opening was followed by his lecture 'Plane facts about the art of paper folding', another humorous title. Among contemporary folders the late Josef Albers was listed as a participant, probably via student work similar to examples we have seen in the historical background chapter (Fig 3.2.8 top left).

An article Huffman wrote in a brochure of UCSC [Atc 79] informs us about his understanding and appreciation of art. 'As an outgrowth of that research, I've developed my own techniques for folding paper into unusual sculptured shapes.' he wrote. 'This was originally only a hobby, but recently I've exhibited my work in an art gallery. I don't claim to be an artist. I'm not even sure how to define art. But I find it natural that the elegant mathematical theorems associated with paper surfaces should lead to visual elegance as well.' Huffman's position relative to art may sound naive, but it reveals that there existed an appreciation on his part and an acknowledgement of disciplinarity, which he wished to cross, but not disturb. 'What is perhaps most exciting of all is to lecture on this work to both artists and scientists,' he continued 'and to have each group understand and appreciate what I have done.'

The question of the relationship to art reappeared in the form of several lectures and we can study his opinion on the matter by reviewing the titles. One of the shows at UCSC was called 'A Scientist Looks at the Art of Paperfolding', a lecture at the Santa Clara Valley chapter of the Information Theory Group 'Presentation on Paper Sculptures by David A. Huffman'. Further lec-

Fig 3.2.8 Exhibition at UCSC in 1978, Huffman's living room (undated, [TG])

tures at UCLA, Cal Tech, UCSC, MIT and at the Sperry Research Center were called 'The Polyhedral Flexing of Paper' or had titles he had used before. The titles indicate that he was willing to publicly show his work and also call it sculptural, not something he did prior to these experiences. We can also observe how he displays his work at home in a photograph of his living room (Fig 3.2.8 right). The titles also declare his attitude toward the spelling of the word paperfolding and the consistent use of folding rather than origami for his own work.

In the 1980s Huffman had 2 more shows, one again at UCSC at Frank and Eleanor Baskin's house called 'Art in Science' and another at Xerox PARC, Palo Alto Research Center. The group show 'Origami Art Show' featured works by Takashi Hojyo, Tom Hull, Robert Lang, John Montroll, Chris Palmer, Jeremy Shafter and Joseph Wu, some of them with backgrounds in computer science, and others invested in figural representations of origami.

Huffman very rarely ventured into the domain of figurative or representational paperfolding, but a trip to Africa in 1989 with Marilyn Homer motivated him to design the head of an elephant. He made several versions of the design and photographed them. He chose a discrete representation with straight creases to approximate a curved shape (Fig 3.2.9). It is unclear if he reverted to this method for ease of achieving the shape or if he studied a specific aspect of discrete representations. Other models in Huffman's archive carry figurative names, but they appear to be the result of a discovery or reading rather than a design goal. The family recalls situations where Huffman would ask what a design looks like and the discovered visual reading would eventually

Fig 3.2.9 Elephant (1989, DAH [DAH]), photo album and itinerary [DK]

determine the name of a design. Such examples include the 'Bird skull', the previously mentioned 'Elephant', the 'Icing on wedding cake' and 'Arches'. Huffman's regular naming convention tends to result in names such as 'Rectangular woven design' or 'Four sections of polyhedral folding'.

In order to investigate Huffman's opinion on the value of his work we can inspect a list of his models, which he compiled in 1978 for the purpose of assessing an insurance amount for his exhibition. The models' values ranged from $350 to $900 and he used the same list in his divorce case from Jane Huffman in 1979. It contains notes on which models have a dollar value and which ones do not. The notes include descriptions on craft and completion. We can clearly observe that Huffman had a value system in place regarding the worth of his work outside of its academic achievement. It also presents an attitude regarding his sense of completion for individual vinyl models.

Photographing

Further evidence of Huffman's sense of aesthetics can be produced via a survey of his photography. He witnessed Resch photographing his own work in Utah, which I am sure was important to Resch as it would be for any artist (Fig 3.2.10).

Huffman produced a small but compelling body of work with his 35mm camera and kept mostly slides. He often used only one light source and produced high-contrast and fairly abstract imagery using his own vinyl models (Fig 3.2.11).

In addition he explored several different effects in his photography and the 3 shown photographs serve as case study (Fig 3.2.12). The left consists of a high-contrast image of a straight crease tiling with the light source coming from the left. The middle example displays a low-contrast detail of one of his discrete domes taken from above. For the image on the right he produced a

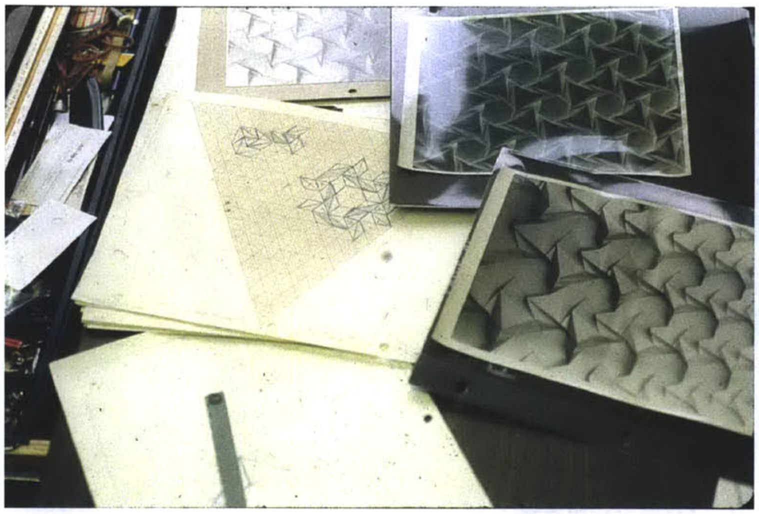


Fig 3.2.10 Black and white photographs of Resch's tilings (probably early 1970s)

dramatic perspective of one of his raised triangle models. It is generally unclear which orientation the photographs are supposed to have, if any.

Huffman examined the visual qualities of his paperfolding work and experimented with photography to explore his options, a clear manifestation of someone who was interested in artistic production and its aesthetic impact. The style he used most are his high-contrast photographs. In some cases, they result in highly abstract images that don't necessarily convey the geometry of the paperfolding in a neutral way. Where applicable, the following taxonomy chapter will use Huffman's own photographs in order to show the reader how Huffman saw his work.

Making objects

The last few examples of Huffman's creations in this chapter consist of various puzzles. He kept a letter by John Munson, Senior Research Physicist in the Artificial Intelligence Group at SRI from May 1971 among his correspondence. The letter is addressed to Martin Gardner, the editor of Mathematical Games and renowned recreational mathematician, and informs him of Munson's work on the problem of the number of ways to fold a packet from (a) a strip of rectangles, such as postage stamps in 'coil' form, or (b) an m-by-n 'map' of such rectangles. Huffman also saved Martin Gardner's puzzles of the 'Mathematical Games' pages from Scientific American between 1971 and 1976. Another letter to Gardner by W.A. Youngblood, most probably a colleague of Huffman's, mentions an earlier solution from 1961 that is related to Huffman Coding.

His passion for puzzles resulted in several physical riddles he made himself. The brain-twisters speak to a side of his leisurely activities, which he most probably engaged in with great pleasure. He used a variety of materials to create his knot puzzles, among them are wooden beads, PVC piping and a recycled egg-shaped container used by 'L'eggs' in the 1970s and 80s

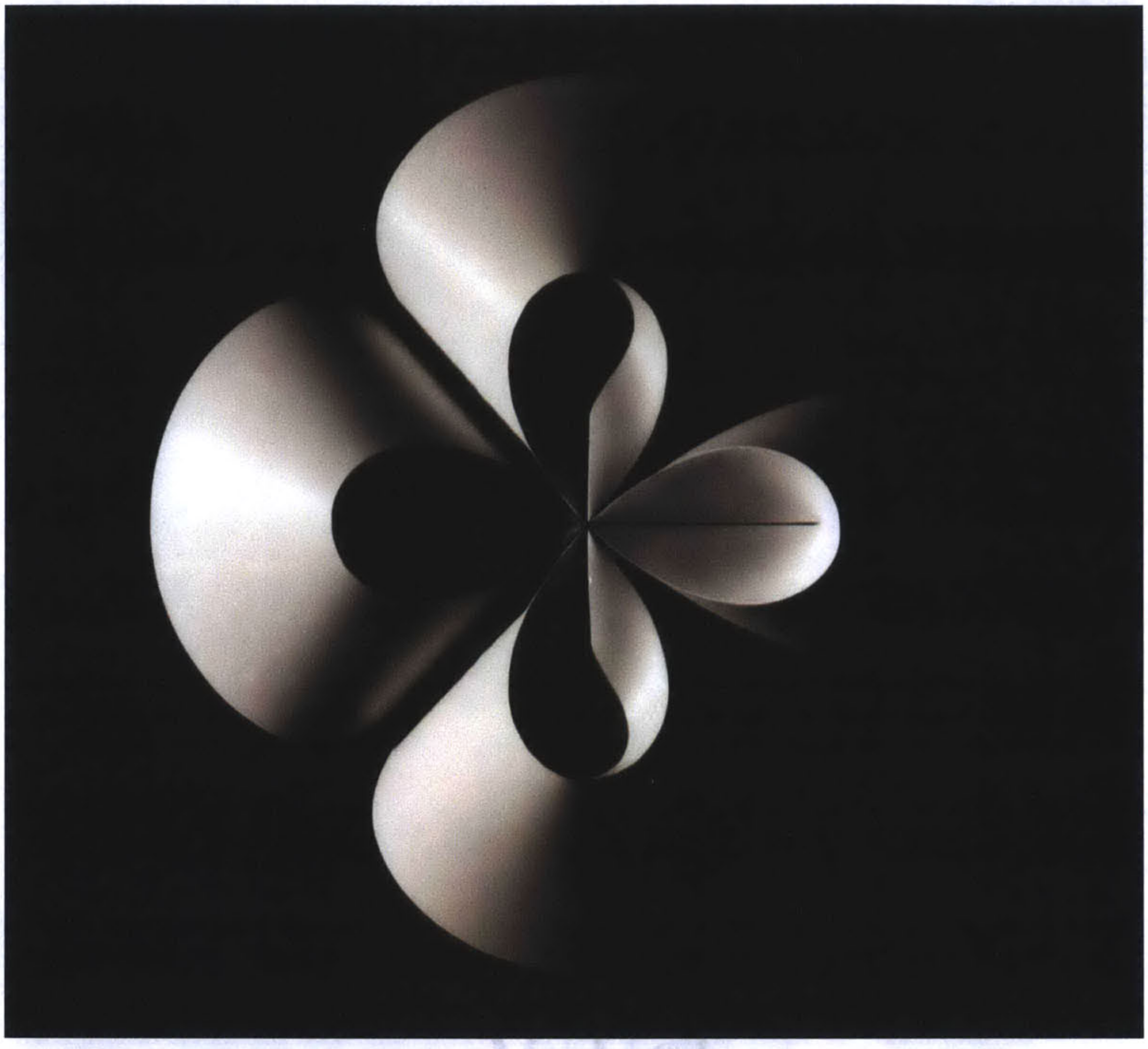


Fig 3.2.11 '4-lobed, cloverleaf design' (probably 1977 DAH [DAH])

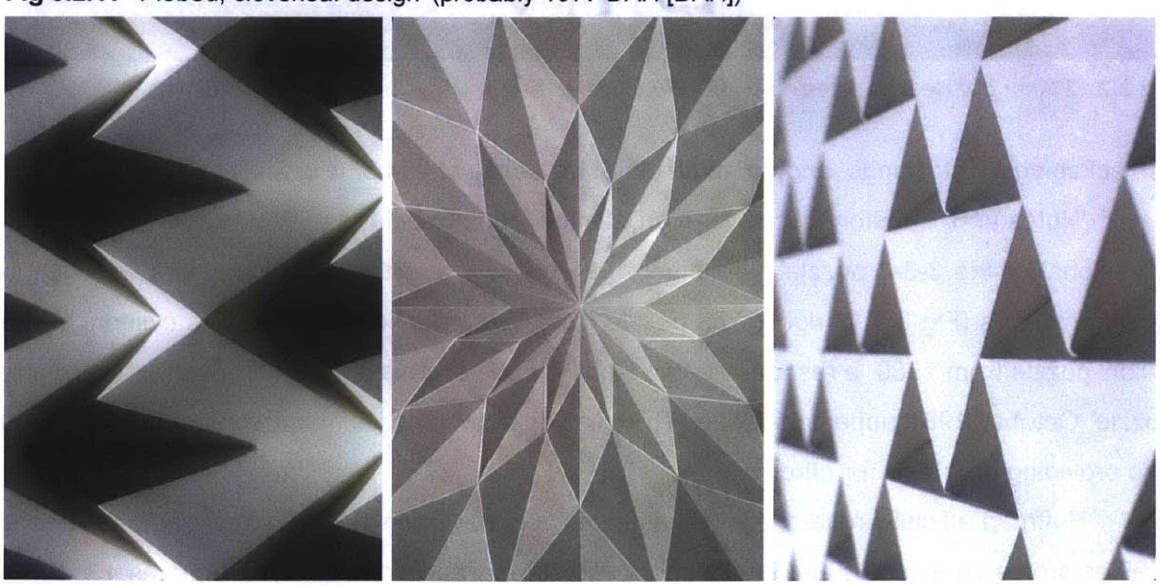


Fig 3.2.12 Detail photographs of straight crease tilings (1978, 1978,1977, DAH [DAH])

Fig 3.2.13 Knot puzzles (undated, DAH [EAH]), puzzles with marbles (undated, DAH [EAH])

as packaging for stockings; another example of his sense of humor (Fig 3.2.13 left).

'Notes and Transformations for my Balls-in-Cube puzzle' from October 1981 and 'Transformations for the 3-3-3 puzzle' November 1981 relate to acrylic glass containers with interior moving marbles (Fig 3.2.13 right). The puzzle displays similarities to Nintendo's cylindrical 'Ten Billion' puzzle from 1980, a response to the Rubik's cube and his notes of 'Solution of the 'Barrel Puzzle' October 1981 appear to relate. The puzzles show that Huffman reveled in solving, but also providing problems for others.

Huffman officially retired from his academic endeavors in 1994, but remained active until 1999 as professor emeritus and taught information theory and signal analysis courses. His retirement from folding paper is hard to estimate as he rarely dated notes. On October 7th 1999 Huffman dies at the age of 74 after a 10 month battle with cancer. The image of Huffman's home office (Fig 3.2.14), taken right after he passed away, testifies to a three decade long passion for making objects and paperfolding.

Designing

In conclusion to this section and as a segue into the taxonomy I would like to point out several observations about Huffman as a creative person.

Huffman looked at, analyzed and re-created or re-built examples as a form of investigation or learning. Copying can be creative and lead to new things and certainly is common practice in

Fig 3.2.14 Office in Huffman's home (1999, [TG])

science as every experiment has to be reproducible. In his mind the attribution of artistic references or inspiration might not have been necessary, which does not match his academic standards. His daughter Elise describes him as an excitable guy, someone, who was quickly inspired by a problem he was confronted with.

It remains unclear why Huffman was shy to publish his paperfolding work. He appears to have been very private about his work, which he pursued in solitary places and times. He received a letter from a colleague to alert him about the potential value of a patent, but it is not clear, if that could represent the main motivation.

He was, in part, a product of MIT's computer science culture that promoted solving problems with a 'divide and conquer' approach, the breaking down of a problem into smaller, possibly its smallest parts, solving those first and then reassembling all parts to solve the initial challenge. Regarding curved creases, he displayed a bias toward creating a design via the use of algorithms or step by step instructions that follow preset constraints. The 'base case' would be a first assumption that forces the paper into a specific configuration. The design would be the result of an algorithm that follows the constraint propagation set up by ray reflection. I elaborate on this interpretation in the final chapter.

Beauty for him is intrinsically related to the mechanism or algorithm that created it; in other words arbitrariness has little potential for beauty. Huffman rarely produced designs without at least one axis of symmetry, which results in a specific kind of aesthetics.

While he produced hundreds of artistic sculptures, he probably would not have called them art. He did not think of himself as an artist and did not elaborate on any general artistic goals or motivations. However, he was also not shy to call his work sculptural.

3.3 DAH, mathematical models as tools

In order to elucidate Huffman's way of working and designing I elaborate on the mathematical models he relied on as they can be seen as his conceptual tools.

Catalog of curves

Huffman used conic sections or conics for many of his designs and established a catalog of curves for himself that mostly consisted of variations of the quadratic curves. The motivation for using conics has to do with Huffman's interpretation of ray refraction for rulings, which I will extensively elaborate on in the taxonomy chapter.

The physical phenomenon of lenses that redirect light began with the ancient Egyptians and the development of glass. The curves themselves date back to the ancient Greeks, who defined the conics to be the ellipse, the parabola and the hyperbola. They were first studied by Menaechmus around the years 360-350 B.C., then by Aristaeus, Euclid and Archimedes. Appolonious of Perga, a 3rd century B.C. Greek geometer, wrote the treatise 'Conics' and was the first to show how all three curves, along with the circle, could be obtained by slicing the same right circular cone at continuously varying angles. The ancient Greeks also described the propagation of light in the abstract form of rays.

No important scientific applications were found for them until the 17th century, when Kepler discovered that planets move in ellipses and Galileo proved that projectiles travel in parabolas. Huffman's use of the curves for paperfolding represents a curiously novel utility for a kind of curve that has served in science for a long time. The combination of 'knowing where the curve is' and also 'knowing where the rays are' must have compelled Huffman to the creative use.

He wrote many notes on the curves for his own reference, which he kept in his collection of index cards (Fig 3.3.1 left). The larger sketches show how he scaled parabolas, a technique he used in a few designs (Fig 3.3.1 top right). A combination of the 3 curves served as comparison of the slopes of their tangents in specific points (Fig 3.3.1 bottom right).

A hand drawn sketch, something rather rare in his archive, shows 2 parabolas and the rulings he had in mind (Fig 3.3.2 left). The combination of curves and lines, turned into creases and rulings, allowed him to predict several aspects of the folding behavior. I call these sets of curves and lines 'gadgets' and will define and explain them in the taxonomy chapter.

Spirals also appealed to Huffman and I present his repertoire of converging curves at the very end of the taxonomy (Fig 3.3.2 right). The curves did not provide him with the same reflection properties and he thus could not predict where the rulings would be. This is also the reason

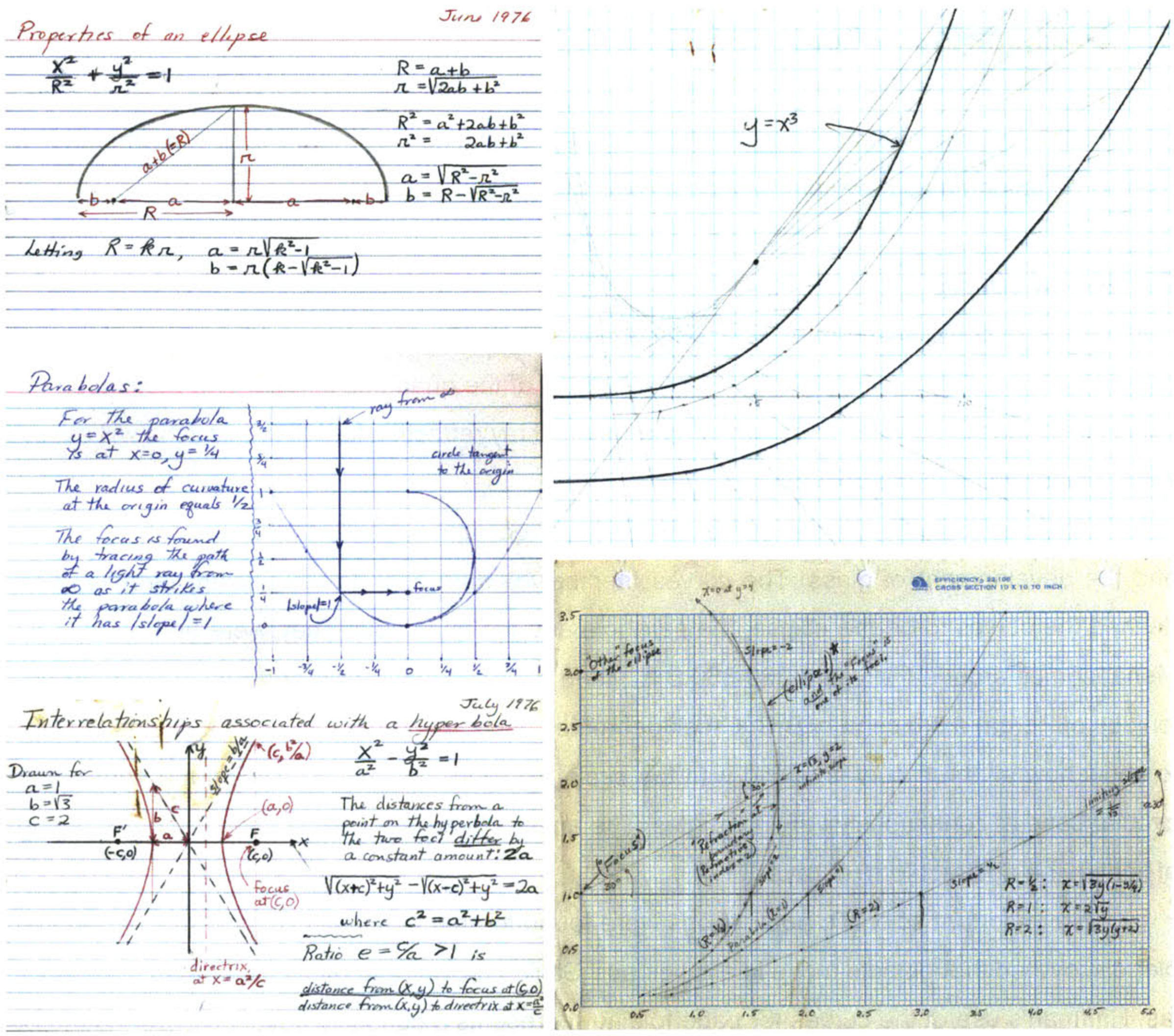


Fig 3.3.1 Conic sections, scaled parabola and a combination of curves (some from 1976 DAH [DK])

why I can not provide any gadgets for the designs that use converging curves. The spiral shown here relies on polar coordinates, which explains Huffman's choice of plain paper versus graph paper. He defines the relation of the curve as $r = \Phi$. He used a few spirals that when used on an appropriate coordinate system, have an angle preserving qualities.

Curve Plotting

Since Huffman did not use a computer for any of his work, he was confined to a pen and a calculator. Incrementally plotting the points of the curves he wanted to work with on graph paper gave him the control he wanted. The examples of an ellipse and a hyperbola testify to the practice (Fig 3.3.3). One can easily imagine that Huffman took a liking for graph paper and integer coordinates as he drew hundreds of curves for his explorations. In some cases he explored variations of a design by altering the graph paper units of the curves he used.

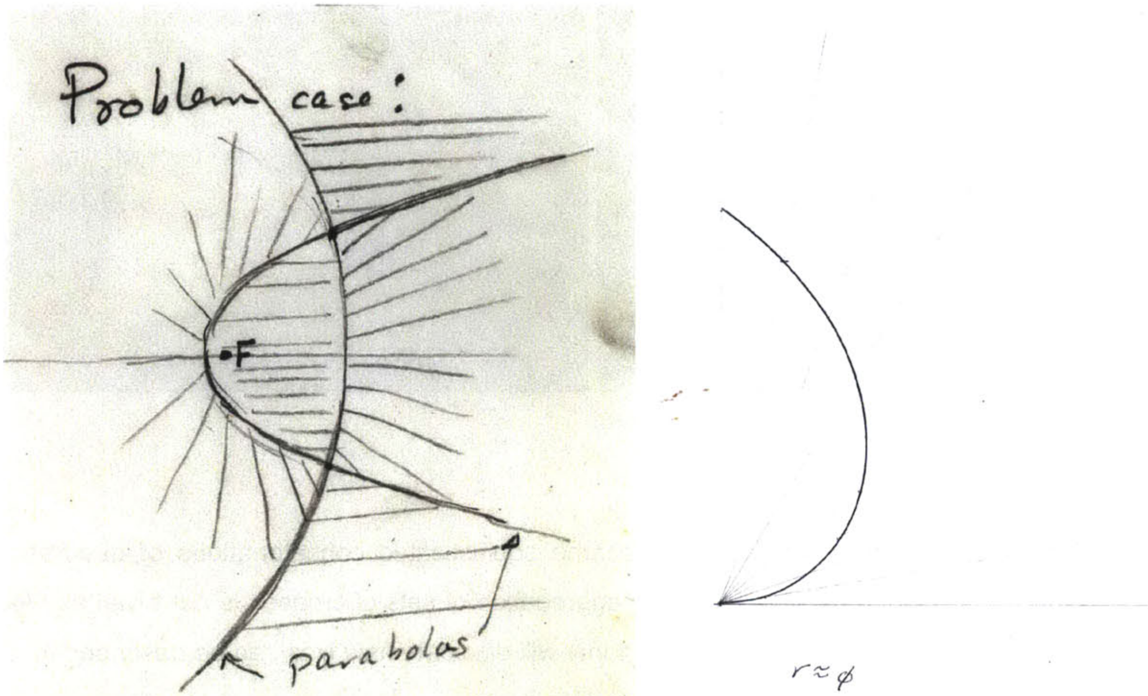


Fig 3.3.2 Sketch (undated, DAH [DAH]), Drawing (undated, DAH [DAH])

Discrete curves

In one of his grant applications Huffman investigated a discrete version of a continuous design. The polygonal approximation of a curve is a technique Huffman used to verify specific aspects of crease patterns. The 2 shown examples stem from an NSF proposal and consist of parabolic curves and 2 different discrete approximations (Fig 3.3.4). Discrete representations of curves may have felt natural to Huffman, as he was a computer scientist and computer science relies heavily on discrete representation. The cases he thoroughly examined with discrete approximations were cyclic tilings, with discrete spirals similar to the design displayed here (Fig 3.3.6 right).

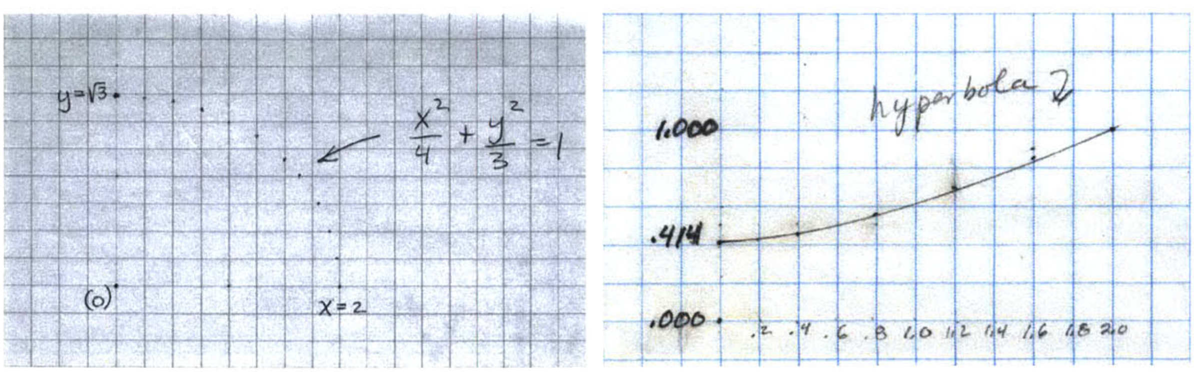


Fig 3.3.3 Sketches of curve plotting on graph paper

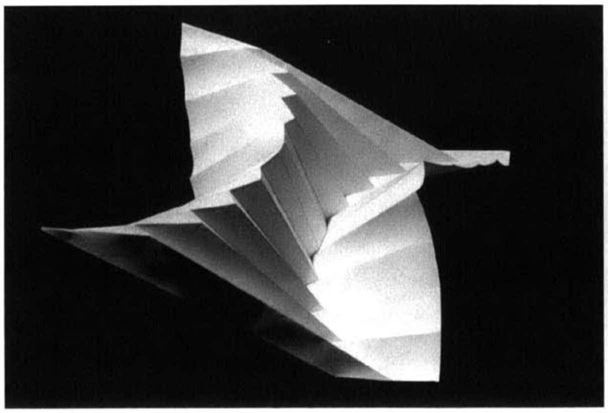
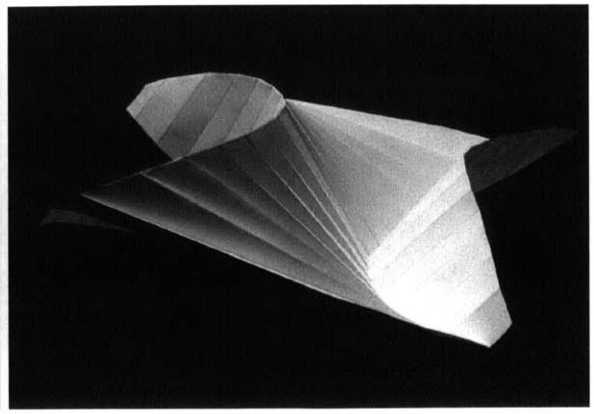



Fig 3.3.4 Models in NSF proposal (1977, DAH [DAH])

Tiling

While some of the curves Huffman used became sophisticated concatenations of quadratic curves, his tilings remained fairly orderly. The aggregation of sets of creases is not trivial as the edges of the set can be ambiguous. The taxonomy will elucidate how tiles can be described in a way that is conducive for curved crease paperfolding.

Huffman deploys 2 kinds of tilings, regular and cyclic. The 2 examples in the figure consist of regular repetitions (Fig 3.3.5). In the case of the left example we can think of one 'column' as a mirrored tile along a horizontal axis and in the right example the tiling, also monohedral, repeats in a staggered way.

The next 2 examples consist of cyclic tilings, one with curved creases and the other with the previously mentioned discrete spirals and a hexagonal single tile at its center (Fig 3.3.6). We can think of the discrete spirals that rotate in the other direction as abstractions of rulings.

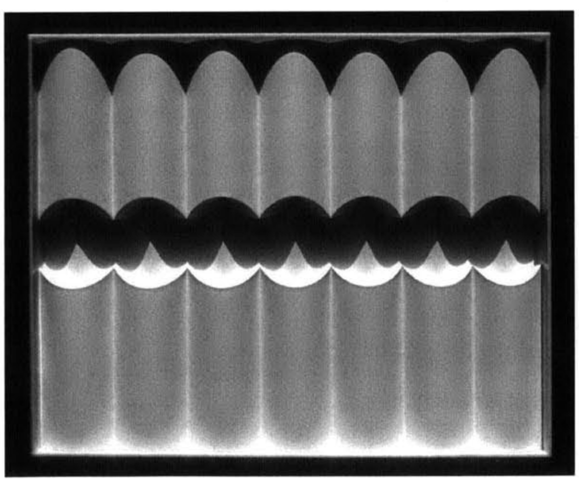
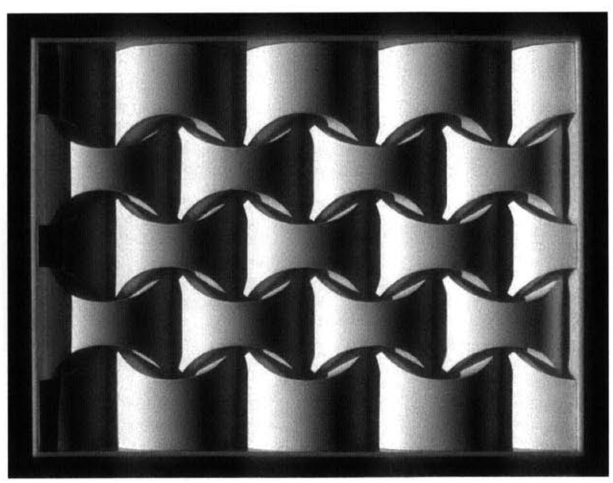



Fig 3.3.5 Tilings (1974, DAH [EAH]), bottom (1974, DAH [DAH])

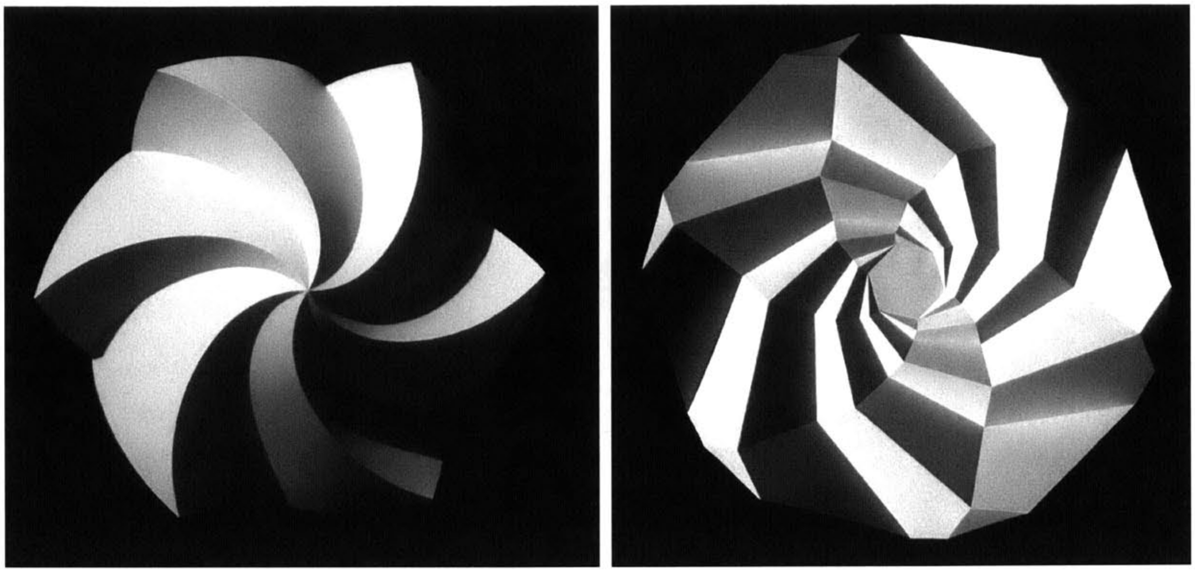


Fig 3.3.6 Tilings (1974, DAH [EAH]), bottom (1974, DAH [DAH])

3.4 DAH and his tool drawer

Fig 3.4.1 Drawer of Huffman's drafting table [EAH]

Similarly to the way Huffman's conceptual tools elucidate his thoughts on designing crease patterns, his physical tools can elaborate on his attitude toward craft. His pristine vinyl models required him to establish a precise work flow with an appropriate set of tools (Fig 3.4.1).

The drafting table

The terminal equipment inventory for NSF grant No. MCS75-12814 mentions 4 items that Huffman put to use for a very long time. The Plan Master Model 350 TW, a drafting table with a wooden top

Fig 3.4.2 Drafting table with drafting machine, desk lamp and chair [TG]

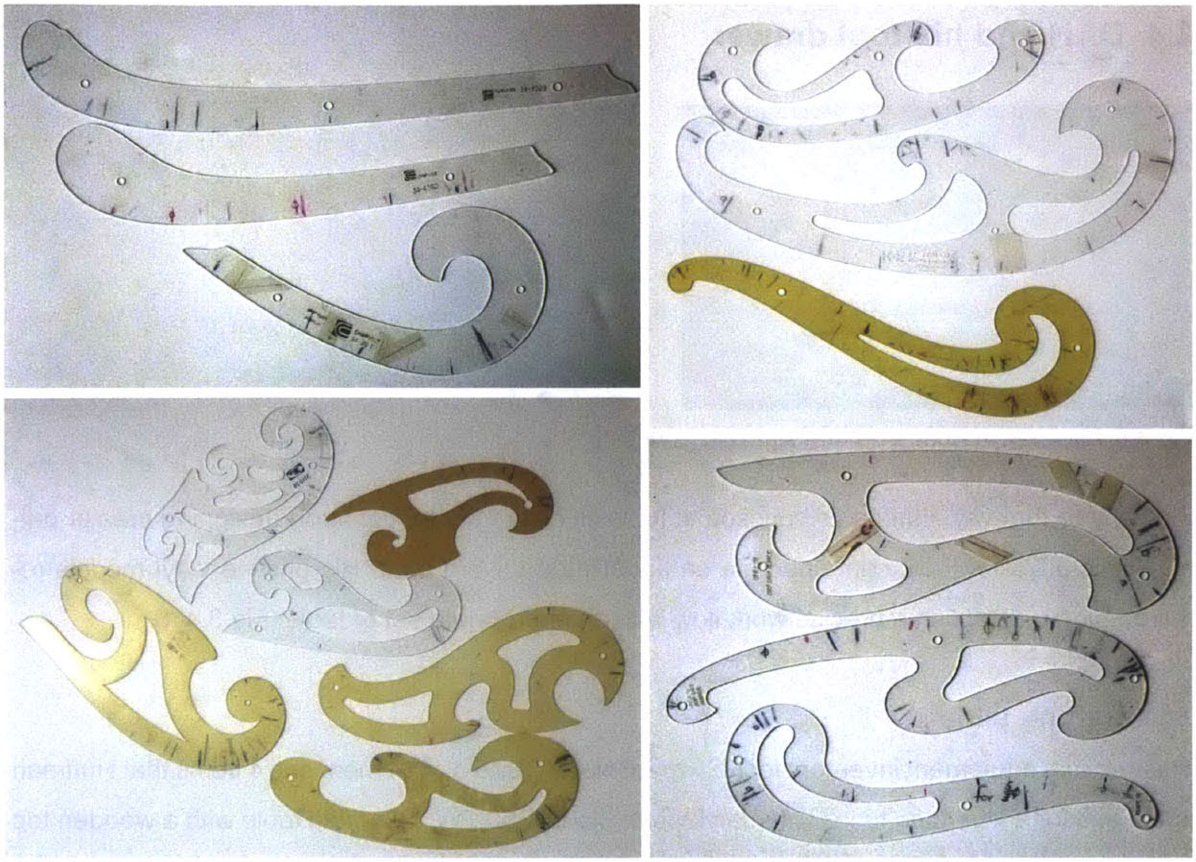


Fig 3.4.3 French curves sets with Huffman's markings [EAH]

covered with soft vinyl was the first item on the list. The K&E Auto-flow Mark II Drafting machine mounted on the left side and a LUXO adjustable lamp mounted on the right were the next essential auxiliary items. Lastly an Interroyal adjustable chair accompanied the set (Fig 3.4.2).

French curves and custom templates

Huffman owned several sets of French curves still available today. The sets were popular until the 1980s for designers in general and some sets were specifically tailored for fashion designers. He

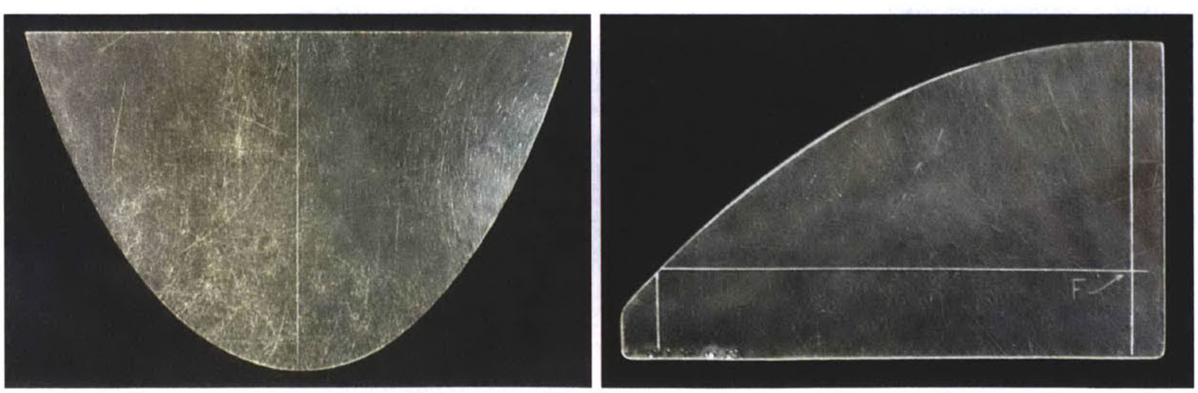


Fig 3.4.4 Custom made templates [EAH]

obviously used these very often and it is remarkable to see how many subsections he marked on his acrylic curve sets (Fig 3.4.3).

Huffman also made his own templates for curves he used frequently such as the displayed parabolic curve templates. He made one version in glass fiber reinforced plastic with a scored center axis (Fig 3.4.4 left). The other template was cut out of a sheet of Plexiglas (Fig 3.4.4 right).

The ball burnisher

In a text Huffman wrote for one of his exhibitions at UCSC he says 'The only tools I use are ball burnishers to imprint the network of creases on the vinyl paper surface and templates.'

The ball burnisher, a spring loaded adjustable tool, specifically developed by X-Acto for rub-on lettering by the Letraset company, enabled Huffman to master the craft of vinyl folding (Fig 3.4.4). The tool was commonly used in the 1970s to transfer printed patterns onto plans was adapted by Huffman to pre-crease his white vinyl sheets. He used ball point pens in a similar way in order to pre-crease his paper models.

Folding aids

Huffman made some of his own tools out of recycled materials and converted several margarine container cut-outs to folding-aids. The nifty tools featured a variety of radii along their edges in order to facilitate gradual creasing must have incurred substantial use over the years (Fig 3.4.6).

Huffman described the process of making his vinyl models: 'The individual creases are then slowly and patiently deepened, a little at a time, while the proper relative angles among the creases are maintained. Finally the sculpted surface must be fastened to a frame or backing to preserve the desired conformation.'

Huffman's compass sets included a channel beam bar compass by Alvin. Finally his many

Fig 3.4.5 X-Acto Letraset Craft Burnisher [DK]

Fig 3.4.6 Folding aids made of recycled margarine containers [EAH], Compass set and knives [EAH]

scalpels and knives fully equipped the tool drawer that was attached to his 350 TW (Fig 3.4.6 right).

4. A taxonomy of DAH's curved crease paperfolding work

Huffman's investigates several different areas of geometry and I present the work in the form of a taxonomy for ease of comparison. The goal consists of abstracting his expansive work into small groups and series of variations. The work is also presented and evaluated in terms of aesthetic considerations. The introduction of this chapter establishes terminology and definitions for the following taxonomy and elucidates the nature of the archival material I present.

Archival material

Huffman's paperfolding work with curved creases consists of hundreds of notes and models, some of which he decided to turn into well-crafted artifacts, often made of white vinyl sheets. A small group of these models have been exhibited, but most of his folding work has not been publicized prior to this dissertation [DDK 11]. Erik Demaine, Martin Demaine and me are very fortunate to have been given access to all of Huffman's work by his family (Fig 4.1).

I have scanned most of Huffman's notes and have established a notation system that relates every digitized file to its current location in the family's estate. Future archival work at the MIT Museum should benefit from the digital archive I have created.

Huffman's vinyl models stand out as the most prominent artifacts of the collection and he includes only a few dozen in his exhibitions. If a design is made in vinyl, I call it a 'vinyl model' in the taxonomy. Huffman draws and often folds a crease pattern into its 3d configuration, but not always. If he has folded a crease pattern, which I can determine by visually examining a high-resolution scan, I call it a 'paper model'. This is significant as we can determine that he is curious enough about a design to fold it. If he omitted this step, I call it a 'drawing'.

Huffman photographs his own work and the images indicate how he might have seen his own work. He may not have excessively documented all his models, but I would argue that he values the aesthetics of a design when he takes the time to photograph it. He extends his evaluation when he decides to also photographically document the making of a model. Some of these photographs display the crease pattern on the rear side of the vinyl model. The lens distortion of his camera makes reconstructions of such models difficult, but the images provide valuable information in terms of the process of making.

Huffman also uses 'index cards' or library cards for his notes. He collects them in two green archival cardboard boxes, one that contains notes on folding, and a second one with notes on his courses. Both boxes show signs of heavy use beyond other artifacts. The crease patterns on index cards appear to be notes to himself, a sort of selection of the most salient discover-

Fig 4.1 Archival material (2010, DAH [EAH]), M. Demaine at Huffman's house (2012, [DK])

ies. He also occasionally includes small examples of flat-foldable designs. If a crease pattern is documented in this format, it most probably served as a record. It might have had the purpose of a folding instruction rather than being an initial sketch. He rarely dates his drawings, but is more diligent in dating index cards, and they thus provide opportunities to understand when he investigates which topic.

The distinction between the terms 'note' and 'sketch' in the taxonomy is vague as both can occur together on an index card or piece of paper. If a crease pattern occupies most of the page, I call it a 'sketch'; if notes and mathematical derivations prevail on the page, I call it a 'note'.

Many artifacts in the collection go through several steps of documentation and digitization prior to appearing in this dissertation. I attribute the author of the artifact as well as the author of the photograph, for example, and expand this convention to the person who scanned an artifact. This allows the reader to distinguish between archival material, documentation, analysis, reconstruction and simulation. A model designed and made by Huffman could have been photographed by his daughter, for example. The analysis of a sketch might start with scanning, continue with drawing a crease pattern as a vector file and end with simulating a discrete version of the crease pattern.

Figure captions use the name of a design in quotation marks if Huffman named the design himself, which he rarely did. The parentheses include the year of the original design, followed by the author of the archival artifact. The square brackets designate the author of a photograph or digitized file. The authors and their initials can be found at the end of this introduction. The following example describes a sketch drawn by Huffman (DAH) in 1971 and scanned by me (DK):

Fig 02.11 Sketch (1971, DAH [DK])

List of authors and their initials:

DAH	David Huffman
EAH	Elise Huffman, Huffman's daughter
TG	Tony Grant, photographer
MM	Matthew Mulbry, photographer
PH	Paul Haeberli, computer scientist
DK	Duks Koschitz
PC	Pauline Caubel, student of DK
AH	Ashley Hickman, student of DK
JH	Jackie Hsia, student of DK
UP	UnJae Pyon, student of DK

Huffman names some of his models, but he seems to have not adhered to any specific convention. He creates a list of his vinyl creations in 1979 for an exhibit at UCSC and documents the event by way of taking photographs. The list and the documentation of the event provide opportunities for dating some of his models. The list also serves as inventory in his divorce case and gives us clues on value and completeness of a design. The taxonomy, however, does not make further use of his naming convention.

Most archival material is shown as a complete artifact, but in some cases I crop drawings and sketches in order to highlight a specific aspect for discussion. This occurs when grouped examples within a category require parametric definitions, for example. His photographs might have much larger black or white backgrounds, which I cut out in order to focus on the object. As altering archival material may violate common practices of using such material I have included an image directory at the end of the dissertation that refers to the locations of every archival scan.

The taxonomy

I choose to call the following structured collection of Huffman's paperfoldings a 'taxonomy' as I divide his work into ordered groups or categories. I classify by surface geometry, curve type, complexity of crease pattern, and tiling. The main meta-categories of the taxonomy follow geometric principles of paperfolding: **I Reflection, II Ruling Refraction, III Forced Rulings** and **IV Converging Curves**.

Each of these meta-categories (I to IV) span across numbered sections with subsections. The numbered sections follow specific crease patterns, some of which are grouped in the same meta-category. The relationship is represented in the table of contents.

Each numbered section starts with a description of the most general version of the operative crease pattern, which I call a 'gadget' and define later in this introduction. Sections and subsection are defined by curve type, number of curves in the gadget and increasing complexity of the gadget. Types of aggregations, so called tilings, can also operate as ordering principle in the taxonomy. Tilings are explained later in the introduction .

Curve types that structure the taxonomy are introduced in the following sequence: ellipses, parabolas and then hyperbolas. Circles play a special role and are only used in a lower hierarchy, which means that a section on parabolas, for examples, begins with a focus on parabolic curves and then segues into a subsection that uses parabolas and circles in combination. Circles are therefore called 'non-dominant'.

Definitions of crease pattern, ruling and gadget

A folding diagram that gives instructions on which lines needs to be folded and in which direction can be defined in many different ways. Given a set of line segments and curves on a flat piece of paper, each assigned a folding direction of mountain (m) or valley (v), the **crease pattern** is the paperfolding diagram that consists of the line segments and curves in the flat state, which become creases when folded. The notational conventions include names of vertices that play a specific role in the crease pattern (Fig 4.2).

A **rule line** is a straight line segment in the 3d folded surface. The theory of developable surfaces in differential geometry says that every noncrease point lies on a rule line. Together these rule lines form the **ruling**. The rule lines are drawn as series of straight black lines in a crease pattern (Fig 4.4).

Huffman uses the refraction properties of conics. The curves refract rays in a converging or diverging way relative to its focus. The rays, the straight line segments Huffman interprets as rule lines, point to or away from a focal point on one side of the curve, and change direction once they pass through the curve (Fig 4.3). He draws the 6 main cases of conics on the upper index card and combines several curves on the lower index card.

p	parabola
p'	confocal parabola
h	hyperbola
s	spiral
C	center of circle
E, F	foci of ellipse
P, P'	focus of parabola
H, J	foci of hyperbola
A	cone apex
f	distance between foci of an ellipse
a	distance from focus to curve on major axis
d	distance from focus to center line
o	distance to center or from curve to a point
l	distance on latus rectum (line through focus perpendicular to major axis with both endpoints on curve)

∘	common tangent point
——	mountain
----	optional mountain
----	tile edge on mountain
——	valley
----	optional valley
----	tile edge on valley
——	rule line
——	paper edge
——	cut paper edge
----	tile edge on ruling
-.--.	tile edge across ruling
——	construction line
——	design tile
	angle in degrees
	perpendicular
	parallel

Fig 4.2 Notational conventions for crease patterns [DK]

A **gadget** is a subset of a crease pattern consisting of a set of straight and/or curved creases together with a defined behavior of the rulings on either side of all creases in the set (Fig 4.4). A gadget that uses parallel rulings or rule lines that intersect in one vertex is called a **reflection gadget** (Fig 4.4 left). The resulting surfaces are cone or cylinder reflections, which are introduced in section 4.1 and 4.2. A gadget that uses the refraction properties of conics is called a **refraction gadget** (Fig 4.4 right). If a gadget only uses one curve, I call it a **single curve gadget.** A gadget often serves as prototile of a tiling and I will elaborate on this issue later in the introduction.

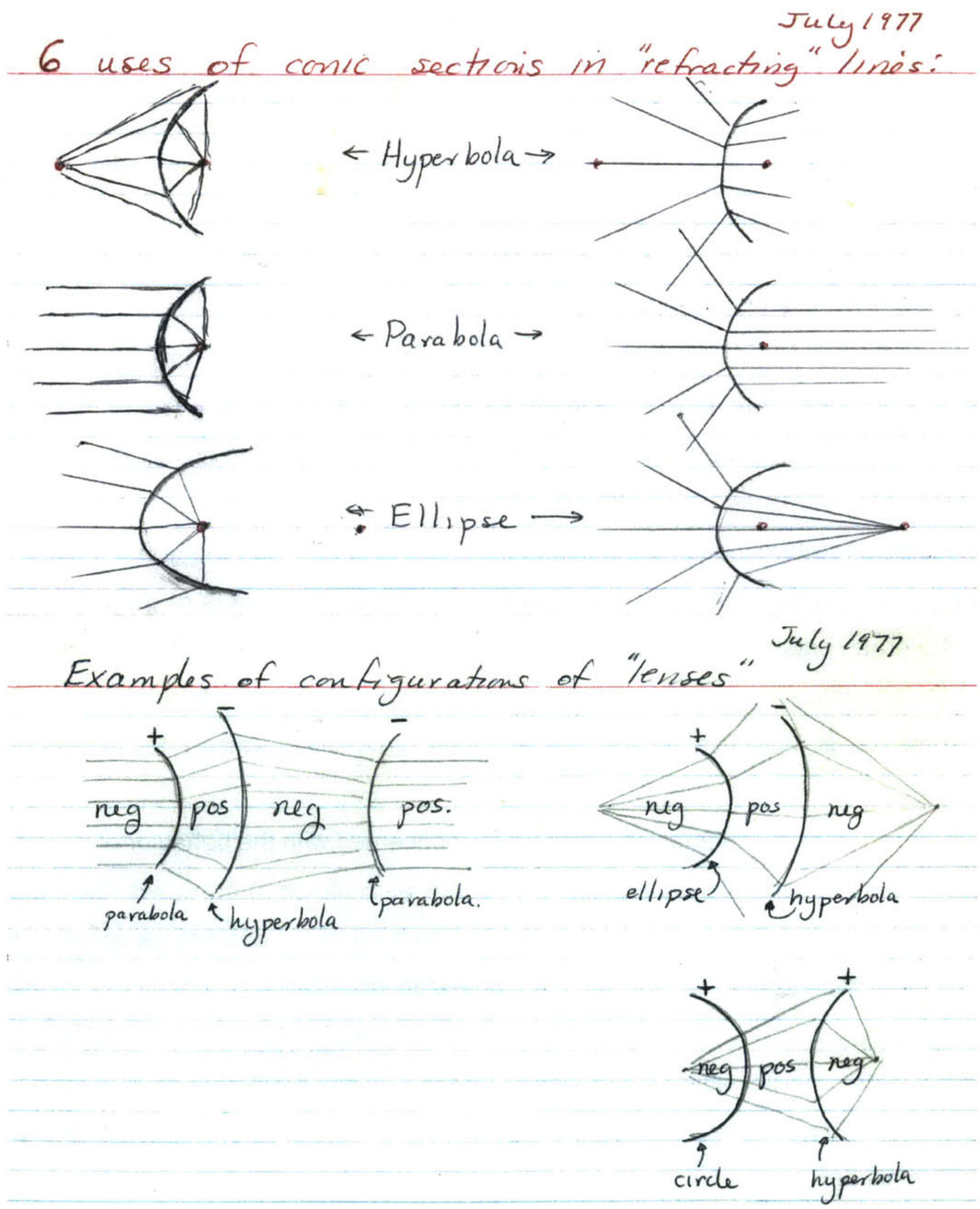


Fig 4.3 Index cards with ray refraction examples (1977, DAH [DK])

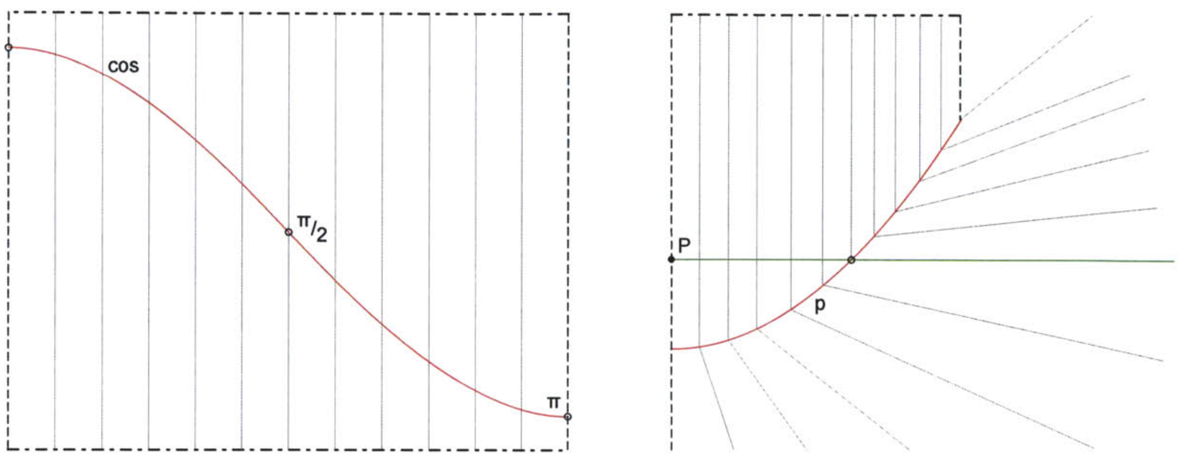


Fig 4.4 Reflection gadget, Refraction gadget [DK]

Individual designs

Each numbered section begins with the introduction of the meta-category, which can include several numbered sections. A section begins with the introduction of its gadget and how it is used in a design. A 'design' is a proposal for a folded model that is based on a specific crease pattern. I describe Huffman's decisions related to a design in the present tense in most cases. I use the small headings in all sections in order to separate geometric descriptions from observations that relate to the making of the designs.

Under the 'crease pattern' heading I describe the types of curves and the general placement of elements on the paper. Huffman most often used 8 1/2" by 11" sheets of graph paper for his designs. The graph paper units range from 1/8" to 1/5". If he decides to cut the paper in a shape I describe the boundary he uses. The tiling analysis describes the kind or type of tiling and its prototiles. In some cases, I discuss 'design tiles' and their symmetries, which I define later in this introduction.

The paragraphs under 'ruling analysis' heading are concerned with the behavior of rulings in the folded state. The separation of this analysis is important, as Huffman rarely draws rule lines in which case I propose a plausible solution. Huffman likes to think of his designs as constraint propagation problems and the ruling analysis often supposes an initialization or 'base case' of his algorithm.

Regarding rulings, one possible assumption is that the rulings don't change throughout the folding motion, which Tomohiro Tachi calls 'rigidly foldable' [Tac 10]. This assumption might be incorrect for some of his designs.

If a crease pattern and its rulings can be drawn, I run a simulation using Tachi's simulation software Freeform Origami and describe the result. If no solution for rulings exist, I reconstruct the design by making and photographing a paper model, which I call a 'paper reconstruction'. In some cases Huffman provides no 2d information and I have to reconstruct a model visually by trial and error.

The sections often conclude with paragraphs under 'notes' that discuss similarities to other designs, possible inspirations, and references to works Huffman has in his library. In some cases, I can point to grant proposals, exhibitions or can make connections to academic papers. A description of the physical qualities of Huffman's models provides insight into the care he used to make them. The making of these artifacts allows me to assess whether a design was valuable to Huffman.

The categories of the taxonomy

A gadget-based visual overview of the sections can be seen on several tables (Fig 4.6 - 4.9).

I Reflection

The taxonomy begins with cylinder and cone reflections that create curved creases based on reflected or mirrored partial cylinders (Fig 4.6 left) or partial cones (Fig 4.6 right). As the geometry is known I can digitally visualize most examples. Huffman's designs with reflection can be abstracted into reflection gadgets, some of which include tucking, which will be described toward the end of each section.

II Refraction Gadgets

The ruling refraction category consists of designs that Huffman constructs via the use of refraction gadgets. The examples in these sections form the largest part of the taxonomy. The taxonomy begins with gadgets that are based on a single curve type such as gadgets with ellipses or gadgets with parabolas (Fig 4.7).

The following group of gadgets consists of conics that are connected to each other (Fig 4.8 left). Huffman uses the curves to make piecewise smooth splines or quadratic splines. The C^1 splines are concatenation of quadratic functions. The required smoothness condition demands common tangency where two curves connect.

The last group of sections that use gadgets with a single curve type explains designs with partial hyperbolas (Fig 4.8 right).

Finally, gadgets with combinations of several conics make up the last three groups of sections in the numbered section (Fig 4.9 and 4.10 left).

III Forced Rulings

Huffman designs a tiling with what appear to be scaled sine curves. The crease pattern exists mathematically speaking and can be drawn with an arbitrary curve, which is not typically the case. This cone and cylinder gadget is a special case, where the rulings are forced into a cone on the convex side of the arbitrary curve, and into a cylinder on the concave side (Fig 4.10 bottom left).

IV Converging Curves

Cyclic tilings with converging curves define the sections with non-quadratic curves such as spirals (Fig 4.10 right). Loxodromic spirals, vortexes and sinks conclude the taxonomy.

This group does not include gadgets as we don't know the rulings for the designs.

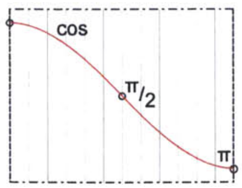
The following small table explains the definitions of captions in the gadget table (Fig 4.5). The first two lines list how many curves and how many line segments a gadget consists of. The '+' indicates that the gadget can be used for several curves of the same type with alternating mountain and valley assignments, which is also known as pleating.

The next two lines describe how often the gadget is used and where the gadget is used for the first time in the taxonomy. The title of a group lists the total number of examples that use the gadgets in the group in square brackets.

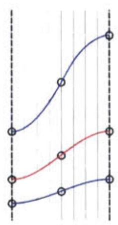
# c.c.:	3	.... Number of curved creases in gadget
# s.c.:	2	.... Number of straight creases in gadget
d/g:	3	.... Number of designs per gadget
des.:	Fig 4.2.3	.... Figure of first design with gadget

Fig 4.5 Table per gadget

Reflection gadgets with cylinders [14]

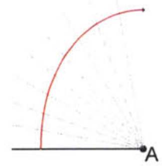


# c.c.:	1
# s.c.:	0
d/g:	9
des.:	Fig 4.1.4

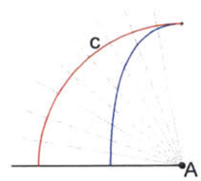


# c.c.:	3+
# s.c.:	0
d/g:	2
des.:	Fig 4.1.6

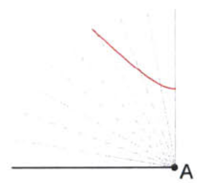
Cone reflection [19]



# c.c.:	1
# s.c.:	0
d/g:	4
des.:	Fig 4.2.3

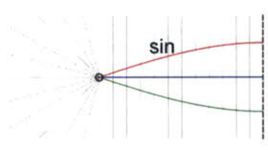


# c.c.:	2
# s.c.:	0
d/g:	10
des.:	Fig 4.2.16

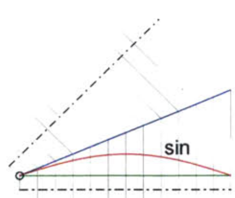


# c.c.:	1
# s.c.:	0
d/g:	2
des.:	Fig 4.2.47

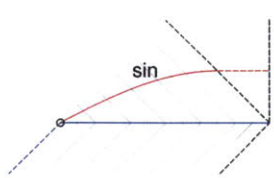
Reflection gadgets with cylinders and tucking



# c.c.:	1
# s.c.:	1
d/g:	1
des.:	Fig 4.1.16

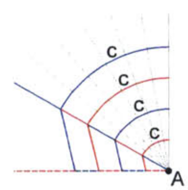


# c.c.:	1
# s.c.:	1
d/g:	1
des.:	Fig 4.1.21

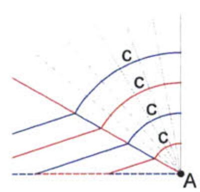


# c.c.:	1
# s.c.:	3
d/g:	1
des.:	Fig 4.1.26

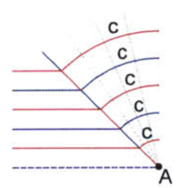
Reflection gadgets with cones and tucking



# c.c.:	1+
# s.c.:	3+
d/g:	1
des.:	Fig 4.2.51



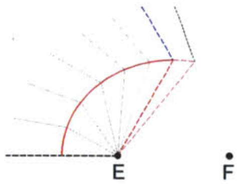
# c.c.:	1
# s.c.:	3
d/g:	1
des.:	Fig 4.2.53



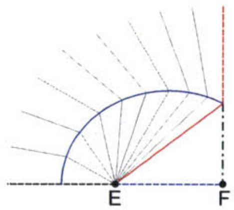
# c.c.:	1
# s.c.:	2
d/g:	1
des.:	Fig 4.2.55

Fig 4.6 I Reflection

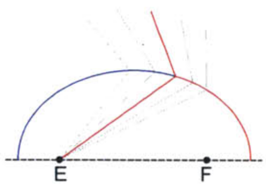
Gadgets with ellipses [13]



c.c.: 1
s.c.: 2
d/g: 1
des.: Fig 4.3.2 left

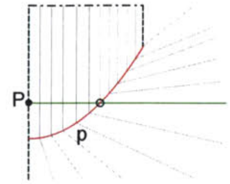


c.c.: 1
s.c.: 3
d/g: 11
des.: Fig 4.3.2 right

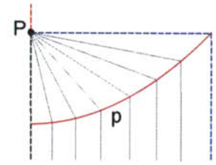


c.c.: 2
s.c.: 2
d/g: 1
des.: Fig 4.3.3

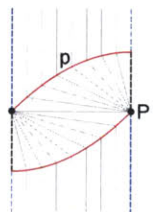
Gadgets with parabolas [33]



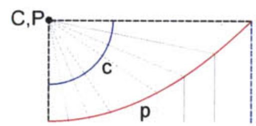
c.c.: 1
s.c.: 0
d/g: 10
des.: Fig 4.4.2 left



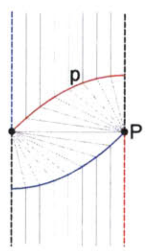
c.c.: 1
s.c.: 2
d/g: 3
des.: Fig 4.4.2 right
(similar: Fig 4.4.58)



c.c.: 2
s.c.: 2+
d/g: 3
des.: Fig 4.4.28



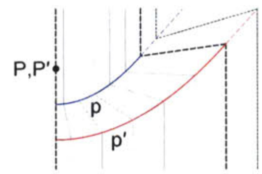
c.c.: 2
s.c.: 1
d/g: 4
des.: Fig 4.4.39



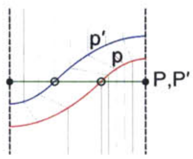
c.c.: 2
s.c.: 2
d/g: 13
des.: Fig 4.4.54
(cont'd: Fig 4.4.137)

Fig 4.7 II Refraction gadgets

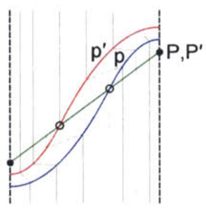
Gadgets with quadratic splines [79]



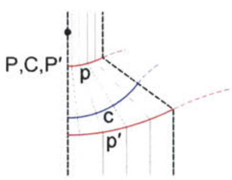
# c.c.:	2
# s.c.:	2
d/g:	6
des.:	Fig 4.4.63



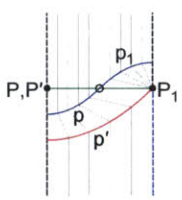
# c.c.:	4
# s.c.:	0
d/g:	11
des.:	Fig 4.4.68



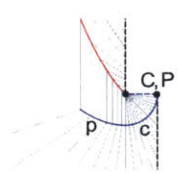
# c.c.:	4
# s.c.:	0
d/g:	10
des.:	Fig 4.4.87



# c.c.:	3+
# s.c.:	0
d/g:	3
des.:	Fig 4.4.102

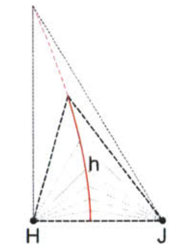


# c.c.:	3
# s.c.:	1
d/g:	47
des.:	Fig 4.4.115

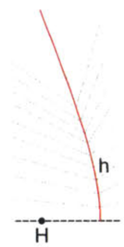


# c.c.:	2
# s.c.:	1
d/g:	2
des.:	Fig 4.4.144

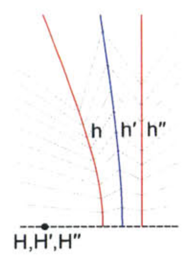
Gadgets with hyperbolas [3]



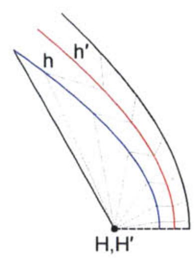
# c.c.:	1
# s.c.:	0
d/g:	1
des.:	Fig 4.5.2 left



# c.c.:	1
# s.c.:	0
d/g:	0
des.:	Fig 4.5.2 right



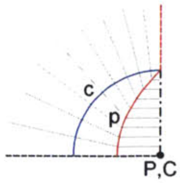
# c.c.:	3+
# s.c.:	0
d/g:	1
des.:	Fig 4.5.3



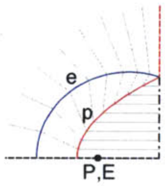
# c.c.:	2+
# s.c.:	0
d/g:	1
des.:	Fig 4.5.6

Fig 4.8 II Refraction gadgets

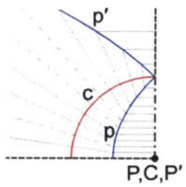
Gadgets with combined curve types [33]



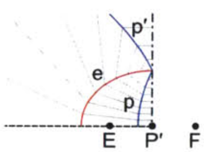
# c.c.:	2
# s.c.:	1
d/g:	2
des.:	Fig 4.6.3 left



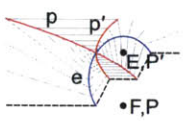
# c.c.:	2
# s.c.:	1
d/g:	6
des.:	Fig 4.6.3 right



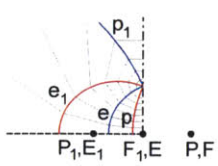
# c.c.:	3
# s.c.:	0
d/g:	11
des.:	Fig 4.7.2



# c.c.:	3
# s.c.:	0
d/g:	11
des.:	Fig 4.7.15

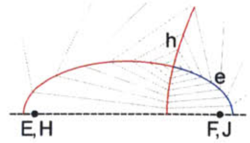


# c.c.:	6
# s.c.:	0
d/g:	1
des.:	Fig 4.7.20

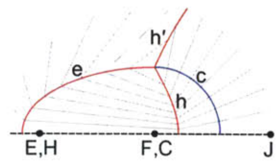


# c.c.:	4+
# s.c.:	0
d/g:	11
des.:	Fig 4.7.22

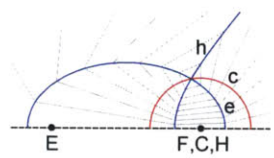
Gadgets with combined curve types [13]



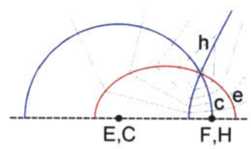
# c.c.:	4
# s.c.:	0
d/g:	1
des.:	Fig 4.8.2



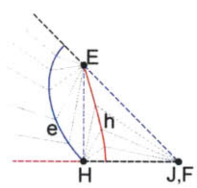
# c.c.:	4
# s.c.:	0
d/g:	1
des.:	Fig 4.8.4



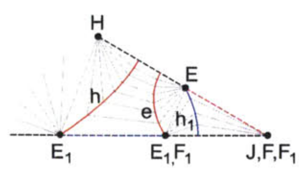
# c.c.:	6
# s.c.:	0
d/g:	6
des.:	Fig 4.8.8



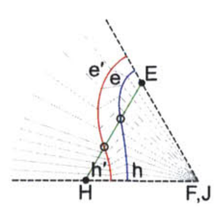
# c.c.:	6
# s.c.:	0
d/g:	1
des.:	Fig 4.8.17



# c.c.:	2
# s.c.:	2+
d/g:	1
des.:	Fig 4.8.18



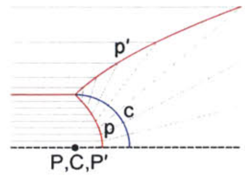
# c.c.:	3
# s.c.:	2
d/g:	1
des.:	Fig 4.8.24



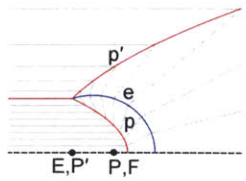
# c.c.:	4
# s.c.:	0
d/g:	1
des.:	Fig 4.8.26

Fig 4.9 II Refraction gadgets

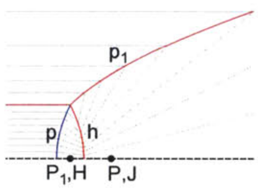
Gadgets with combined curve types [6]



\# c.c.: 3
\# s.c.: 1
d/g: 2
des.: Fig 4.9.2

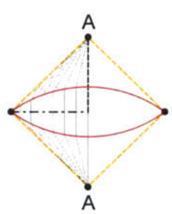


\# c.c.: 3
\# s.c.: 1
d/g: 1
des.: Fig 4.9.4



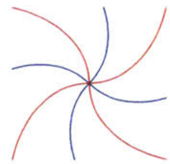
\# c.c.: 3
\# s.c.: 1
d/g: 1
des.: Fig 4.9.8

Forced Rulings

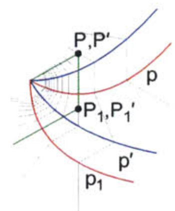


\# c.c.: 2
\# s.c.: 0
d/g: 11
des.: Fig 4.10.3

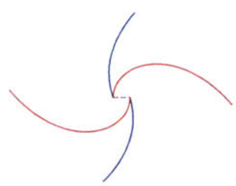
Converging curves [22]



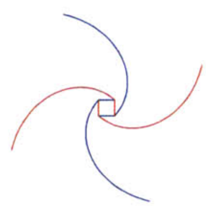
\# c.c.: 8+
\# s.c.: 0
d/g: 7
des.: Fig 4.11.1



\# c.c.: 12
\# s.c.: 0
d/g: 1
des.: Fig 4.11.8

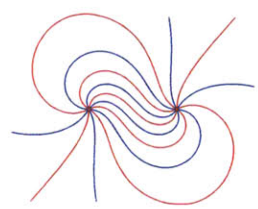


\# c.c.: 4
\# s.c.: 0+
d/g: 4
des.: (Fig 4.11.16)
Fig 4.11.19

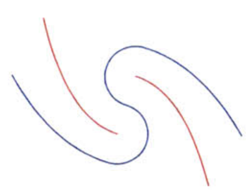


\# c.c.: 4+
\# s.c.: 4+
d/g: 6
des.: Fig 4.11.23

Loxodromic spiral



\# c.c.: 12+
\# s.c.: 0
d/g: 2
des.: Fig 4.12.1



\# c.c.: 3
\# s.c.: 0
d/g: 2
des.: Fig 4.13.11

Fig 4.10 II Refraction gadgets, III Forced Rulings, IV Converging Curves

The tile of a crease pattern

Many terms exist for tilings, such as tessellation, pavement, mosaic and parqueting, and patterns. Grünbaum and Shephard [Grü 87] make the distinction between tilings that have edges and patterns that consist of the repetition of a motif. Their book provides definitions and terminology which I adhere to as much as possible.

Huffman often modifies gadgets such that he can use them in different kinds of tilings. In order to describe the differences of the tiles, definitions of the edges of a tile are needed and such definitions are not trivial. The gadget is defined such that it is the smallest possible set of curves, similar to the way a **prototile** is the smallest of the necessary shapes of a tile in a tiling according to Grünbaum and Shephard [Grü 87].

It may seem counter intuitive to discuss the 'tiles' of a 'crease pattern', but I propose to expand some of the definitions by Grünbaum and Shephard in order to describe specific aspects of tiles for paperfolding. As the edges of the repeated parts need to be defined for curved creases I prefer to use the term tiling. 'Teilung' the word for division in German captures the aspect of dividing a crease pattern into smaller parts.

The tiling of a crease pattern is not always obvious as the edges of the tile can be rule lines as well as creases. Grünbaum and Shephard define a tiling as: "Any 'partition' of the plane into 'regions' (the tiles), regardless of whether or not this partition is realized (or can be realized) by physical objects" [Grü 87, p11]. They further define a tiling as a countable family of closed sets, $T = \{T_1, T_2, ...\}$, which cover the plane without gaps or overlaps.

A folding gadget can be cropped or extended in order to become a tile. We need to distinguish between edges that are aligned with a rule line or a crease, and edges that cut through rulings. These traversing edges can be a line or curve as long as it is symmetrical around its midpoint, the crossing rulings are not altered and no new crease is introduced. The three edge types have the following characteristics (Fig 4.11):

1. A straight edge can follow a straight crease or a ruling (dashed line).
2. If the edge is collinear with a ruling, the adjacent tile must connect smoothly (dashed line).
3. If the edge is a line or curve that crosses the rulings, it must not disturb a ruling in the flat state and during folding. A straight edge can be replaced with a curve that is symmetrical along its midpoint (dash-dotted line).

The prototiles are drawn with dashed and dash dotted lines (Fig 4.11). However, the prototile is perhaps not always the one that is visually apparent and I believe it is useful to group prototiles such that they create a visual unit. Multiple prototiles can create such a **design tile** (yellow) and in some cases their edges may not coincide. The coarsening of the 'prototile tiling' to a figure is

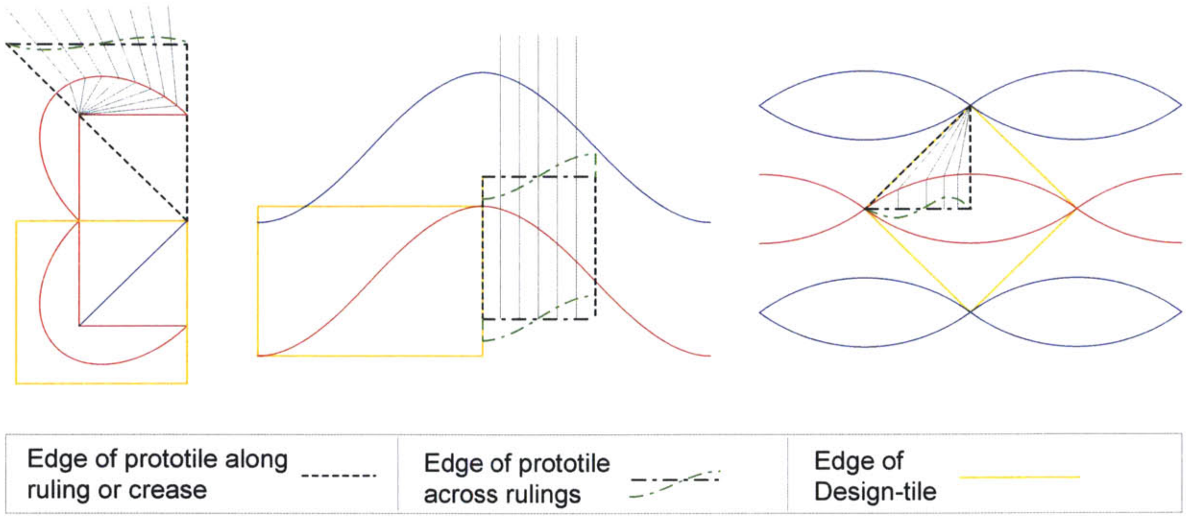


Fig 4.11 Prototiles and design tiles [DK]

productive when discussing Huffman's work in visual and aesthetic terms.

Grünbaum and Shephard define four types of isometries, mappings of the Euclidean plane E^2 onto itself, which preserve all distances:

1. Rotation about a point O through a given angle θ. When θ = 180°, the rotation equals a 'half turn'.
2. Translation in a given direction by a distance.
3. Reflection in a given line *L* which can be thought of as the 'mirror' or 'line of reflection'.
4. Glide Reflection in which reflection in a line *L* is combined with a translation by a given distance parallel to *L*.

The symmetry groups are defined as:

cn, the cyclic group of order *n* or *n*-fold rotational symmetry

dn, the group of all isometries of *cn* together with all reflections or *n*-fold dyhedral symmetry

Further definitions:

The elements of a tiling are vertices, creases, rulings, tiles and tile edges. The degree of a vertex is the number of endpoints of creases that end in the vertex. A tile adjacent to another tile shares an edge. Congruence shall be understood to include flipping. A tiling that consists of a single prototile is monohedral. A tiling is finite or complete, when all prototiles can be drawn and no tile can be added. Unbounded tiles have an open edge. Confocal means that several conics share identical foci.

The summary at the end of this chapter provides findings related to the taxonomy. There I provide assessments in terms of aesthetic considerations and summarize observations and point toward possible open problems.

4.1 Cylinder Reflection

One simple way of designing with curved creases consists of reflecting or mirroring a cylinder. We start with a general cylinder, cut it with an intersecting plane and subsequently 'reflect' the cut-off part through the plane. We obtain a mirror image and the curvature of the paper flips from concave to convex or vice versa. The straight lines in the paper, the rulings, lay on a line in the flat state and remain parallel to each other within one surface patch in the 3d state (Fig 4.1.1). Cylinder reflection behaves similarly for all developable surfaces, such as planes, cylinders, cones and tangent surfaces, in the sense that the ruling does not change direction in the flat state.

Huffman invents an alternate way of achieving cylinder reflections by drawing sine curves in 2d in his crease patterns. He owned a photocopied version of Herbert Yates' 1947 edition of 'A handbook on curves and their properties', in which the author describes the relationship between a cylinder that is cut at an angle and its developed surface (Fig 4.1.2). Yates writes: 'The Sine (or Cosine) curve is the development of an elliptical section of a right circular cylinder.' Huffman scaled sine and cosine curves that he constructed by hand on graph paper.

By definition, the curved crease starts and remains in a plane parallel to the initial one during the folding process. In his 'Curvature and Creases: A Primer on Paper' [Huf 78] Huffman describes the behavior of a more general case, in which the rulings remain in a plane perpendicular to the osculating plane, the surface defined by the tangent and normal vectors. The rulings are not parallel to each other in the flat state in this case, but the crease itself remains in a plane throughout the folding motion. The rulings remain on the same line as they cross the crease in 2d (Fig 4.1.3). Huffman draws the crease and its dual representation, its trace on a Gaussian sphere, for the left and right side of the crease.

Cylinder reflection has been exploited for software simulation as it is relatively easy to determine the position of the rulings in the 3d state [Mit 11]. Regarding available software a designer can choose to work with curved input surfaces or simulate the known crease pattern. Huffman expands his analysis on cylinder reflection by drawing discrete versions presented at the end of this section. When making a physical model based on cylinder reflection, we might have to force the paper on both sides of the crease into the same surface type, which can pose a challenge. The final chapter elaborates on several design approaches with the geometry that include digital

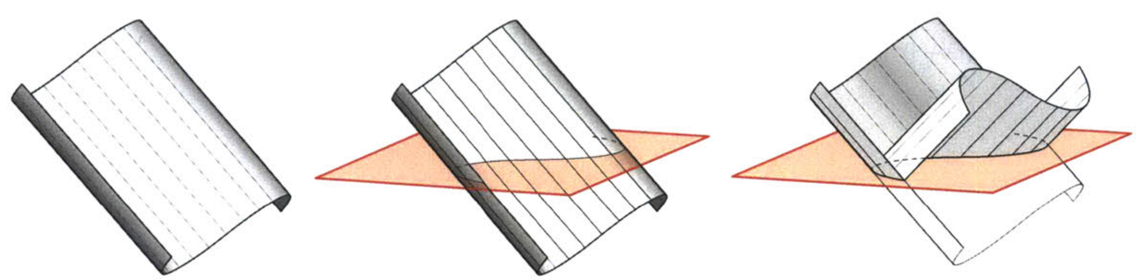
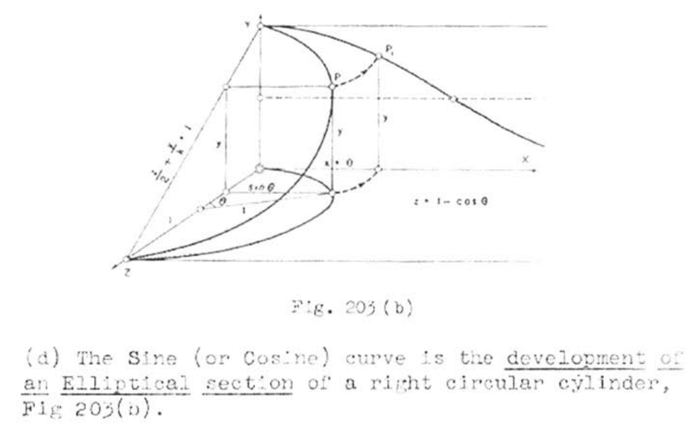


Fig 4.1.1 Cylinder reflection through a plane



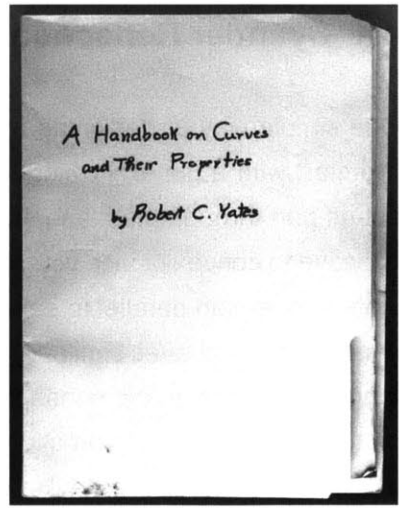


Fig 4.1.2 Diagram by Herbert Yates, Huffman's copy of Yates' book [DK]

and analog ways of working.

The following section presents designs Huffman created using reflected cylinders and displays the variety one can obtain with this relatively simple folding method. The section is sub-divided into subsections according to curve type.

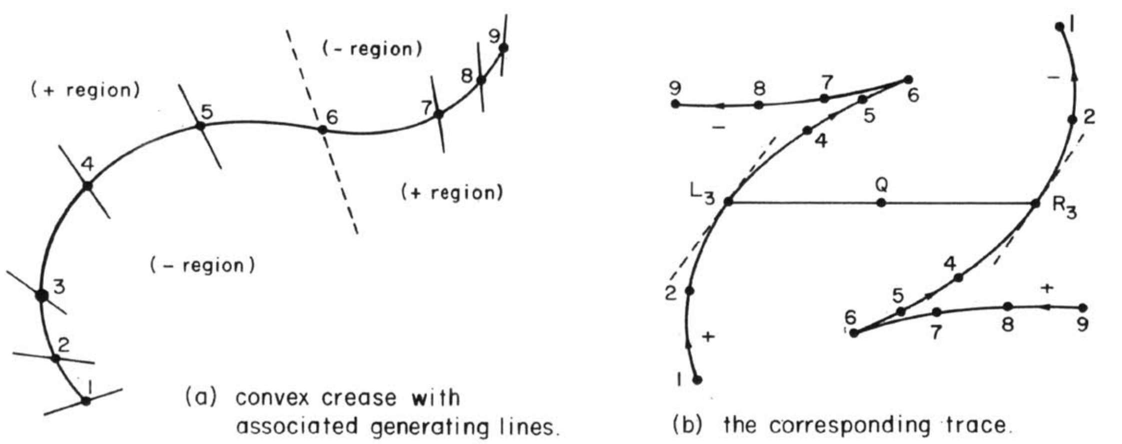
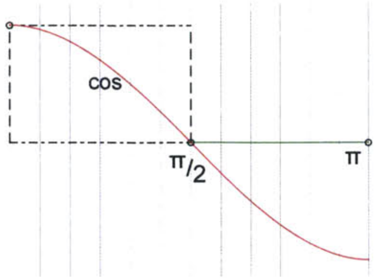


Figure II. Representation of a convex crease contained in a single osculating plane.

Fig 4.1.3 Diagram (DAH)

Cylinder reflection along sine curve and parabola



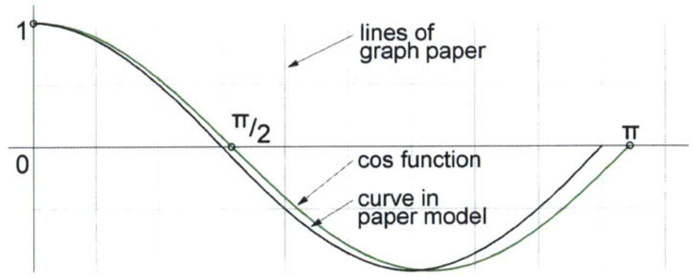


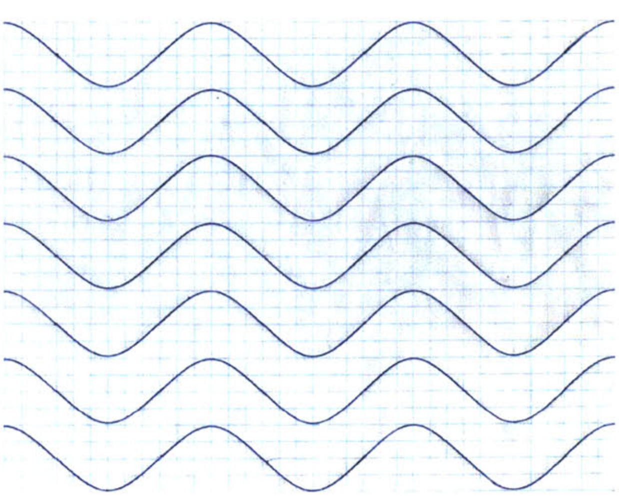
Fig 4.1.4 Sine or cosine gadget [DK], Scaled cosine curve on graph paper [DK]

Huffman most likely uses cylinder reflections for the designs in this section and draws them with sine and cosine curves. The basic gadget shows a cosine curve and straight parallel rulings and demands alternating mountain and valley assignments, if used in a tiling (Fig 4.1.4 left). The rulings at 0, $\pi/2$, π and so forth present good opportunities for such tilings.

Since he is plotting points manually, I assume that he exploits the efficiency of graph paper lines by scaling the functions to match the graph paper's increments as this accelerates the process of drawing the crease pattern. The figures shows the discrepancy between a regular cosine curve and the curve Huffman uses in the following design (Fig 4.1.4 right).

Crease pattern and ruling analysis

The cosine curve can be broken down into a prototile consisting of half the curve, but the design tile might consist of the entire undulating crease (Fig 4.1.4). The monohedral tiling results in 7 cosine curves (0 to 12π) with alternating mountain and valley creases. Huffman omits the rulings in his paper model (Fig 4.1.5). The assumed rulings are parallel in the flat state and the simulation on the right shows the resulting 3d configuration. Huffman uses scaled sine curves in the next 2 examples.



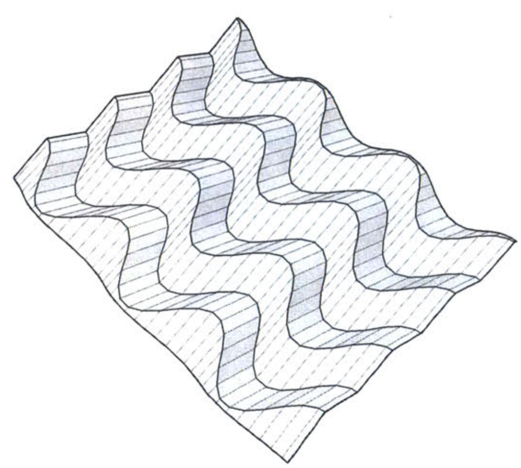


Fig 4.1.5 Paper model (undated, DAH [DK]), Simulated model [JH]

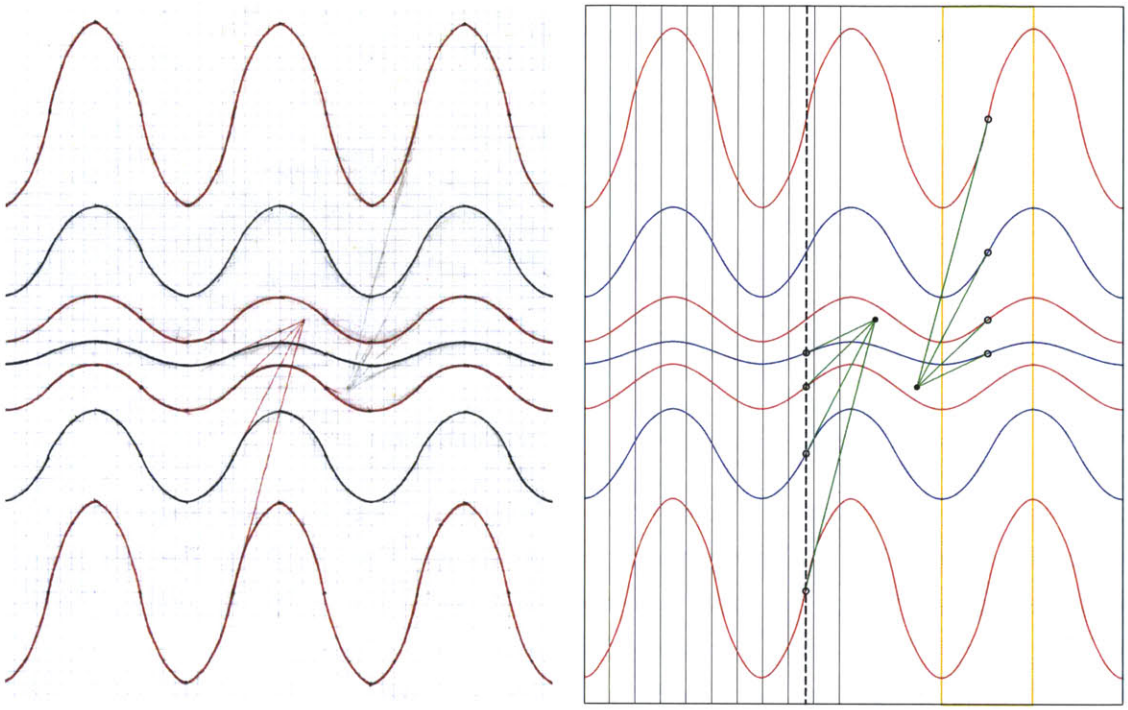


Fig 4.1.6 Paper model (undated, DAH [DK]), Crease pattern [DK]

Crease pattern and ruling analysis

The design starts with a shallow sine curve (0 to 6π) at the center and continues with scaled copies with a factor of 2 along the amplitude (Fig 4.1.6). The curves are placed apart by half of the vertical distance of their amplitude upward and downward. Huffman marks the common tangent points, where the sine curve changes direction and a tile edge exists. A total of 12 prototiles that are also the gadget define the vertically symmetrical tiling.

While Huffman does not draw rulings it appears very likely that they are all parallel in the flat state. The simulation is based on the same assumption (Fig 4.1.7).

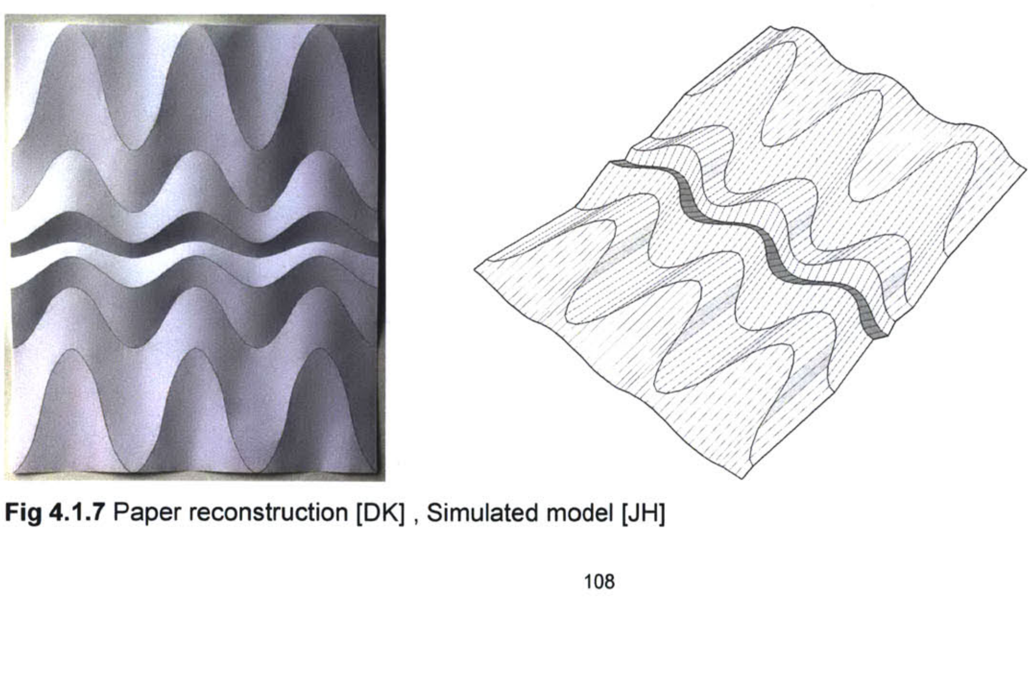


Fig 4.1.7 Paper reconstruction [DK] , Simulated model [JH]

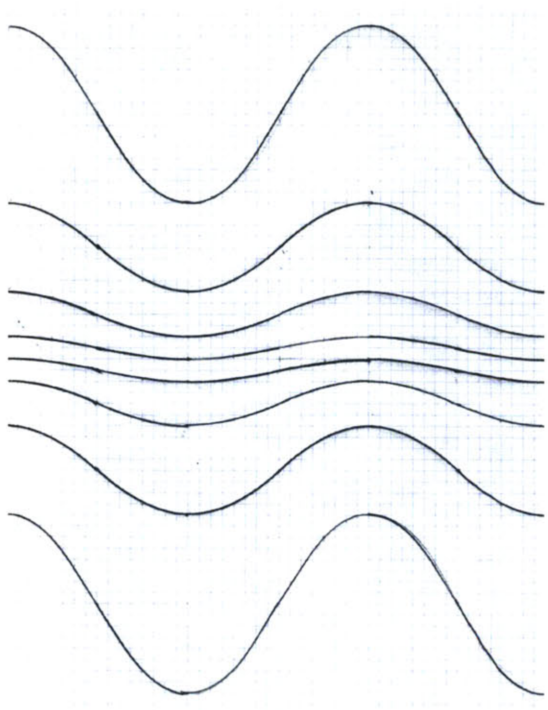
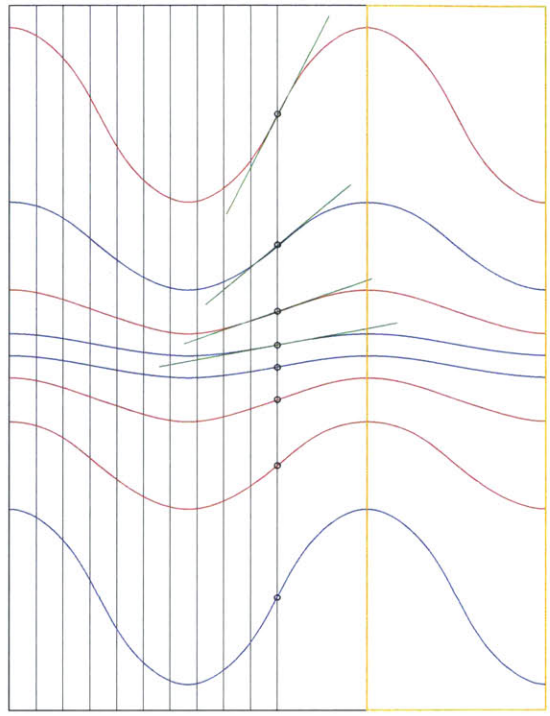



Fig 4.1.8 Paper model (undated, DAH [DK]), Crease pattern [DK]

Crease pattern and ruling analysis

We can observe several subtle differences in the above design (Fig 4.1.8). It begins with 2 shallow cosine curves (0 to 3π) at the center. They occupy the 8 1/2" by 11" in a similar fashion with scaled copies that have a factor 2. The curves are placed apart by half of the vertical distance of their amplitude in the same directions. Here, a total of 6 prototiles define the vertically symmetrical tiling in 2d, but the mountain and valleys are mirrored.

The rulings, also assumed to be parallel in the flat state, allow for a similar simulated model (Fig 4.1.9).

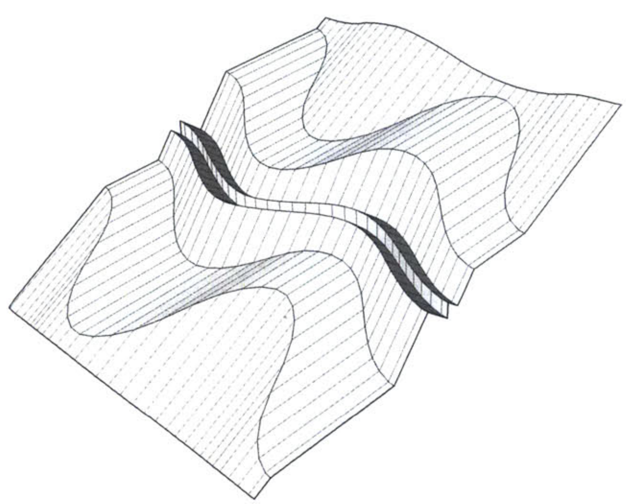



Fig 4.1.9 Paper reconstruction [DK] , Simulated model [JH]

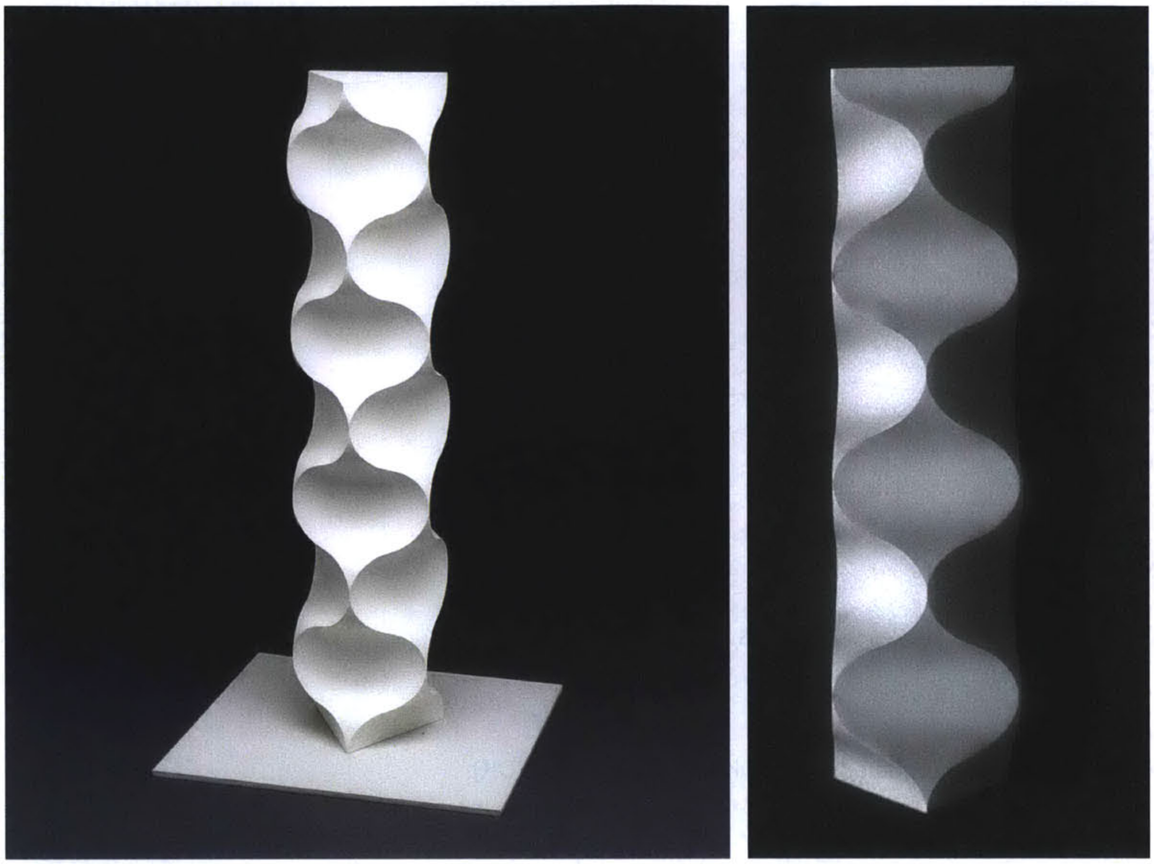


Fig 4.1.10 'Horizontally-fluted column' (1977, DAH [TG]), Paper model (1977, DAH [DAH])

Huffman calls the above design 'Horizontally-fluted column' in one of his inventory lists, and photographs both paper and vinyl versions. The paper model consists of less tiles than its pristine vinyl relative (Fig 4.1.10). Concave and convex surfaces alternate in the tiling that consists of mountain folds only. The reflection plane can be seen on the left and right side of the image of the paper model as the curved crease collapses into a line in perspective.

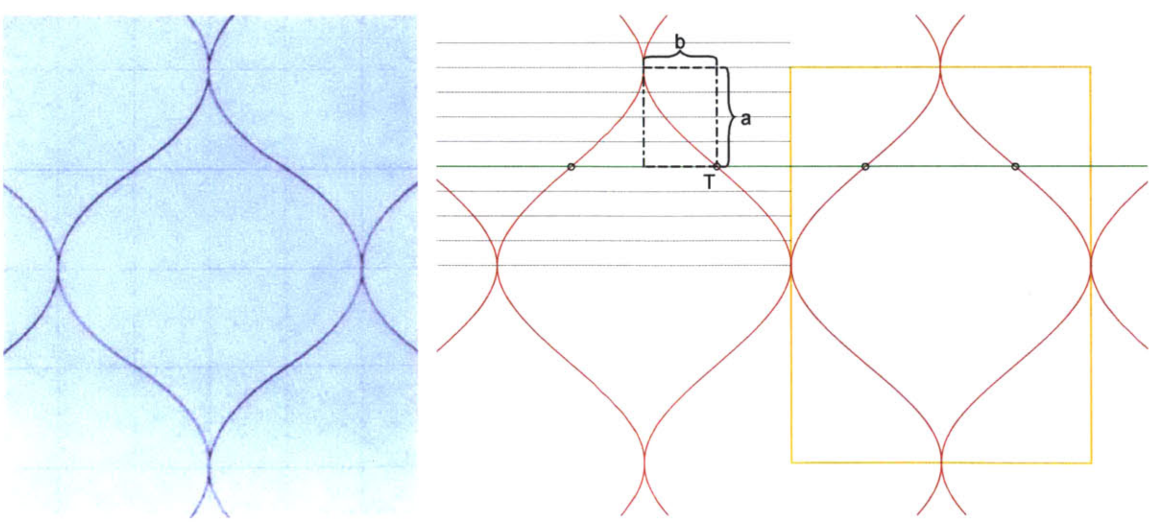


Fig 4.1.11 Vinyl model (1977, DAH [DAH]), Crease pattern [DK]

Crease pattern and ruling analysis

Huffman scales the sine curve and the gadget occurs rotated 90° in this case. His photograph of the traces on the inside of the design prior to folding reveals an *a* to *b* ratio of approximately 1 to 1.3 (Fig 4.1.11), which is smaller than 1 to $\pi/2$. The prototile remains the same as in previous examples, but he mirrors it along the vertical axis into 6 undulating sine curves. We can visually interpret the resulting crease pattern as a monohedral tiling made of the design tile drawn in yellow.

Due to the symmetries and regularities of the design the parallel rulings remain within a plane during folding. All tangent points where the sine functions change curvature lie on the same ruling. As a result the 6 creases can be folded such that the surface to the left of crease 1 and right of crease 6 can become congruent and form a cylindrical enclosure.

Notes

Huffman might take a similarly undulating example in 'Forms of paper' as inspiration. However, it is unclear when he obtains the 1971 edition by Hiroshi Ogawa [Oga 71]. The design uses smoothly connected arc segments, but is not depicted as a crease pattern that creates an enclosure (Fig 4.1.12 left). The creases also do not touch one another, which is a salient feature of Huffman's design.

Regarding the vinyl version, Huffman adds columns of incomplete tiles in order to be able to glue congruent surfaces to each other. He documents the visible side of the model prior to folding it (Fig 4.1.12 right).

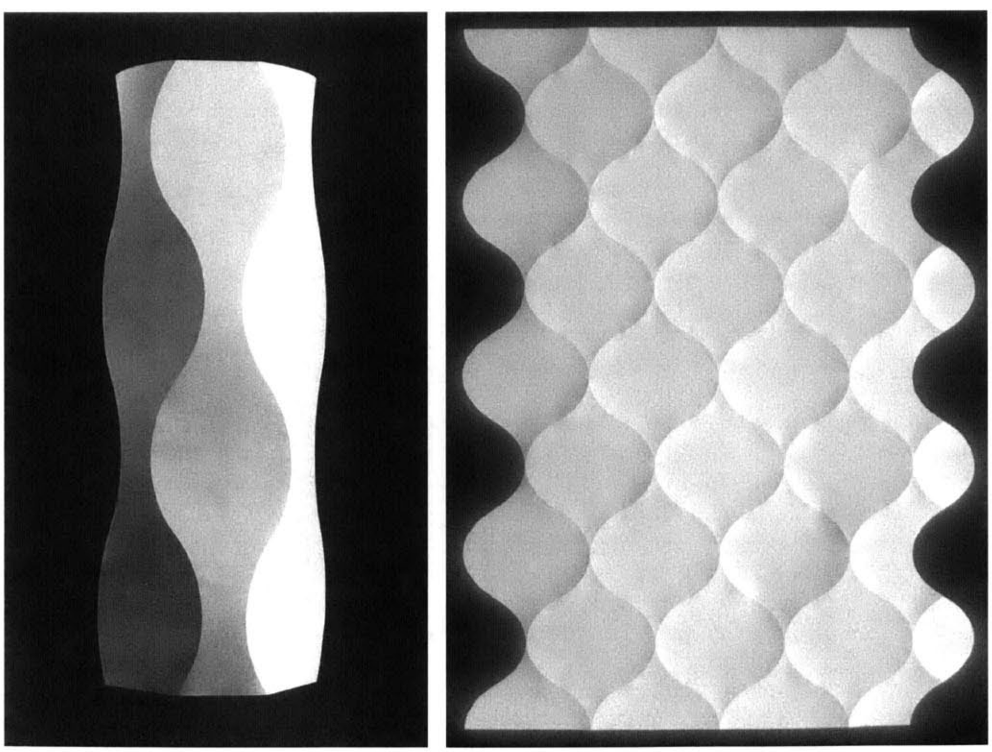


Fig 4.1.12 Model by Ogawa, Vinyl model in flat state (1977, DAH [DAH])

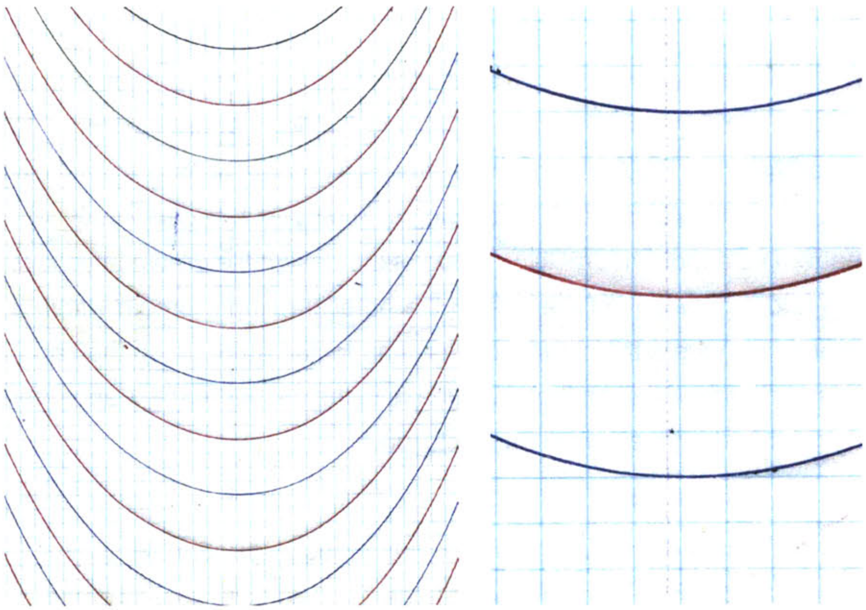


Fig 4.1.13 Paper model (undated, DAH [DK]), Detail of identical model

Huffman uses parabolic curves in conjunction with cylinder reflection for the following 2 designs. He omits focal points or tangents in the paper model. A very faint line to the left of the symmetry axis could be a ruling for the design. The detail shows a line lightly drawn with a ball point pen (Fig 4.1.13 right). Both designs appear to have been drawn with one of Huffman's custom templates mentioned in the previous chapter.

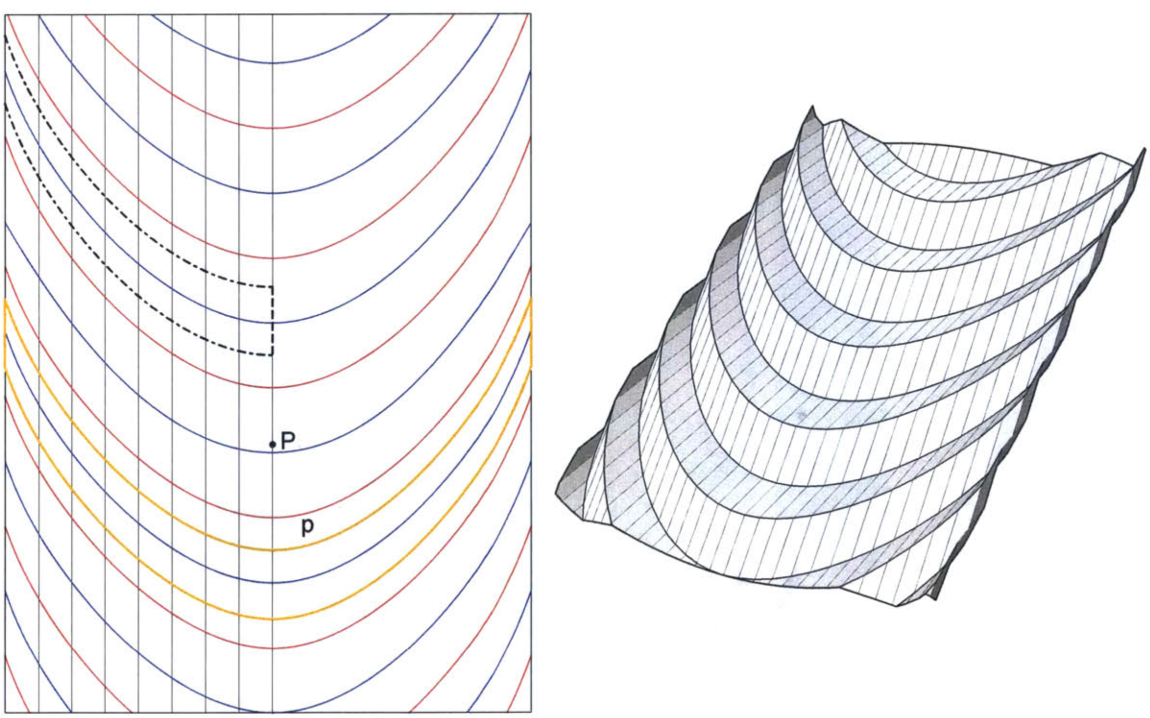


Fig 4.1.14 Crease pattern [DK], Simulated model [JH]

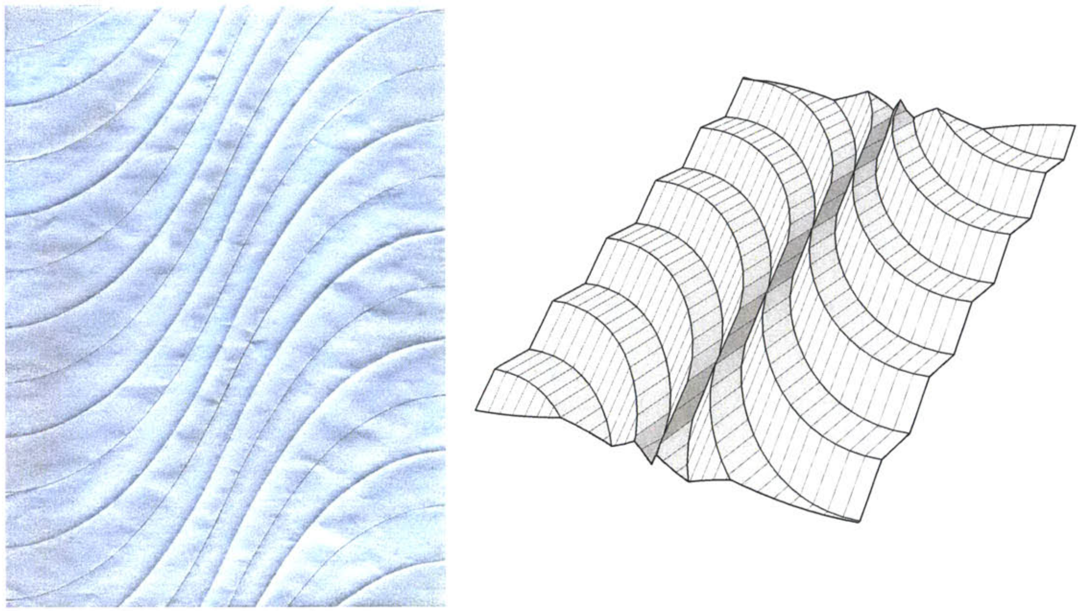


Fig 4.1.15 Paper model (undated, DAH [DK]), Simulated model [JH]

Crease patterns and ruling analysis

One parabolic curve forms the prototile for both crease patterns with alternating mountain and valley assignments (Fig 4.1.14 and 4.1.15). The first tiling is regular and symmetrical along the vertical axis and the second design is rotationally symmetrical around the center.

The simulations show the parallel rulings clearly and fold well.

Notes

Huffman uses an unusual paper for the design, which appears to be similar to gift wrapping paper (Fig 4.1.15).

Cylinder reflection along sine curve and tucking

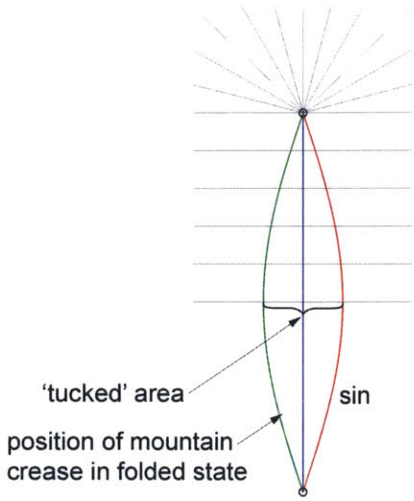


Fig 4.1.16 Gadget [DK]

When designing a paperfolding, we might want to remove material from the center of the sheet or hide it on its opposite side, for example. The term used for such a technique is called 'tucking' paper and this section analyses 2 of Huffman's examples that accomplish that. The gadget has to obtain an additional straight crease. The rulings cross this line without changing direction. The new crease allows both surfaces on either side to lie on top of each other in the folded state. The overlapping area allows us to tuck away that part of the paper.

We can achieve this by using a gadget that consists of a straight valley fold and a curved mountain crease (Fig 4.1.16). The mountain crease touches the paper on the left side along the green curve in the diagram. A regular sine curve will result in a 90° angle turn between the visible surfaces. Scaling the sine curve allows for control of the turn.

Paul Haeberli photographed Huffman during a visit at UCSC and later uploaded the imag-

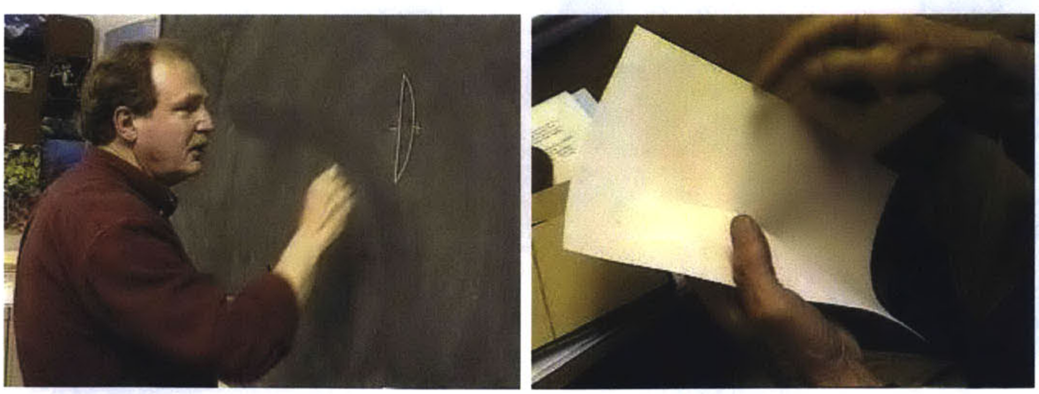


Fig 4.1.17 Huffman drawing tuck fold and holding the paper model [PH]

es on his website [Hae]. Huffman demonstrated the technique and drew the 2 necessary curves on a black board (Fig 4.1.17 left). He is holding he resulting saddle shape made of paper in the second image (Fig 4.1.17 right).

The next design takes advantage of this technique twice (Fig 4.1.17).

Crease pattern and ruling analysis

The mirrored pair of the gadget is connected via a valley fold at the top and bottom (Fig 4.1.19). Huffman appears to scale the sine curve in this design by a factor of 1/3. The prototile consists of a quarter of the crease pattern and includes half of a flat triangle.

A designer might think of the design tile as being half the model indicated in yellow.

Ruling analysis

The rulings of the general cylinder in the center are parallel to the 2 valley folds above and below. The two tuck folds on either side create a saddle shape made of 3 convex cylinders along the horizontal. It is unclear if the top and bottom part of the model incorporate the previously mentioned flat triangle exactly the way I draw them, but we can see a triangular area in Elise Huffman's photo (Fig 4.1.18 left).

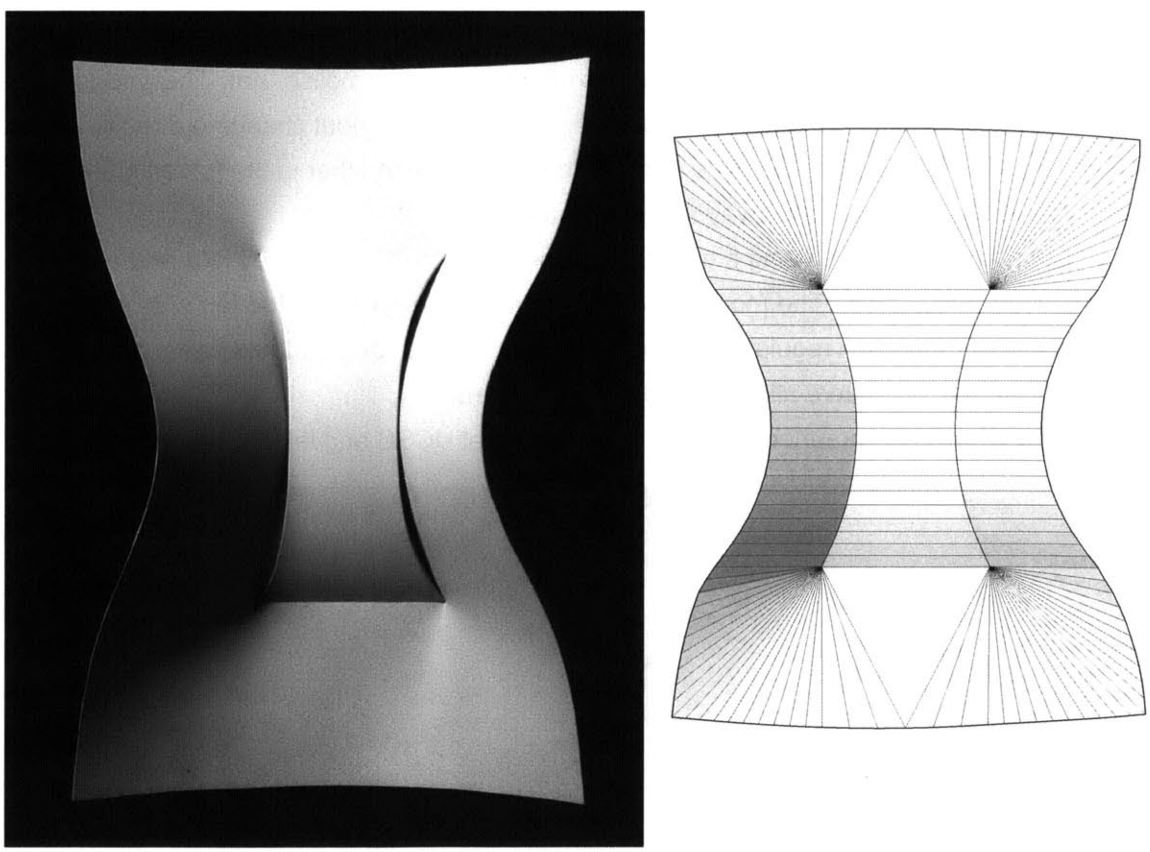
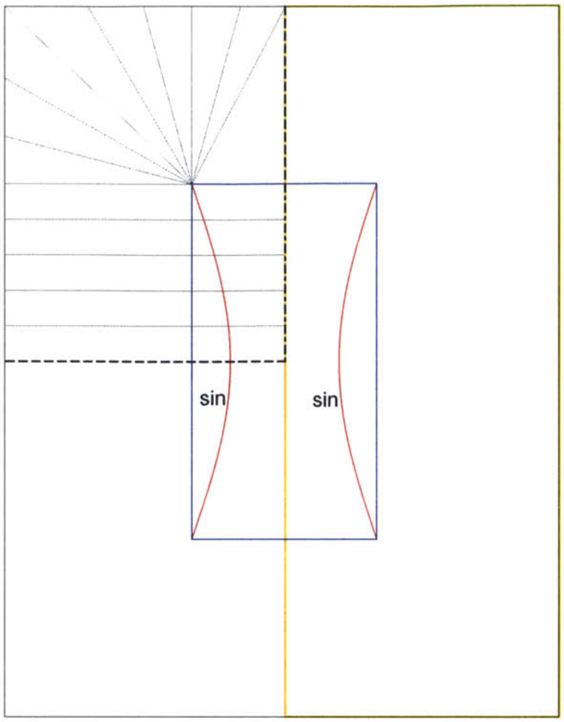


Fig 4.1.18 Vinyl model (undated, [DAH] [EAH]), Simulated model [JH]



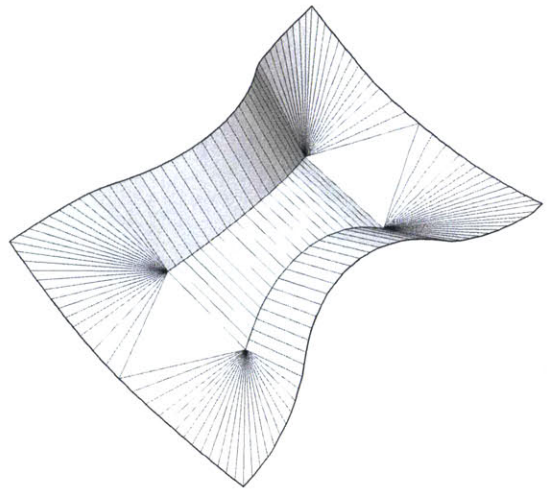


Fig 4.1.19 Crease pattern [DK], Simulated model [JH]

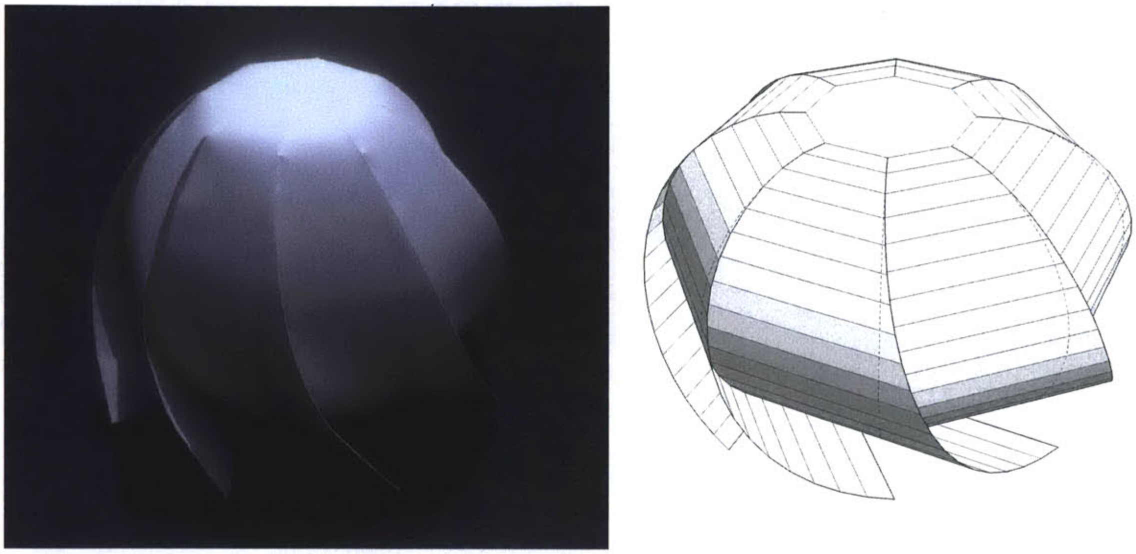


Fig 4.1.20 Paper model (1977, DAH [DAH]), Simulated model [AH]

Huffman's photographs of the above design provide the only source of information as the model does not exist any longer (Fig 4.1.20). The eight partial cylinders, each with a tuck fold, appear in 3 of his slides in 1977. It seems likely that Huffman uses sine curves to achieve the tuck folds, but it is unclear exactly which scale factor he uses.

Crease pattern

The gadget is also the prototile in this case, which consists of a sine curve, traversed by rulings, and a straight line segment (Fig 4.1.21). The crease pattern consists of 8 prototiles with an octa-

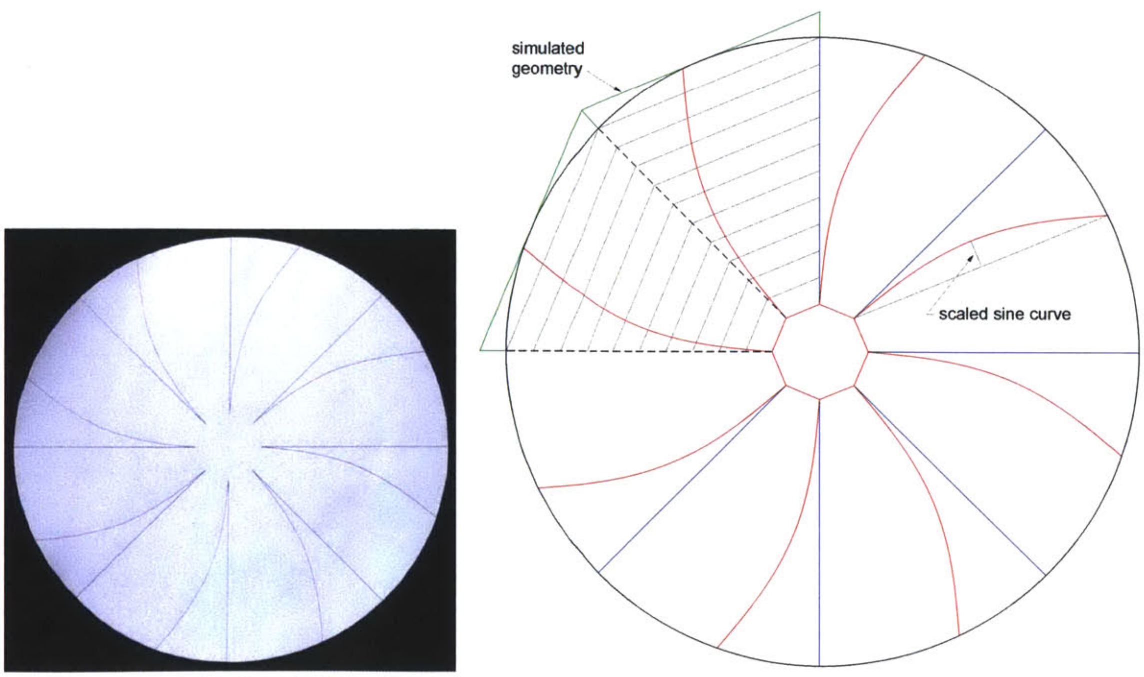


Fig 4.1.21 Paper model (1977, DAH [DAH]), Crease pattern [DK]

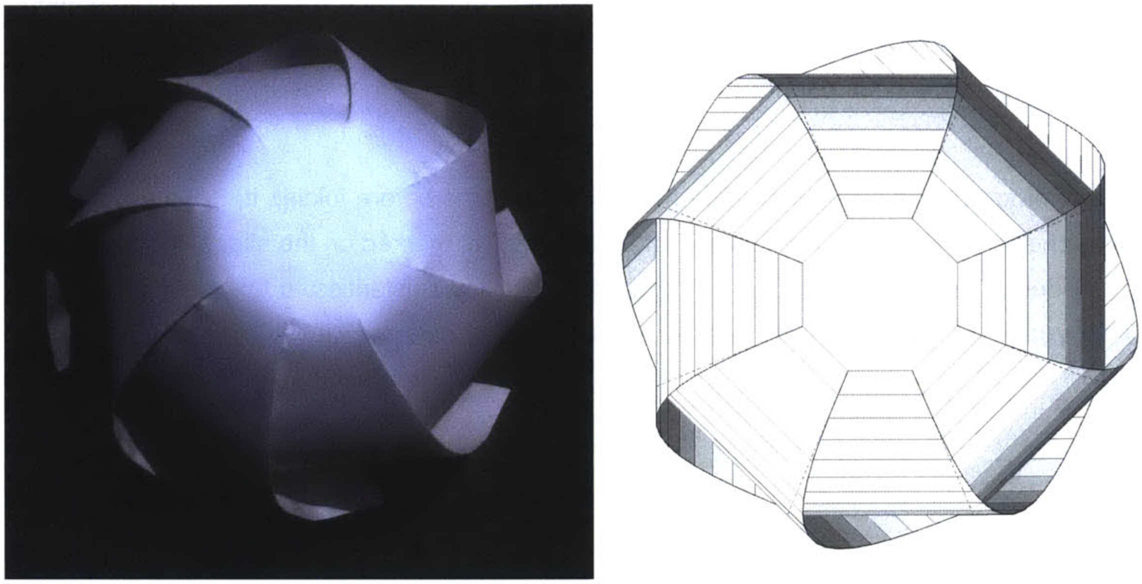


Fig 4.1.22 Paper model (1977, DAH [DAH]), Simulated model [AH]

gon at the center. The sine curves appear to be scaled, but it is unclear by how much.

Ruling analysis

The simulation uses an octagonal outer edge, shown in green, in as opposed to the circular edge of Huffman's crease pattern (Fig 4.1.21). It is unclear whether Huffman wants to create a smooth transition at the edge of the small octagon. However, the sine curve would not be scaled appropriately in order to achieve a smooth transition. The tuck folds behave in similar ways to previous examples with sine curves.

The simulations use a scaled sine curve by tan 22.5° and result in regular half cylinders.

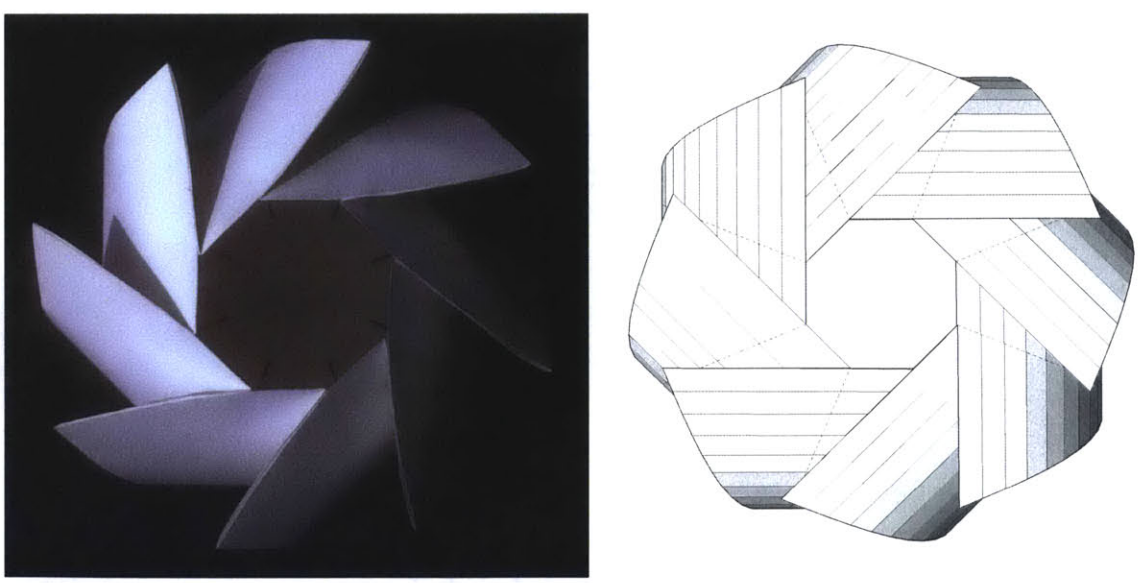


Fig 4.1.23 Paper model (1977, DAH [DAH]), Simulated model [AH]

The resulting shape appears wider than Huffman's models (Fig 4.1.20, 22 and 23). Simulations created by approximate tracing of his photographed crease pattern do not create a closed octagon at the bottom, which his paper model seems to suggest.

Notes

Huffman photographs the model and the flat crease patterns before folding it, but there exist no other sketches or drawings of the design. The date '1977' is marked on the slides. Robert Lang's 'uncut duodecagon' and Cheng Chit Leong's vases display similarities to this design, but date from later years. Jun Mitani discusses a very similar crease pattern [Mit 09b].

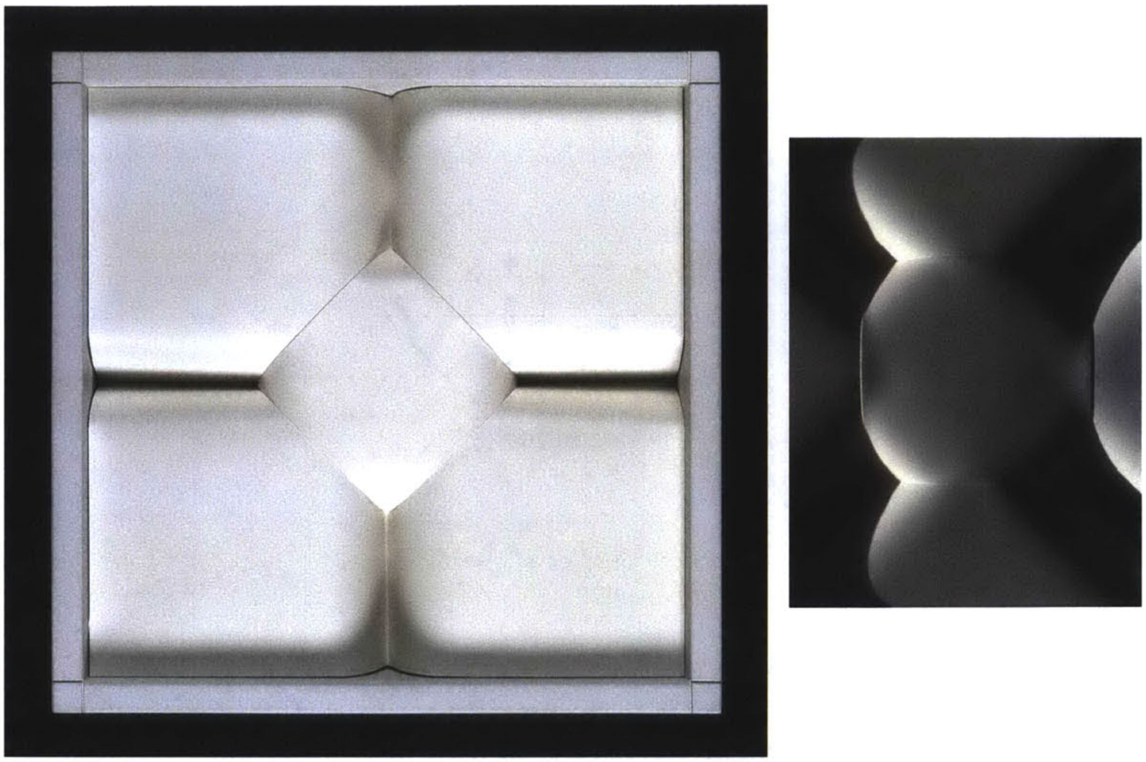


Fig 4.1.24 'Four-quadrants with centerpiece' (1979, DAH [TG]), Detail (1979, DAH [DAH])

Huffman designs the above framed model using the same gadget with sine curves and tucking (Fig 4.1.25). The model gives the illusion of a convex cylinder turning 90° twice within each of the 4 crevasses.

Crease pattern

The sine curves, here scaled to fit a 5 by 3 ratio (from 0 to $\pi/2$), form part of the eight necessary prototiles (Fig 4.1.25 right). The prototile is equivalent to gadget in this case. The gadget can be

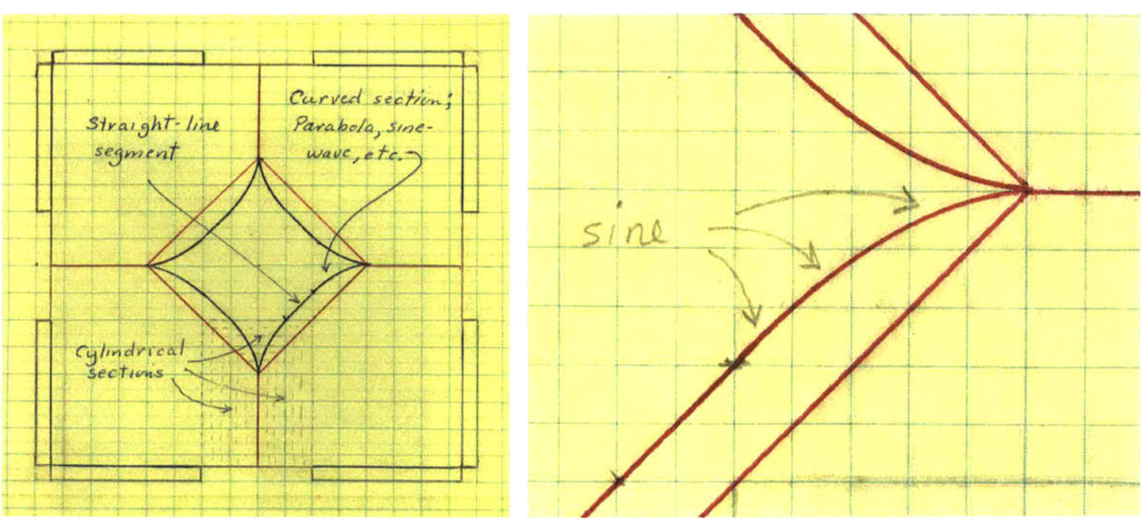


Fig 4.1.25 Crease pattern (undated, DAH [DK]), Detail of paper model (undated, DAH [DK])

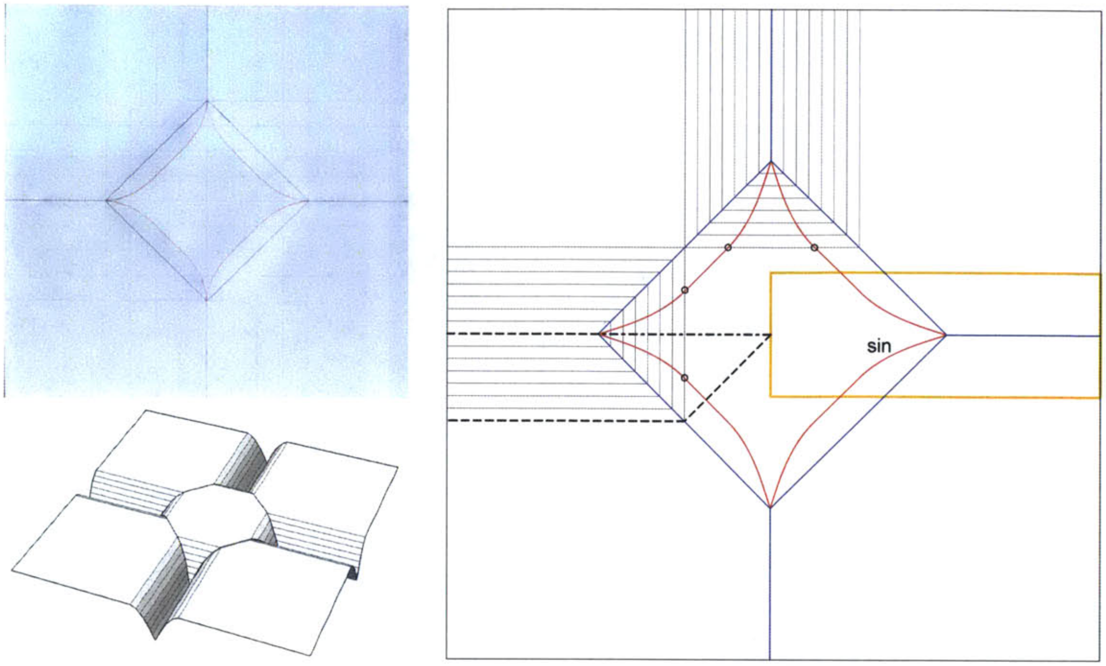


Fig 4.1.26 Vinyl model (1978, DAH [DAH]), Simulated model [PC], Crease pattern [DK]

seen where the rulings traverse the sine curve in a straight line (Fig 4.1.26 right).

Ruling analysis

Huffman uses tuck folds such that the rulings turn 90° at the straight valley crease. The tile edge along the horizontal must be such that the cylinder formed by the sine curve remains continuous (Fig 4.1.26 right).

Notes

Huffman documents the making of the model in 1977 and its completion with an unpainted wooden frame in 1978. He paints it white in 1979. The bottom part of the model consists of a wooden strut that keeps the surfaces in place (Fig 4.1.27 right).

Huffman might have studied a similar model in his copy of 'Forms of Paper' (Fig 4.1.27) [Oga 71]. Ogawa's does not use tucking and appears to be made of circles.

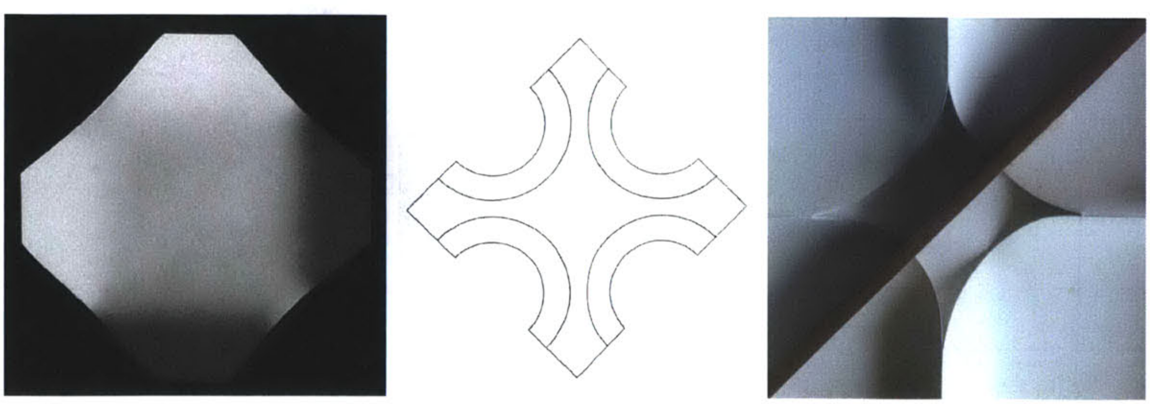


Fig 4.1.27 Figures in book Huffman owns, Vinyl model (undated, DAH [EAH])

Discrete cylinder reflection

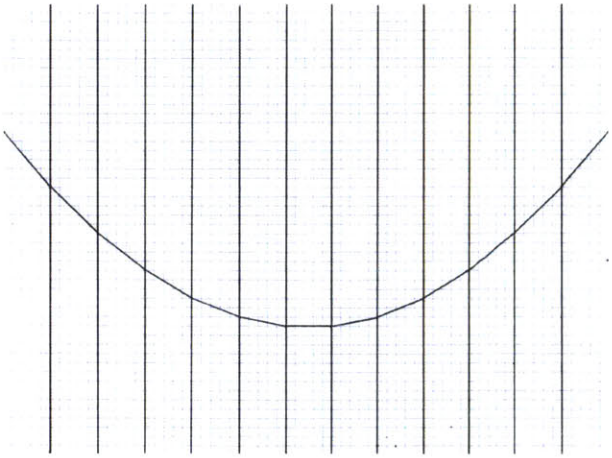
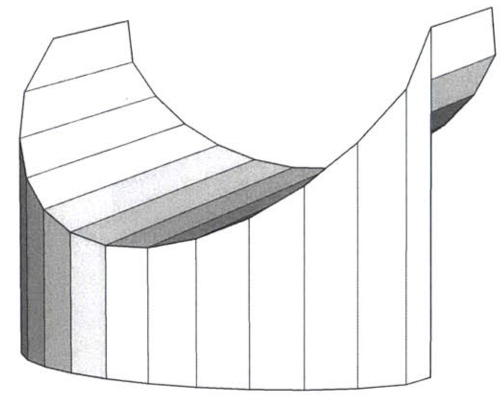



Fig 4.1.28 Paper model (undated, DAH [DK]), Simulated model [AH]

The last examples in this section consist of discrete crease patterns, which appear to be approximations of sine curves and parabolas. The study models are related to cylinder reflection as the rulings are straight. The first 2 examples use sine curves and the third model consists of a parabola.

Crease pattern and ruling analysis

The above design approximates a regular sine curve using graph paper units and cycles from 0 to π (Fig 4.1.28). The simulation results in an approximation of a cylinder as expected. The design below consists of a partial sine curve that does not cycle from 0 to π (Fig 4.1.29). Huffman uses paper clips to hold the model in place.

The simulation matches and allows for coplanar surfaces (Fig 4.1.30). The resulting cylin-

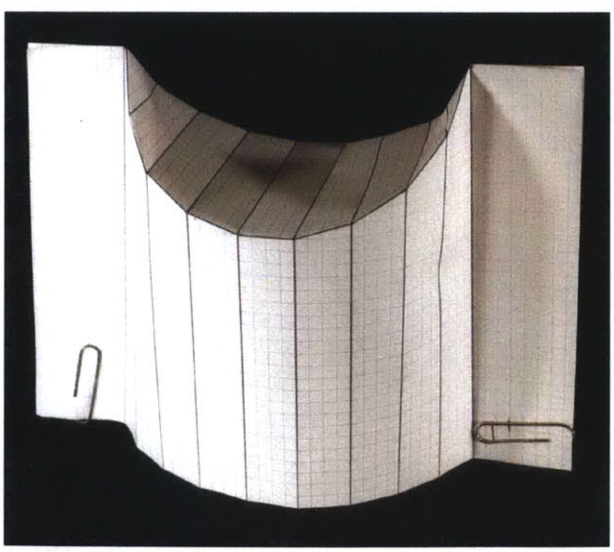
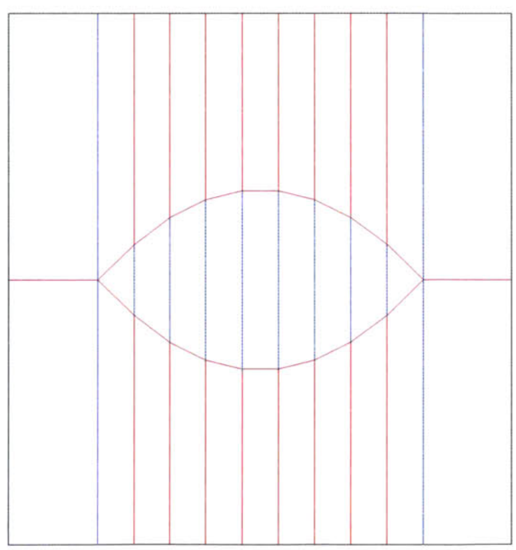



Fig 4.1.29 Paper model (undated, DAH [EAH]), Crease pattern [DK]

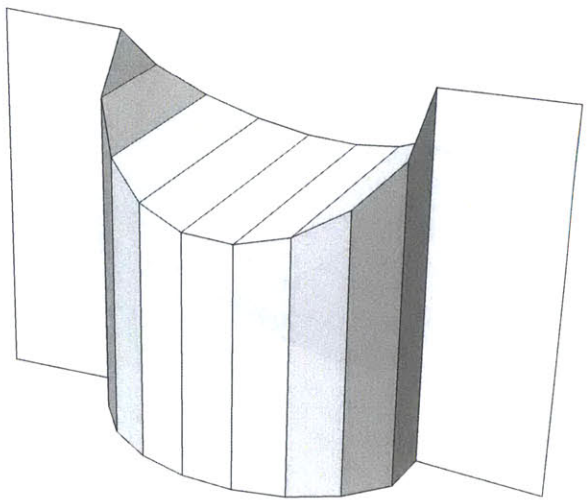


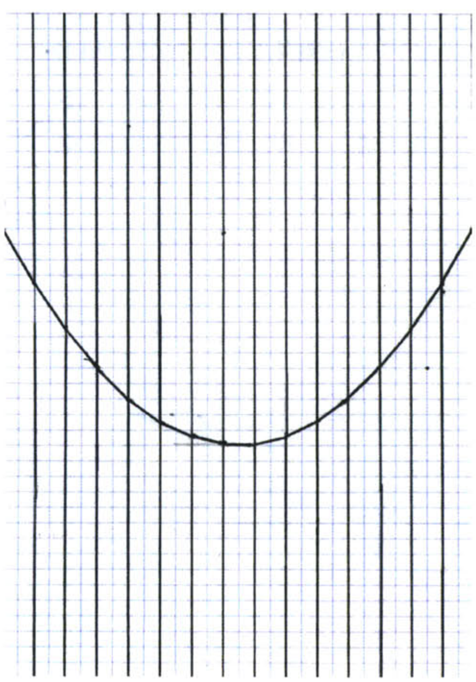
Fig 4.1.30 Simulated model [AH]

drical surface does not resemble a regular cylinder just like in the physical model.

Crease pattern and ruling analysis

In the third example a parabola is represented as polygon with 15 segments and a focus that is 4 graph paper units up from the lowest point of the parabola (Fig 4.1.31). Huffman does not use integer steps of the graph paper grid for the y-value of the parabolic function.

The rulings are parallel and indicate that Huffman is studying cylinder reflection. The simulation folds as well as the two previous examples.



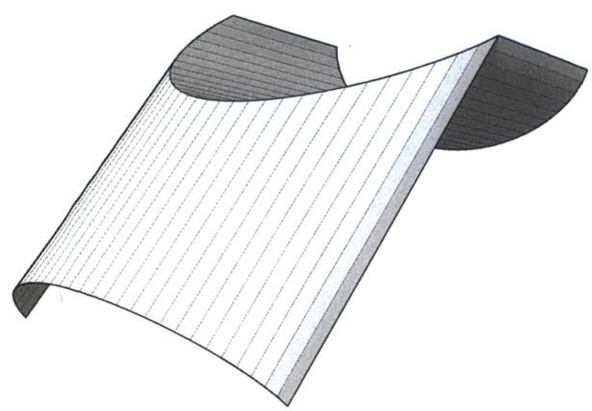


Fig 4.1.31 Paper model (undated, DAH [DK]), Simulated model [JH]

4.2 Cone reflection

[I Reflection]

We further investigate reflections in this section, but cones take the place of the previous cylinders. The section is structured according to design constraints Huffman chooses to work with, and elaborates on 'cone reflection between two planes', 'cone reflection with rotating axis', 'cone reflection parallel to axis', 'cone reflection and tucking' and 'cone reflection of general cones and tilings'.

Every group starts with a description of the constraints and continues with an analysis of the section, a projected drawing with the traces of all creases on the initial cone before folding. In some cases, Huffman draws the section before and after folding in one drawing.

Huffman's documentation of cones reveals how he designs in 3d. He draws sections for every design and derives formulas for the flattened crease pattern, which leads me to believe that he visualizes the designs in his mind and produces sections and other projected drawings to clarify his thoughts. He constructs the crease pattern only as final output for the paper model.

The individual examples present these sections that convey the design process first. I then segue to the folded sections of reconstructions, and finally, discuss crease patterns with brief descriptions of the rulings. The convention for numbering reflections starts at the base and works its way up step by step. Tiling descriptions can largely be omitted as Huffman only takes interest in tiling partial cones in very few examples at the end.

In some cases I discuss 'reflection gadgets' as Huffman investigates tucking, which is a useful technique to hide paper. The gadgets assume that all rulings stay on their path when traversing curves.

The following example provides a simple case study to explain the analysis, the reconstruction and the design process (Fig 4.2.1). Huffman draws the cutting plane in projection as a line and uses relationships between angles to find the regularities he wants to work with. He documents the cone in its un-reflected and reflected configuration and calculates the polar coordinates for the crease pattern in the table on the top left of the figure.

The single reflection mirrors the axis of the cone such that the apex touches the base and the rulings in the plane of reflection become adjacent to each other. The triangle on the lower right of the section formed by the base, the right edge of the cone and the mirrored edge on the left have the dimensions *1*, *2* and $\sqrt{3}$ to form a right angle.

The known definitions of all curves on a cone enable me to reconstruct Huffman's designs as digital 3d models in similar ways to working with cylinders. These '3d models', usually represented in parallel projected top views or perspective views, accompany his sketches throughout this chapter. The images do not include rulings as the geometry is constructed with CAD software that represents continuous surfaces with smooth shading (Fig 4.2.1 top right and bottom right).

In order to plot the developed curve on a cone Huffman derives the formula for the length

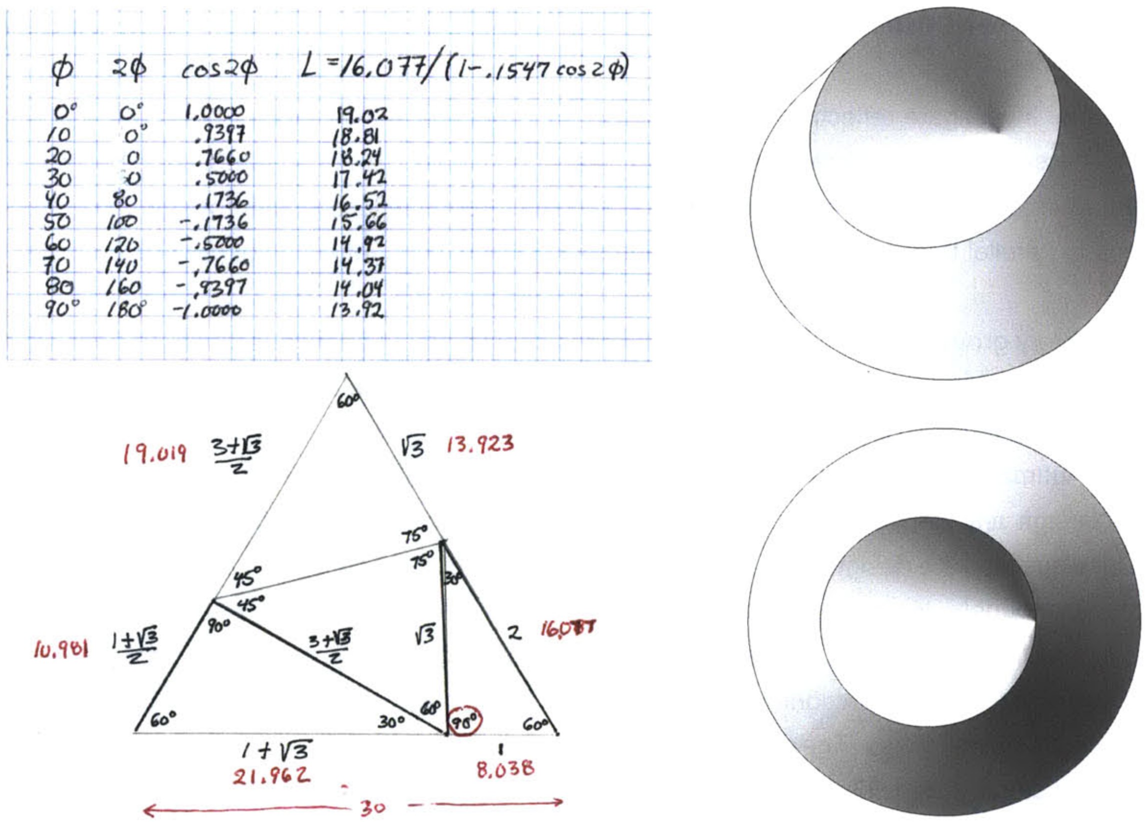


Fig 4.2.1 Section and polar coordinates (undated, DAH [DK]), 3d model [UP]

R, the ruling on the cone that starts at the apex and ends at the crease, on one of his index cards (Fig 4.2.2). He calls the same length *L* in the above figure and appears to keep that notation for other designs (Fig 4.2.1 left).

The mountain crease in the crease pattern is constructed in the same manner (Fig 4.2.3).

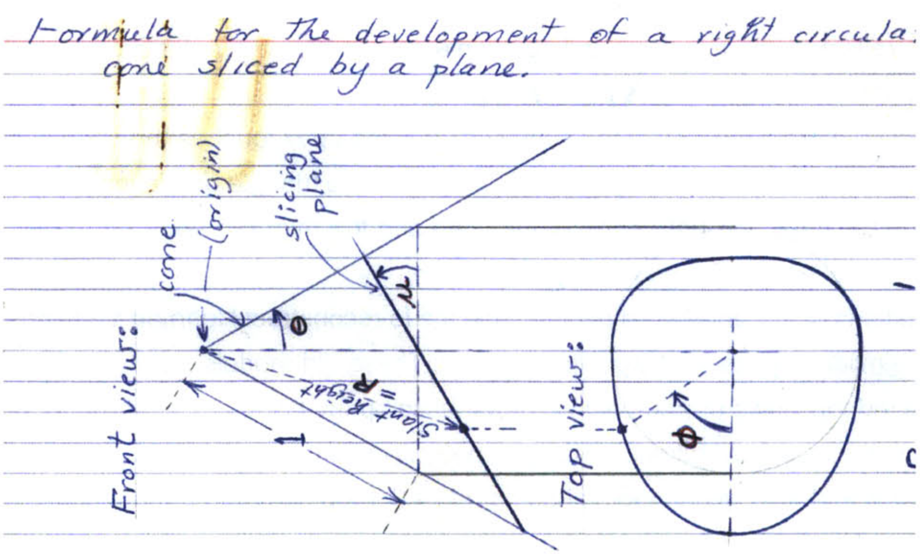


Fig 4.2.2 Index card (1978, DAH [DK])

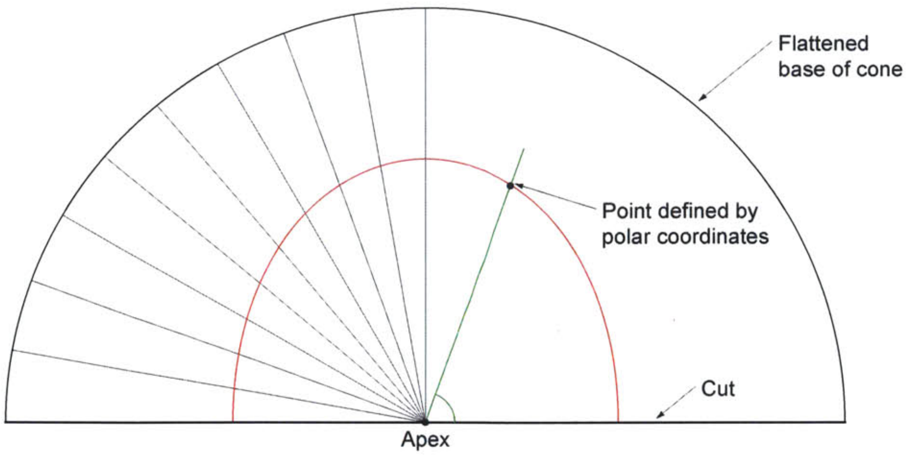


Fig 4.2.3 Crease pattern [DK]

We can plot points of the curve using its polar coordinates that start at the apex of the cone. The cut lines at the bottom, drawn sightly thicker, are the edges that meet in the folded configuration.

An example by Ogawa might have served as inspiration. It consists of cone reflections along 2 planes perpendicular to the axis. It can be thought of as alternating between the 2 parallel surfaces similar to the following design by Huffman (Fig 4.2.4).

Fig 4.2.4 Figure in 'Forms of paper' (1971, I.Ogawa)

Cone reflection between two planes

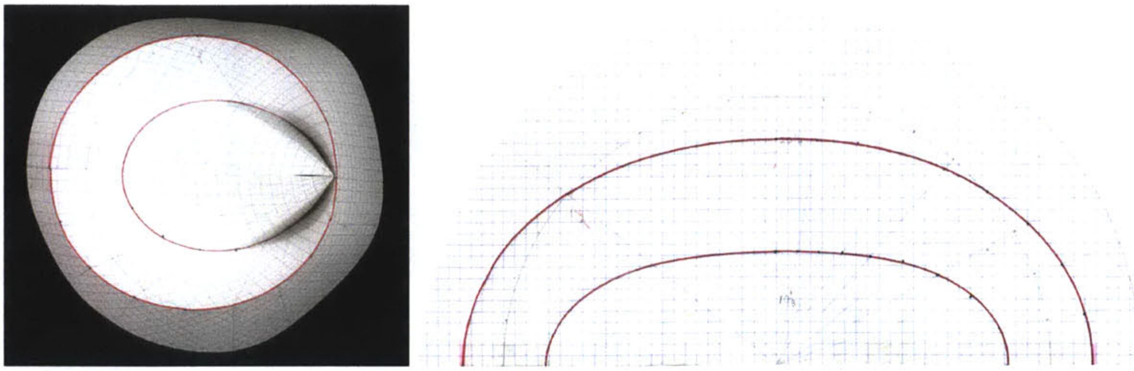


Fig 4.2.5 'Crushed cone #2' (1998, DAH [EAH]), Paper model (undated, DAH [DK])

Huffman uses cone reflections such that all truncated cones fit within the volume given by the base and the first reflection (Fig 4.2.5). First, in this case, means the reflection that starts closest to the base. His drawing is dated 1998, which indicates that he revisits this topic as he began his investigations in the 1970s (Fig 4.2.6). He defines relationships between the edges of the reflected truncated cones and aligns rulings to make them adjacent to rulings of other truncated cones. He also includes the derivation of L_1 and L_2, the previously discussed lengths on the ruling, which he needs to plot points. The 2 distances are marked in green in the crease pattern (Fig 4.2.8).

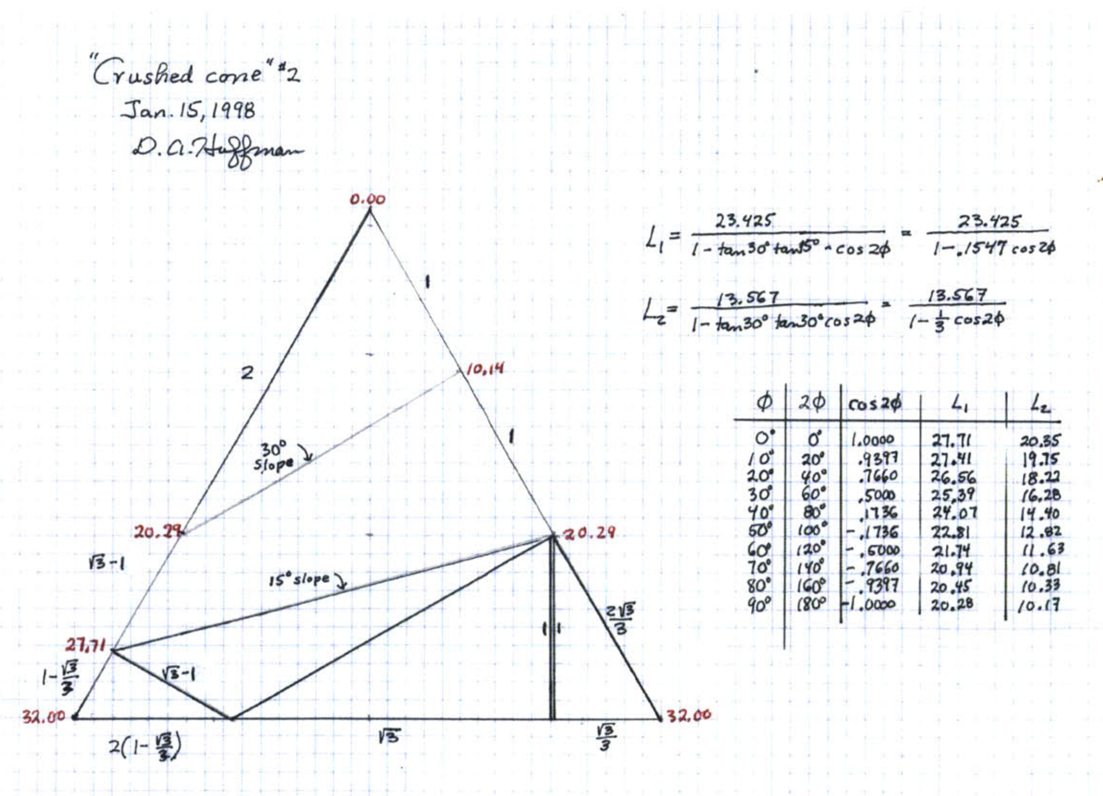


Fig 4.2.6 Drawing (1998, DAH [DK])

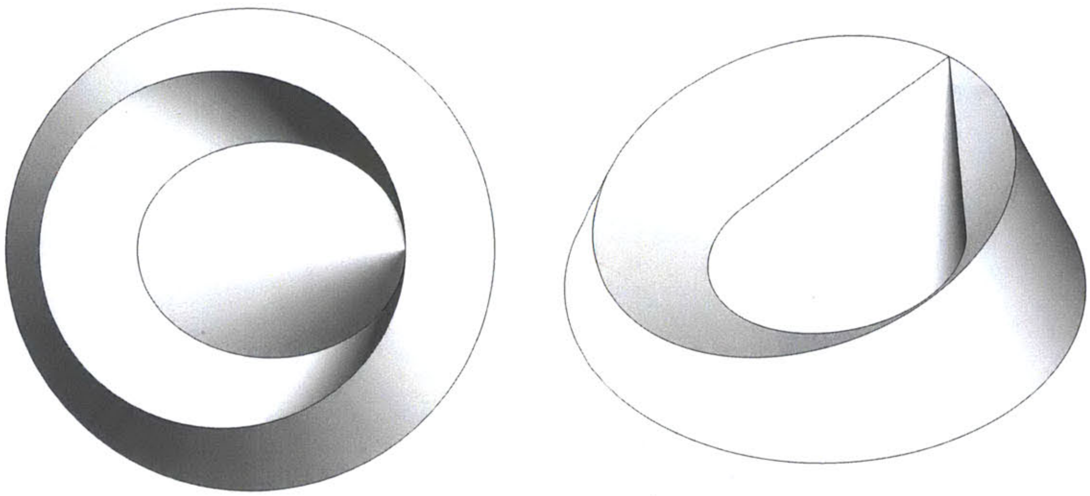


Fig 4.2.7 3d model [UP]

Section and crease pattern

'Crushed cone #2' features 2 reflected cones with adjacent rulings that are parallel to the axis. The right edge of the original cone is divided into 3 parts, the upper 2 of equal length (Fig 4.2.6). The first reflection occurs along a 15° slope. The right edge is now perpendicular to the base. The subsequent reflection along the base plane aligns the right edge of the last truncated cone with the right edge of the previous cone.

The crease pattern consists of a mountain and a valley crease (Fig 4.2.8). Huffman plots both curves in his crease pattern in only one color (Fig 4.2.5 right).

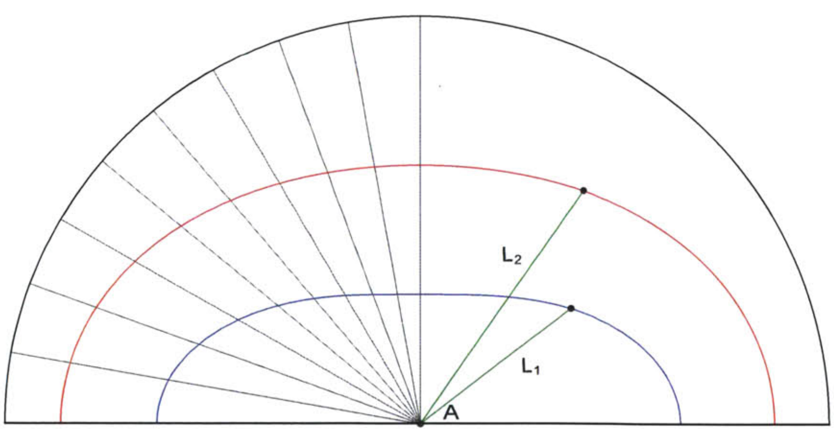


Fig 4.2.8 Crease pattern [DK]

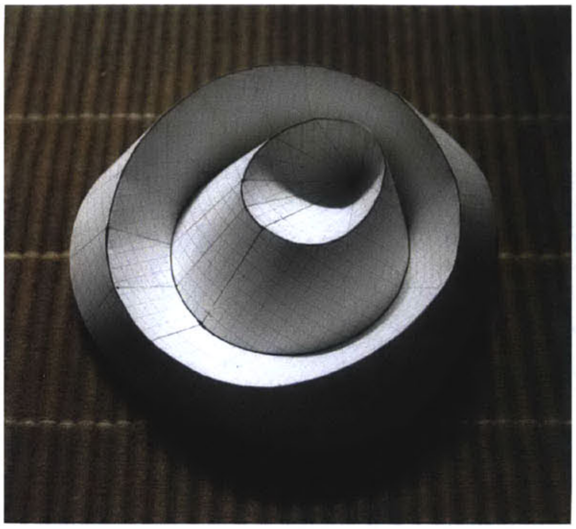


Fig 4.2.9 Paper model (undated, DAH [EAH])

Huffman calls the above model 'Crushed cone #1', which follows the same constraint as the previous example, where all reflections occur within the volume given by the first truncated cone (Fig 4.2.9). The difference between the 2 designs consists of the number of reflections, 3 in this case.

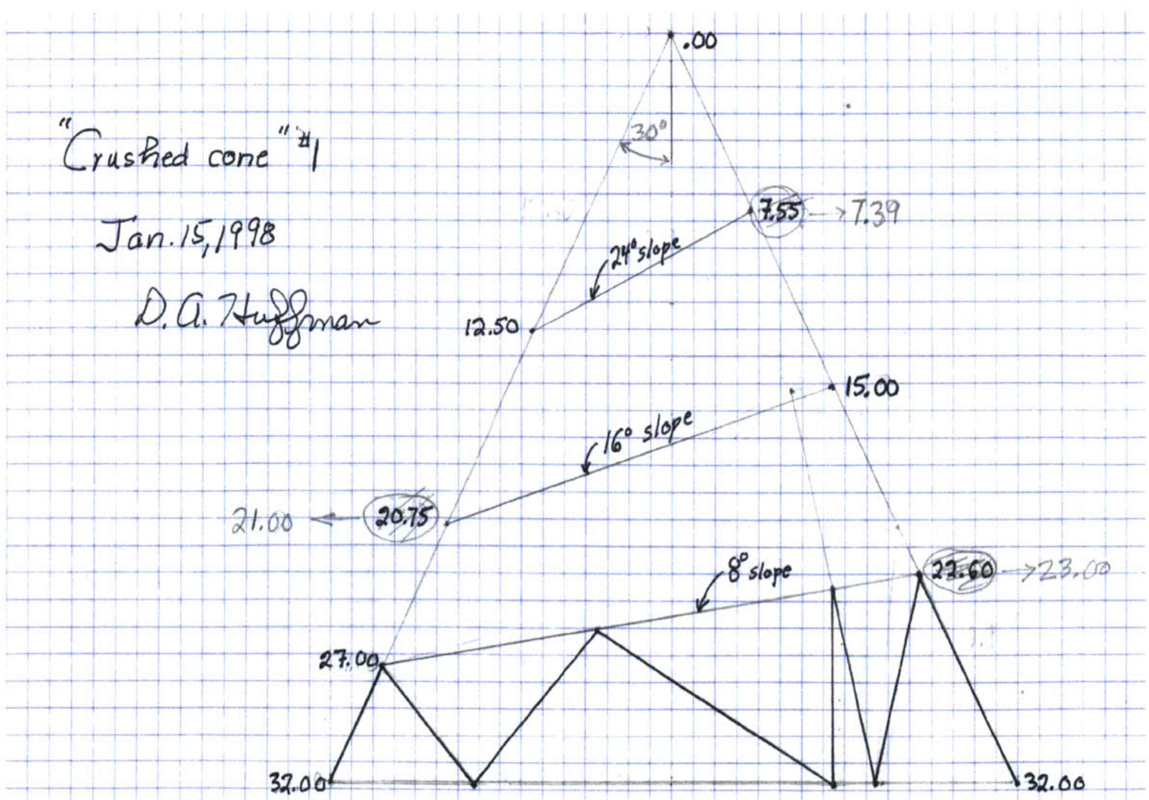


Fig 4.2.10 Section of 'Crushed cone #1' (1998, DAH [DK])

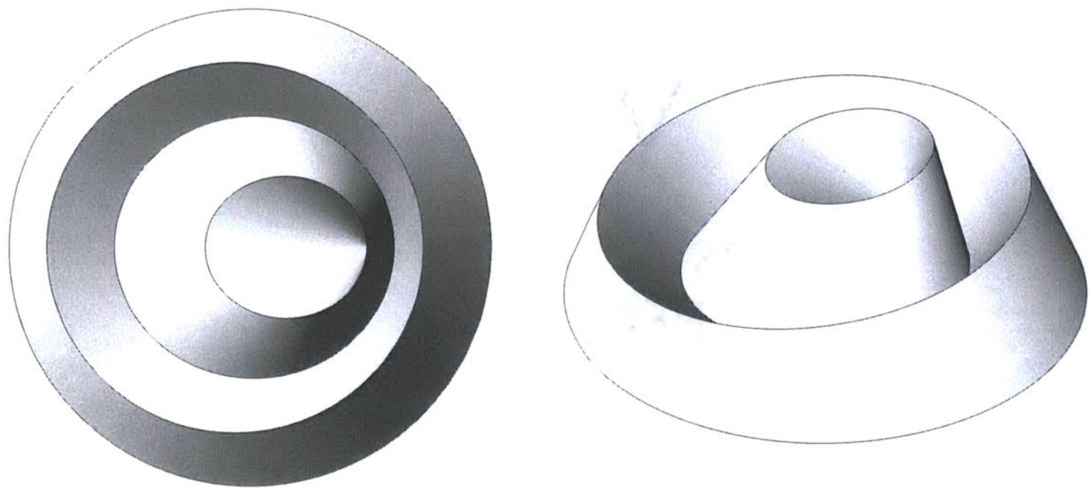


Fig 4.2.11 3d model [UP]

Section analysis and crease pattern

The first reflection occurs at an 8° slope and the subsequent 2 reflections at 16° and 24°. Huffman 'stretches' an equilateral cone by lengthening the left and right edges, but keeps all angles the same. The operations align the right edge of the last truncated cone in section such that it is perpendicular to the base. We can observe that the apex of the cone lies in the projection of the vertical ruling (Fig 4.2.11 right).

The design is comprised of 3 mountain and valley creases, but Huffman only uses one color (Fig 4.2.12).

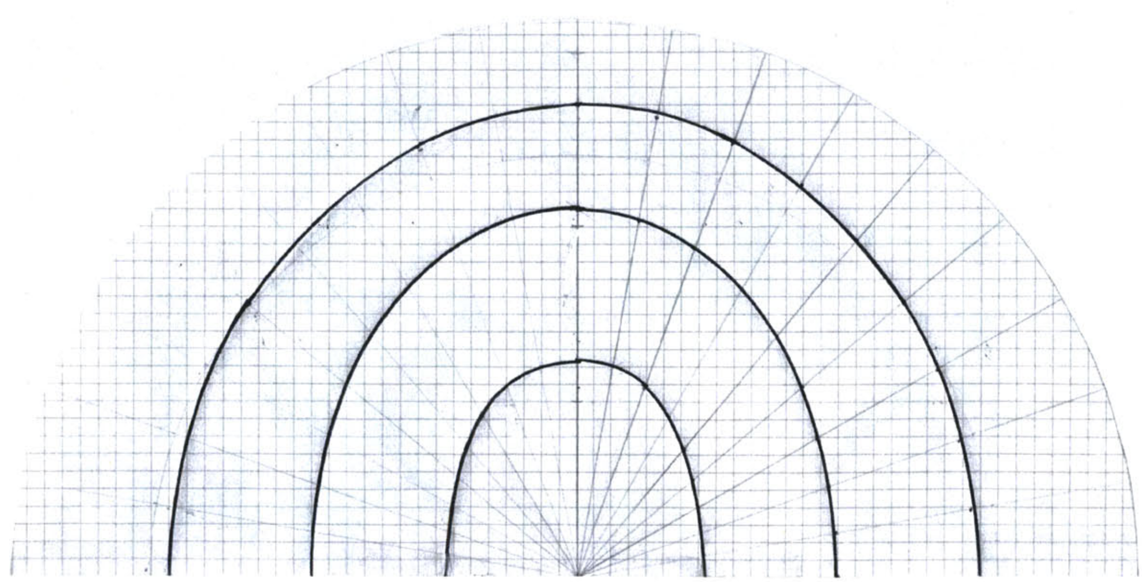


Fig 4.2.12 Paper model (undated DAH [DK])

Cone reflection with rotating axis

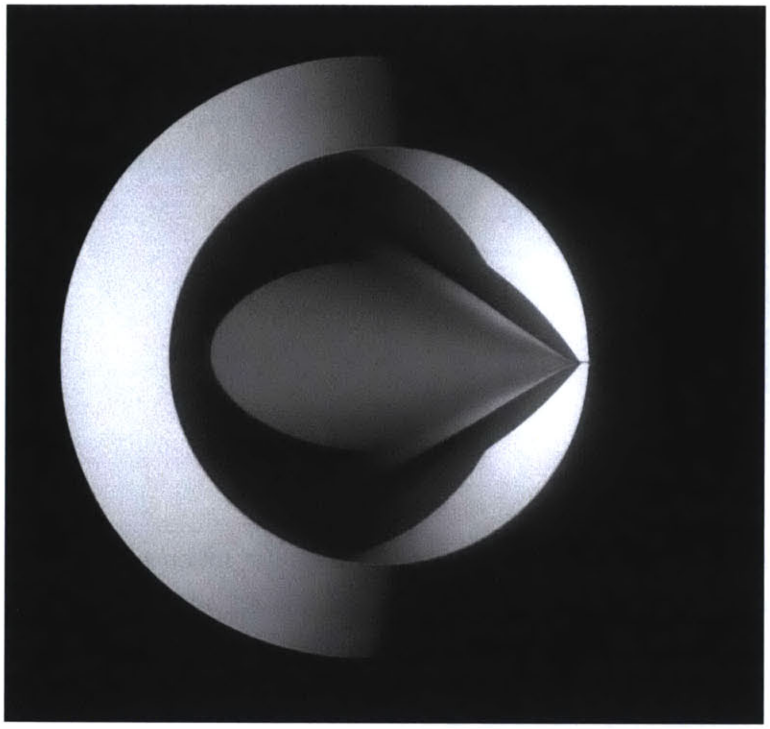


Fig 4.2.13 'Cone, reflected' (1978, DAH [DAH])

This subsection presents cone reflections that gradually rotate the cone axis in one direction. The design approach controls the parameters of the relationships between the edges of the truncated cones and their axes at every step. The following 2 examples, which Huffman makes as a set in 1978, appear to be related to the previous category as we can identify an upper bound. The above 'Cone, reflected' does not however have a lower bound, which we can study further in the in section (Fig 4.2.13).

Section analysis and crease pattern

Huffman reflects the equilateral cone only twice and turns the cone axis such that the left edge in the section of the 3d digital model becomes horizontal (Fig 4.2.14). As a result the apex touches the crease of the first reflection. The second and third truncated cones share the ruling of their right edge.

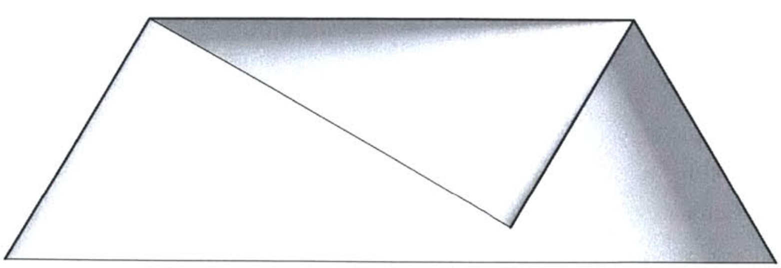


Fig 4.2.14 Folded section [DK]

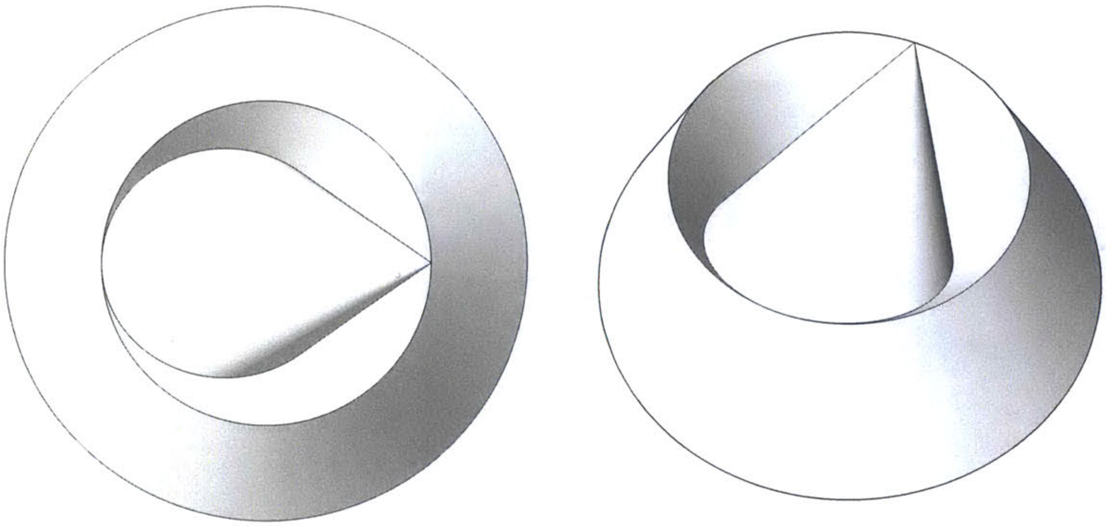


Fig 4.2.15 3d model [UP]

The crease pattern consists of a mountain and a valley crease that touch each other, which allows the vertical ruling to become horizontal in the folded state (Fig 4.2.16).
The distance between *A* and the valley crease along the horizontal cut line in the crease pattern needs to be equal to the distance between the valley crease and the mountain crease on the same line.

Notes
The design is surprisingly dynamic if we consider that it consists of only 2 creases. Huffman photographs the model after taking great care in assembling it.

The design also displays similarities to a design by Ron Resch, in which he examines cones as a special case of his more general work with curved creases (Fig 4.2.17). Resch writes on his former website: 'This particular model 'Yellow Folded Cones: Kissing (1969-1970)' was

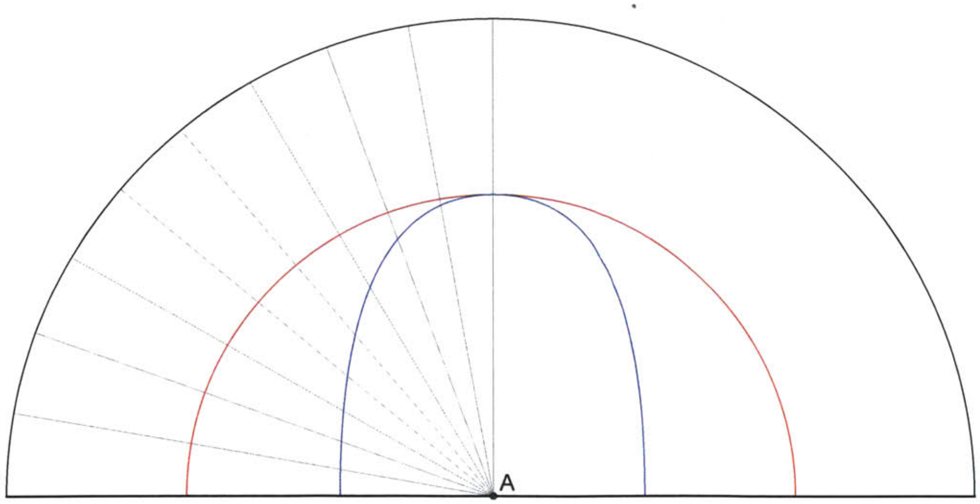


Fig 4.2.16 Crease pattern [DK]

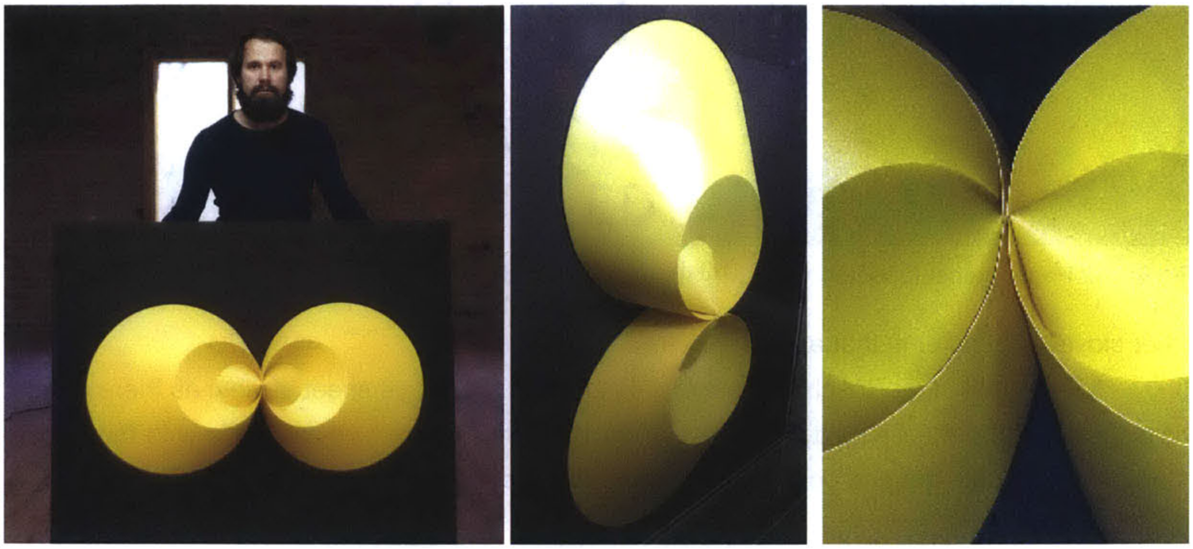


Fig 4.2.17 'Yellow Folded Cones: Kissing' by Ron Resch, 1969-1970

executed using computer aided scoring with a custom made CAD Scoring machine.' [Res]

Ephraim Cohen, one of Resch's collaborators, calculates the crease configurations via his software (Fig 4.1.18). The work is exhibited in the 1972 'Ron Resch and the Computer show at the Utah Museum of Fine Art, University of Utah, Salt Lake City. As mentioned in the biographical chapter, it is likely Huffman sees the work during his visit in 1973.

If we consider a single set of reflected cones and imagine the horizontal arrangement in the above image, then the 3 edges of the left 3 truncated cones in the symmetry plane align at the center where the 2 sets of cones meet. The 2 reflections occur along planes perpendicular to these edges, not the slanted base of the first cone. Huffman's and Resch's designs relate to each

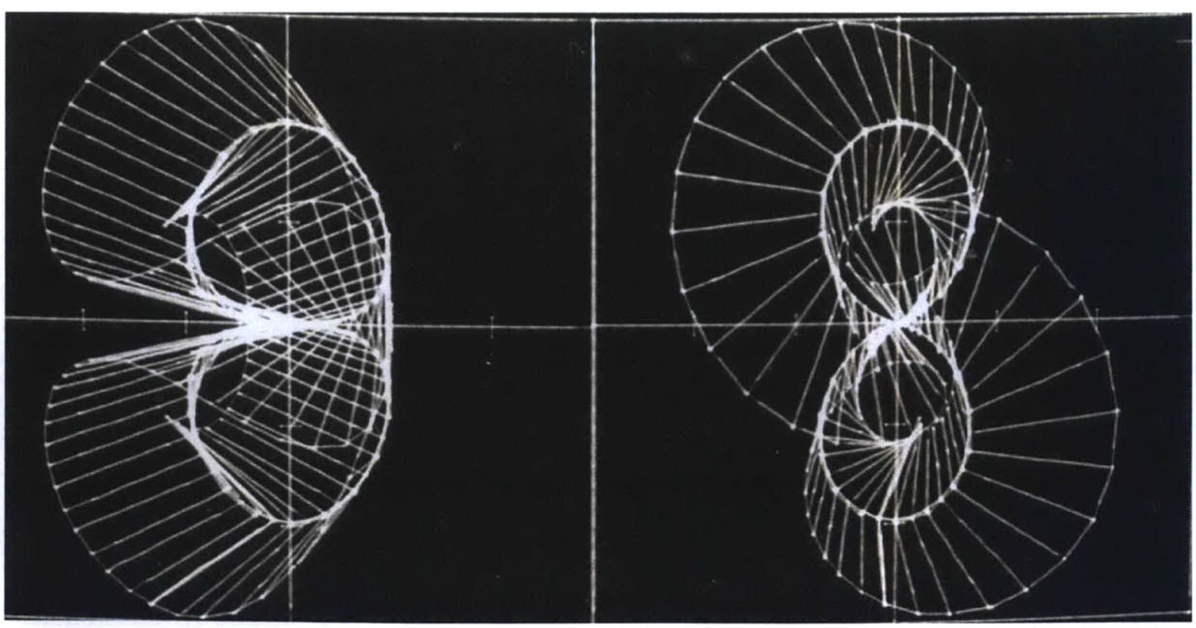


Fig 4.2.18 Screen shots of similar design, Ephraim Cohen, around 1970

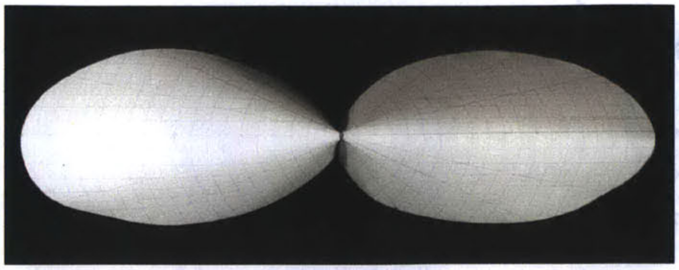
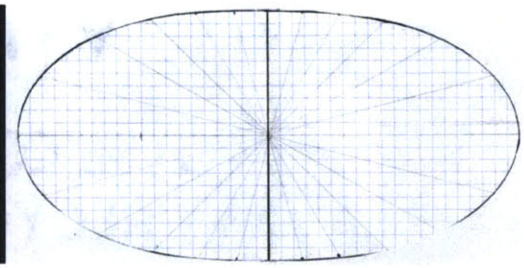



Fig 4.2.19 Paper model (undated, DAH [EAH]), (undated, DAH [DK])

other closely, but differ in that aspect.

It is possible that Resch made the sculpture out of a single sheet of vinyl as there are no visible seams (Fig 4.2.17 right). Huffman might have investigated similar goals with the above model in which both general cones are made of one sheet of paper (Fig 4.2.19).

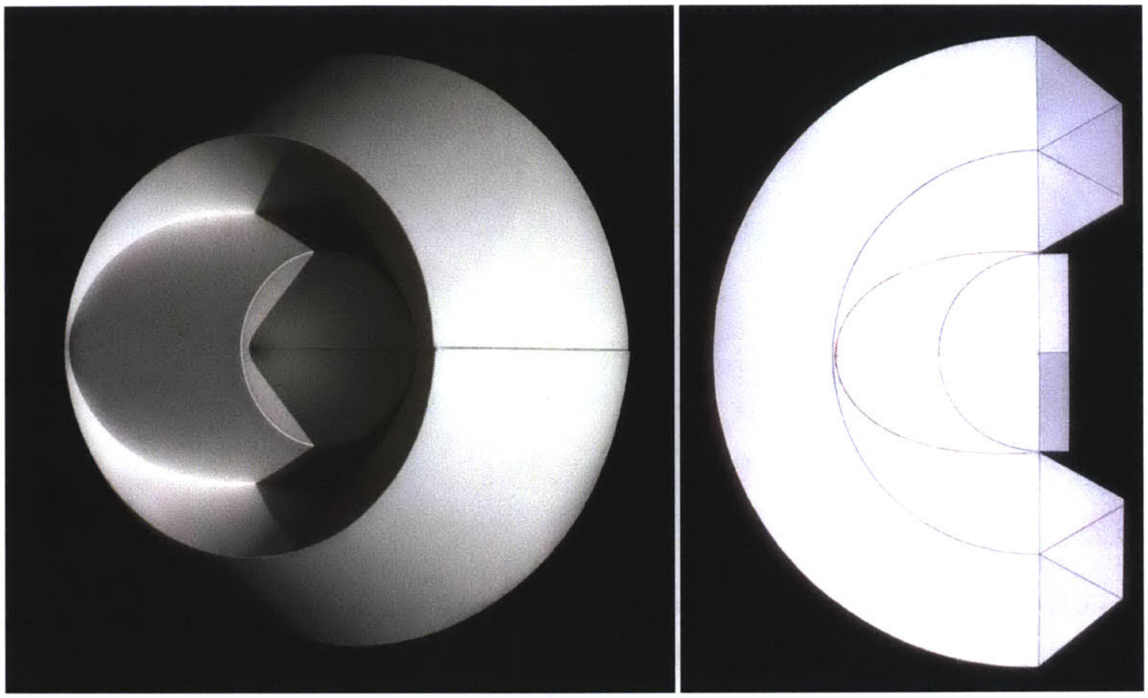


Fig 4.2.20 'Bent / reflected cone' (1978, DAH [DAH])

The second model of the pair is called 'Bent / reflected cone' in which Huffman adds a reflection to the previous example. He documents the process of making the model and takes a picture of the inside prior to folding it (Fig 4.2.20 right).

Section analysis and crease pattern

Huffman reflects the same regular cone twice to turn the cone axis such that the left edge of the second truncated cone becomes horizontal similar to the previous example (Fig 4.2.21). The 2nd cone that includes the apex in the first model gets reflected one more time such that its right edge is parallel to the base.

The crease pattern consists of a mountain and 2 valley creases. The first and second creases touch each other similar to the previous example, which allows the ruling along the vertical in the crease pattern (Fig 4.2.23) to turn horizontal in the folded state (Fig 4.2.21). The

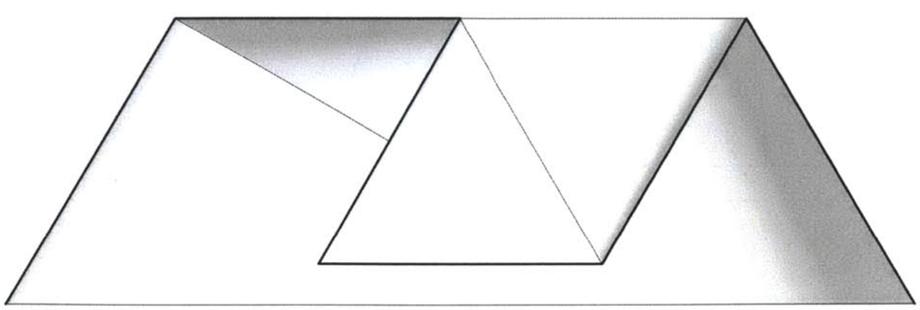


Fig 4.2.21 Folded section [DK]

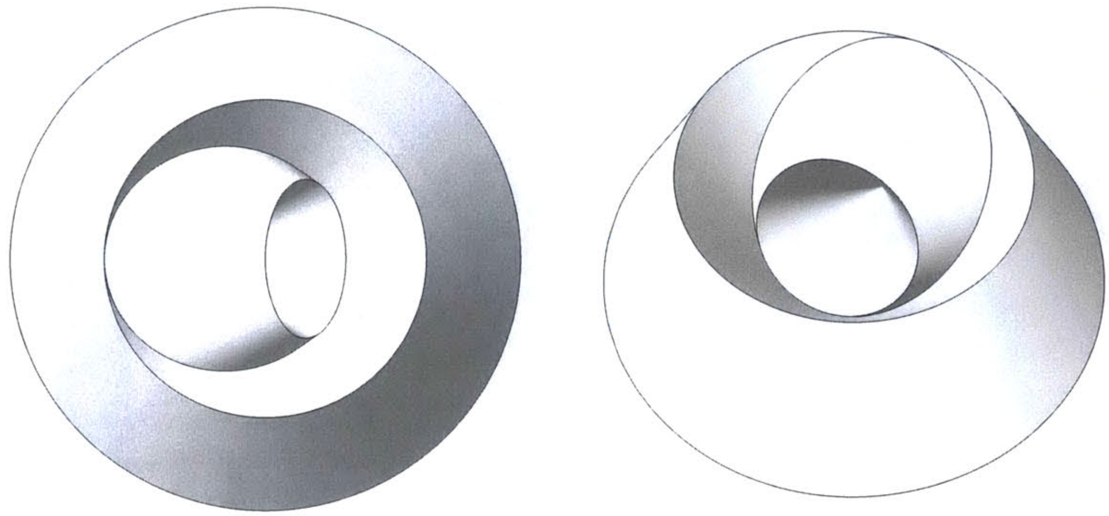


Fig 4.2.22 3d model [UP]

distances between the first and second crease and between the apex and the last crease are equal, which gives the design a very regular appearance.

Notes

Huffman builds the model using a flap with a triangular guide to stabilize the first and second truncated cones at the seam. He also adds a rectangular flap between the last crease and the apex that ends up on the back side of the folded model.

He takes great care in assembling this model as well.

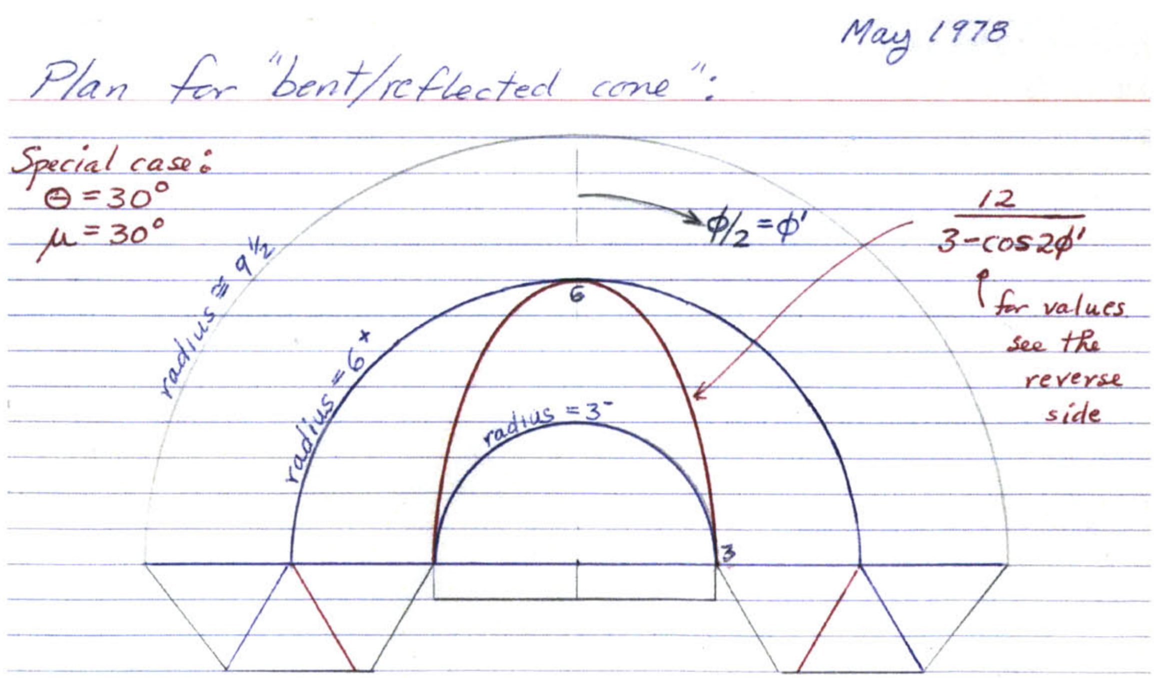


Fig 4.2.23 Index card (1978, DAH [DK])

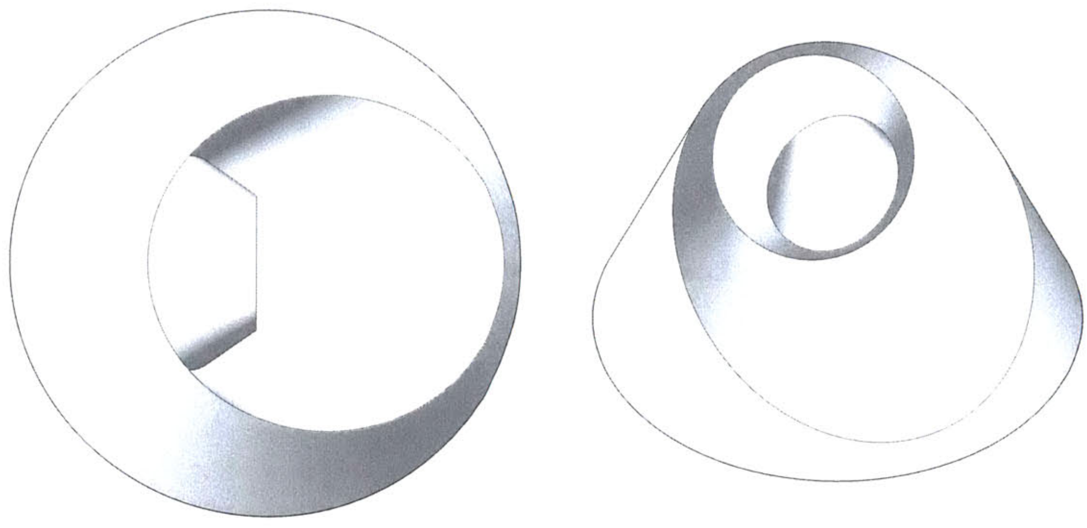


Fig 4.2.24 3d model [UP]

The following 5 designs turn the cone axis continuously in one direction and we will see several iterations of the same idea. Huffman designs this reflected cone example with only 3 reflections and turns the plane of the last reflection normal to the base (Fig 4.2.26). The last reflection plane collapses into an edge in the center of the plan projection (Fig 4.2.24 left).

Section analysis and crease pattern

The reconstructed sections follow Huffman's conceptual drawings, where I trace the creases on the unfolded initial cone. The 3 reflections occur twice at $\pi/6$ and once parallel to the base (Fig 4.2.25). We can describe the design in the form of an algorithm that consists of an uneven number of reflections. Here, the first and last reflection use the sloped plane.

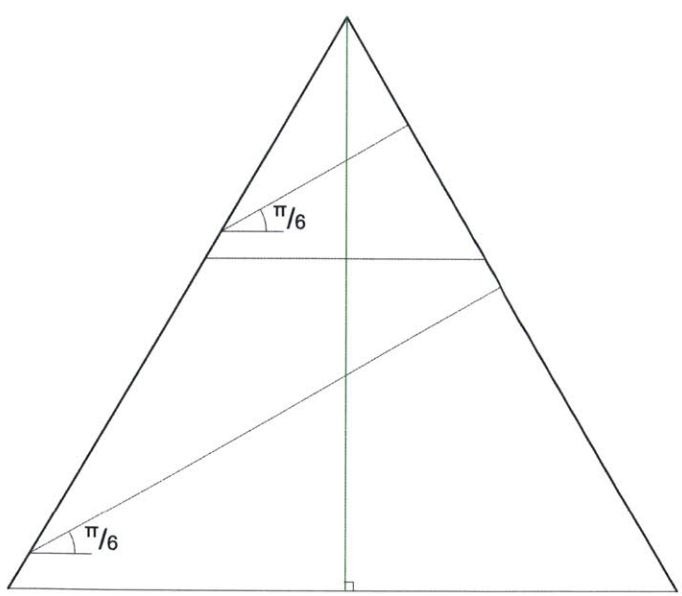


Fig 4.2.25 Section [DK]

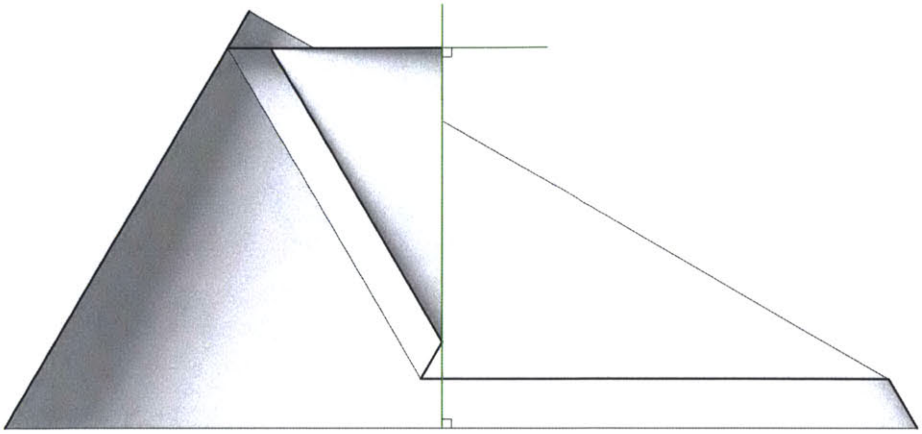


Fig 4.2.26 Folded section [UP]

Most of the following examples take an equilateral cone as a point of departure and hence all have a half-disc as crease patterns. The crease pattern in this case consists of 1 arc as a valley fold and 2 as mountain folds.

Notes

Huffman makes several of these models and appears to think of them as sketches as he does not make them in vinyl. He marks 2 angles as α and β. A matching angle in this case could be the slope of the reflection plane (for α), but his notation appears to refer to different angles in later examples (Fig 4.2.27).

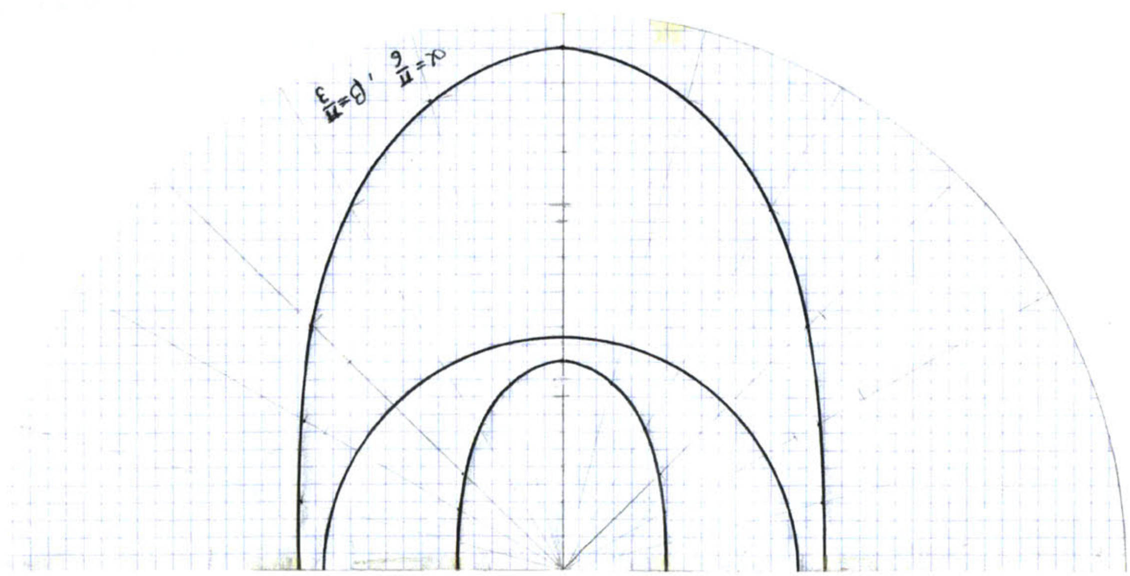


Fig 4.2.27 Paper model, (undated, DAH [DK])

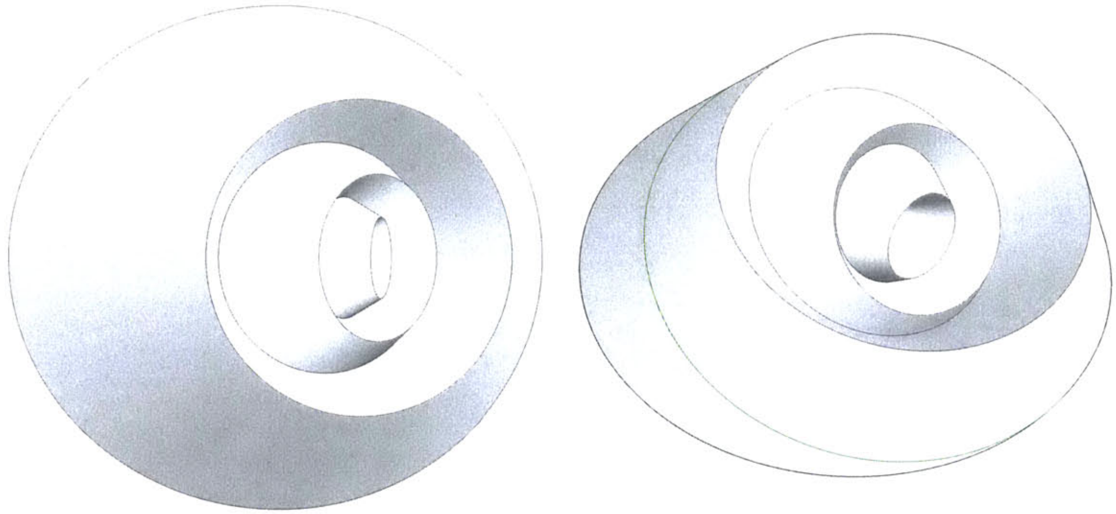


Fig 4.2.28 3d model [UP]

Huffman makes the next few examples in this series with 5 reflections and seems to be interested in rotating the cone axis horizontally. He also appears to want to turn specific rulings vertically.

The above design differs from the other examples in this series as it uses a tilted base (Fig 4.2.28). Huffman might have wanted to provide enough space for all reflections to occur without any truncated cone passing through the base.

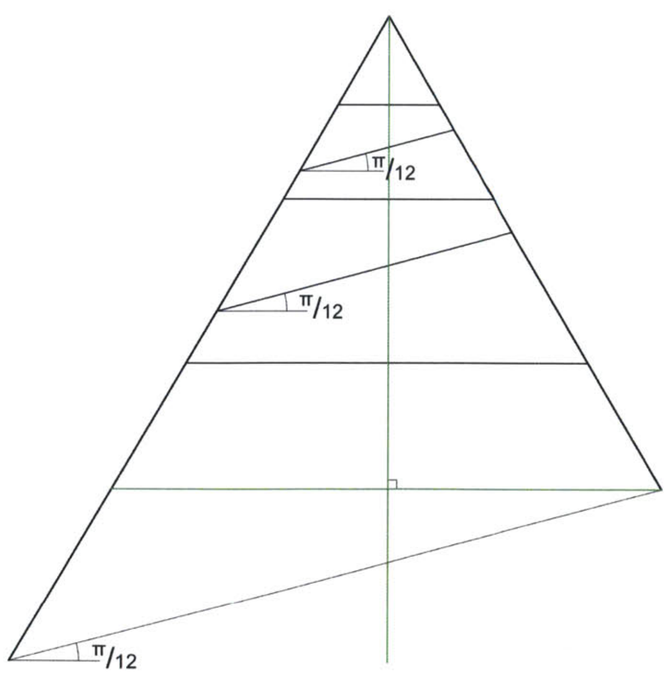


Fig 4.2.29 Section [DK]

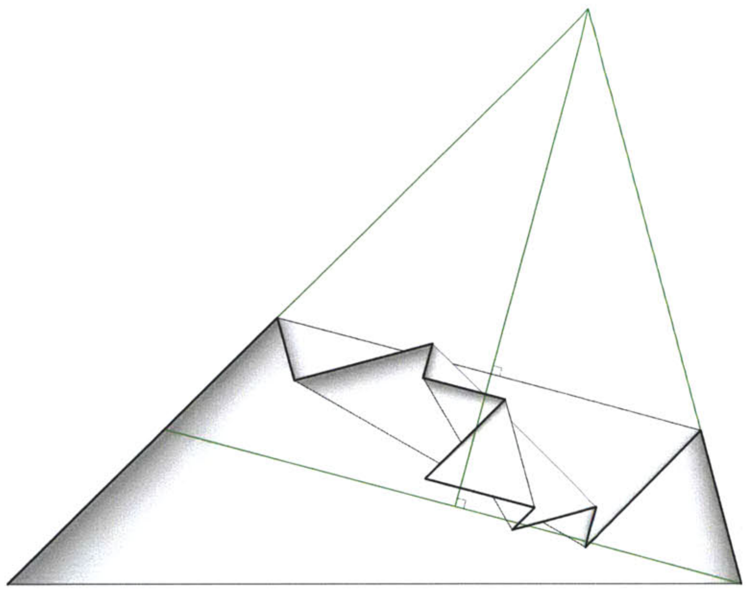


Fig 4.2.30 Folded section [UP]

Section analysis and crease pattern

Huffman uses a slope of $^{\pi}/_{12}$ for each of the 2 sloped reflections (Fig 4.2.29) and once to cut the cone at the bottom. The algorithm would again consist of an uneven number of reflections. The second and fourth reflections use the sloped plane.

These examples take a truncated equilateral cone as the initial step, which results in an elongated boundary in the crease pattern with 3 mountains and 2 valleys (Fig 4.2.31).

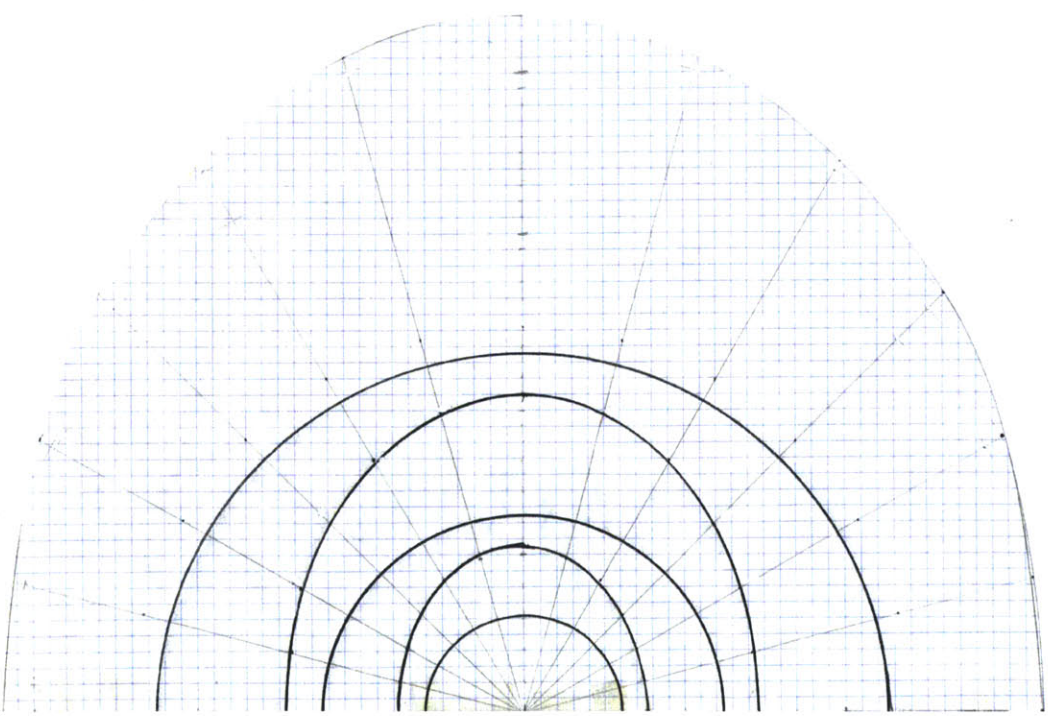


Fig 4.2.31 Paper model (undated, DAH [DK])

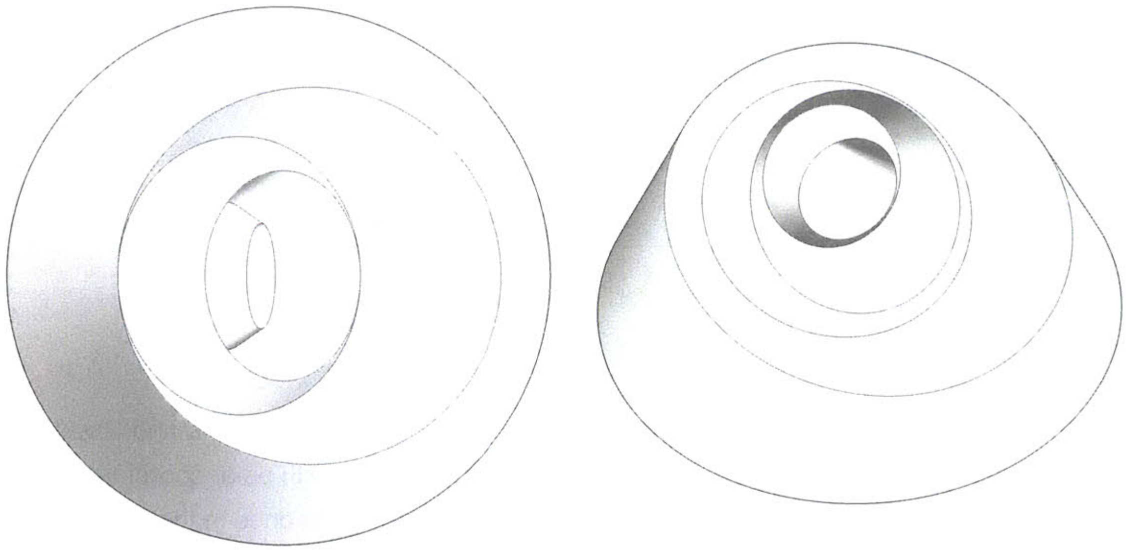


Fig 4.2.32 3d model [UP]

Huffman makes this model called 'Folded cone with $\alpha = \pi/6$ and $\beta = \pi/12$' in 1998 (Fig 4.2.32). It is likely from the same series as the previous examples. It uses the same graph paper and the models are all in similar condition. The salient feature of this design is the horizontally rotated cone axis of the last truncated cone (Fig 4.2.34), but none of the reflections turn perpendicular to the base as we can see in the projected top view (Fig 4.2.32 left).

Section analysis and crease pattern

The rotating reflections are based on a slope of $\pi/12$ and occur 3 times, including the first and last reflection (Fig 4.2.33). The algorithm would consist of an uneven number of reflections, but here the first, third and last reflection use the sloped plane.

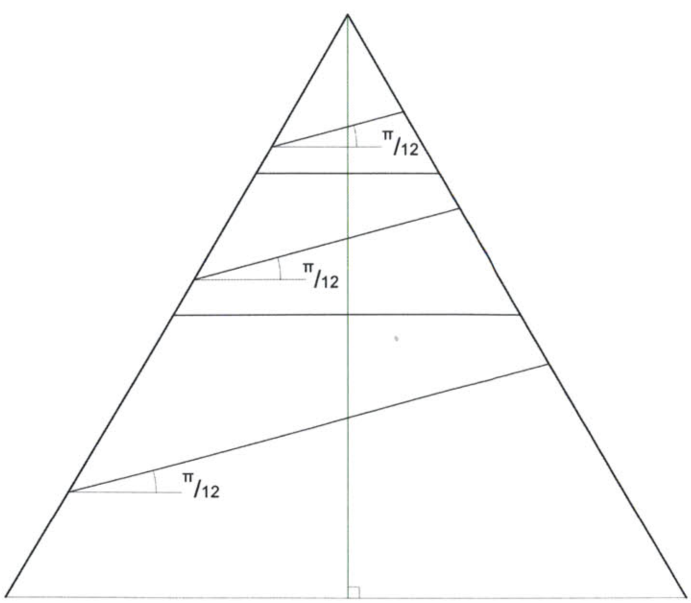


Fig 4.2.33 Section [DK]

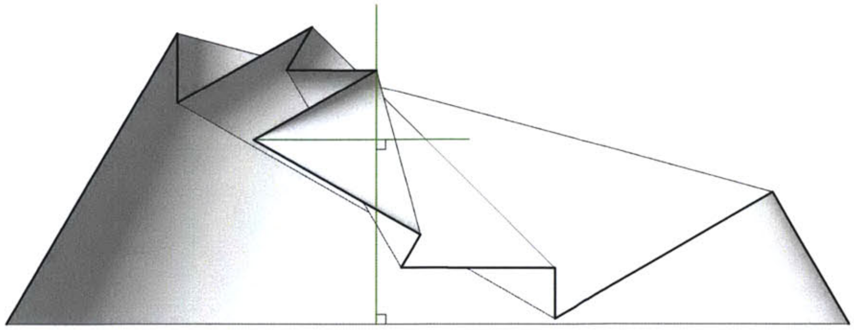


Fig 4.2.34 Folded section [UP]

The equilateral cone results in a regular crease pattern with 3 mountains and 2 valleys. We can clearly see Huffman's working method of plotting curve points with polar coordinates (Fig 4.2.35). The design also exists as a sketch model similar to the previous ones in the series.

Notes

Huffman appears to make the series in 1998, 20 years after his 'Cone reflected seven times' shown at the end of this section.

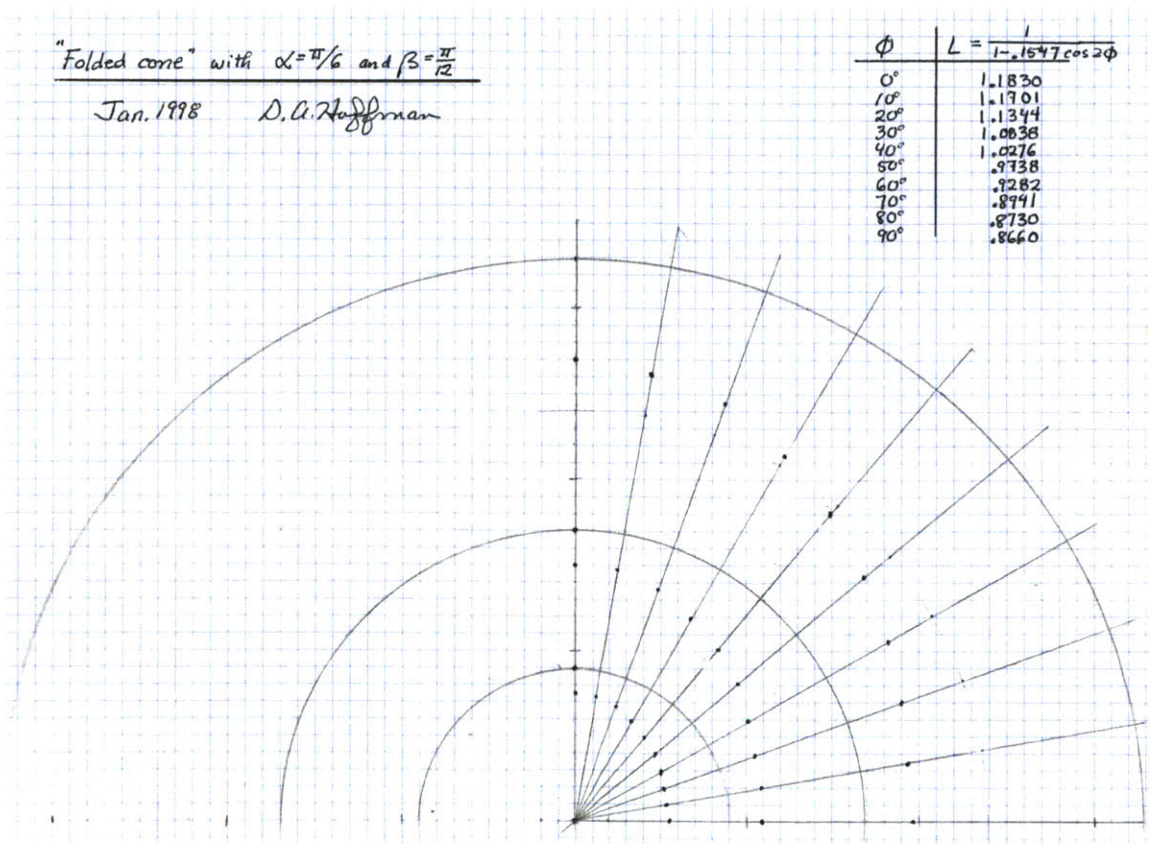


Fig 4.2.35 Drawing (1998, DAH [DK])

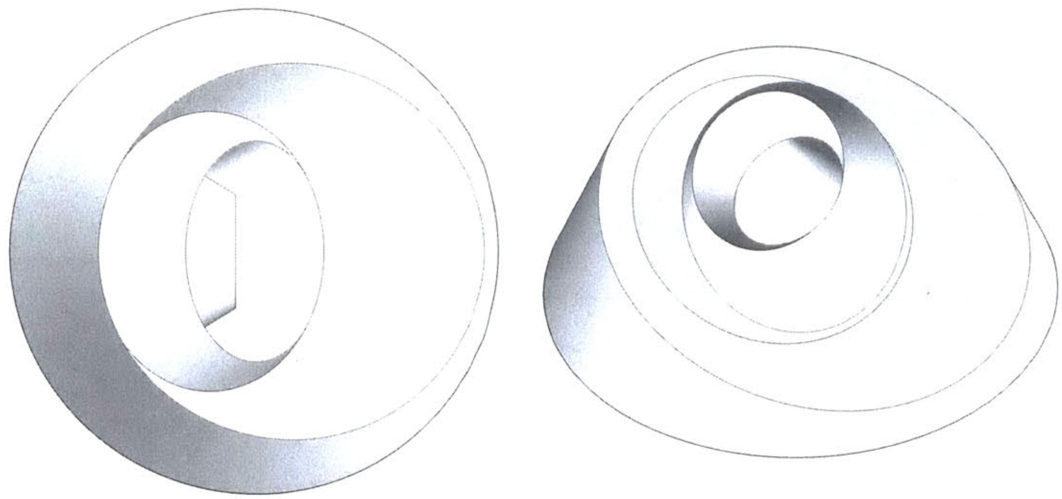


Fig 4.2.36 3d model [UP]

The 6 truncated cones are mirrored in an alternating way just like the previous examples in this series (Fig 4.2.36). The rotation is synchronized such that the last reflection becomes normal to the base in the folded state. It collapses into an edge in the projected view from the top (Fig 4.2.36 left). The salient 'flaw' of this design consists of truncated cones passing through the base. The consequence for the paper model is that it can not rest on the base like all other previous models do (Fig 4.2.38).

Section analysis and crease pattern

This example uses rotating reflections with a slope of $\pi/10$, which occur 3 times, again including the first and last (Fig 4.2.37). Here, the algorithm would consist of an uneven number of reflections with the first, third and last reflection as sloped plane.

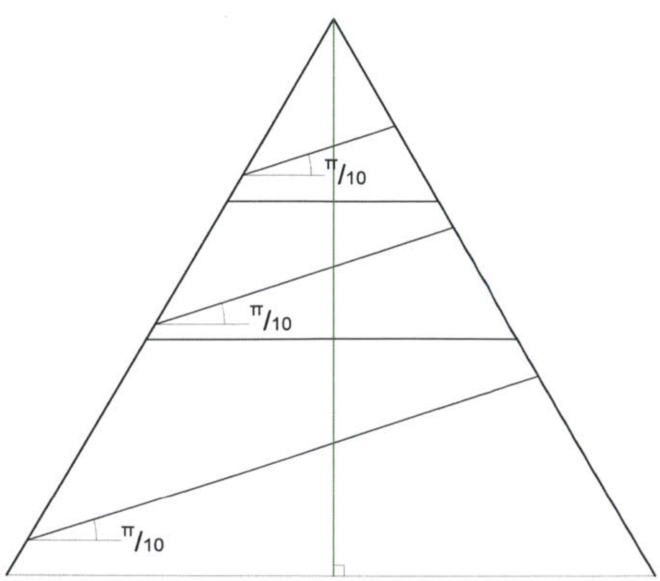


Fig 4.2.37 Section [DK]

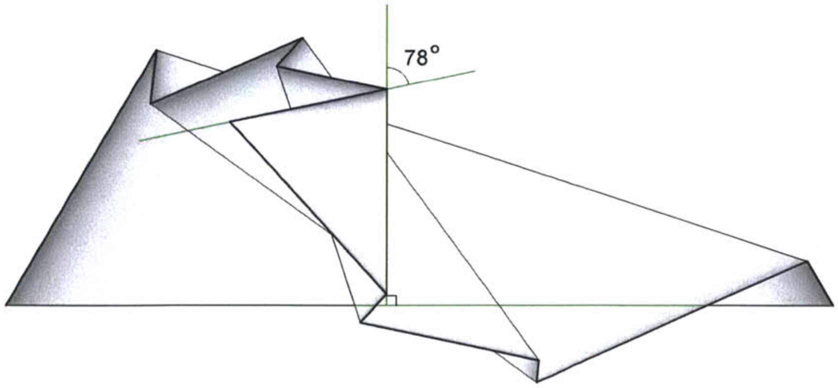


Fig 4.2.38 Folded section [UP]

The crease pattern of the equilateral cone consists of 3 mountains and 2 arcs as valley folds (Fig 4.2.39). Huffman adjusts the first arc when he redraws the crease with a ball pen and shrinks the diameter slightly. The angle designations in this case could mean that α is half the angle at the top of the cone and that β is the slope of the reflection plane.

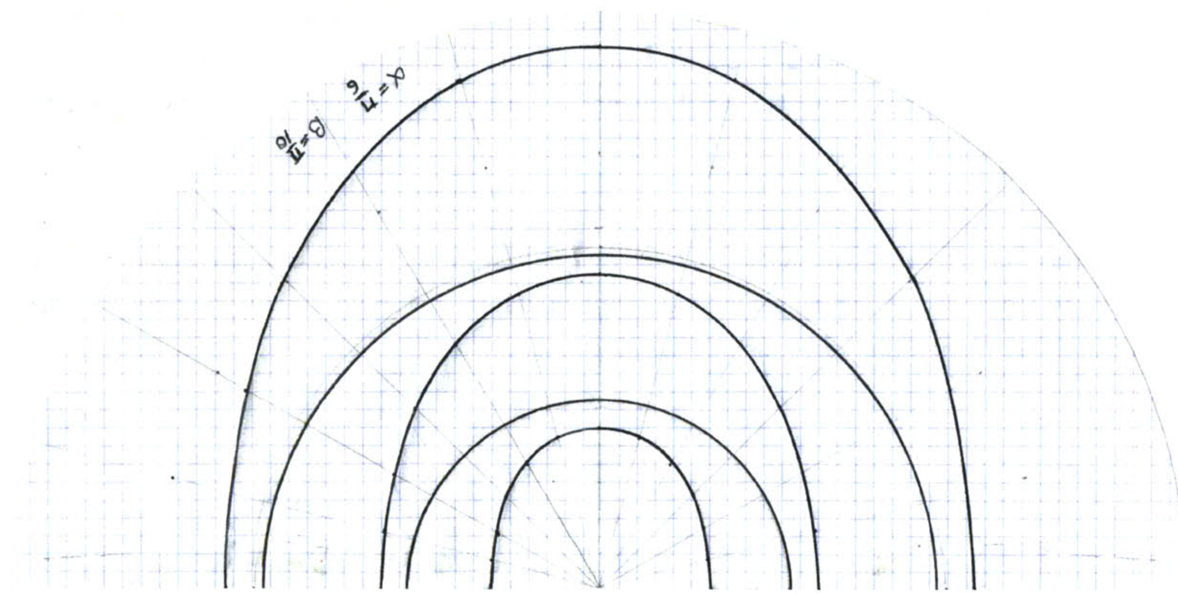


Fig 4.2.39 Paper model (undated, DAH [DK])

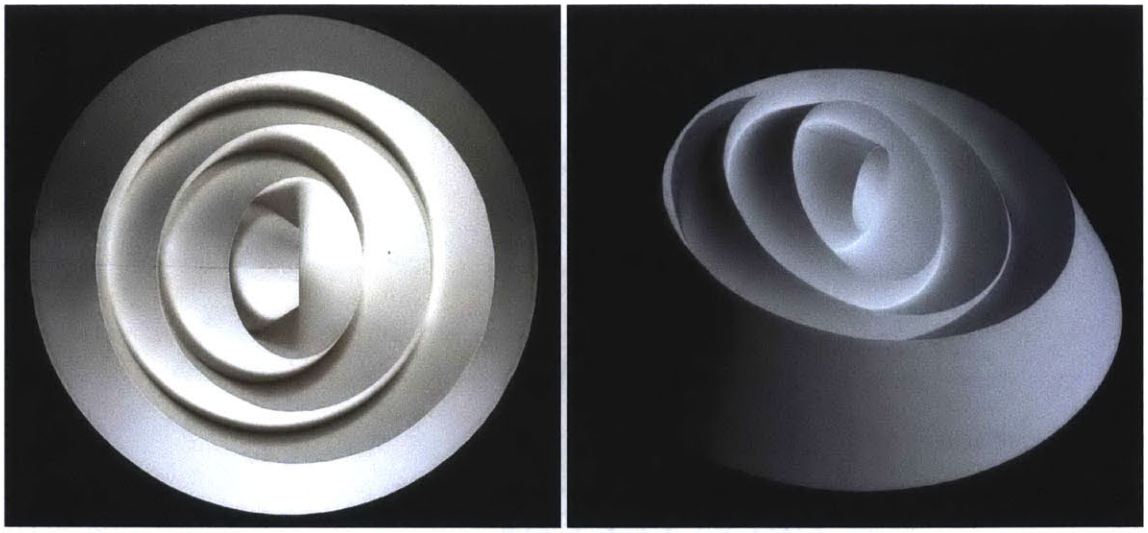


Fig 4.2.40 'Cone, reflected seven times' (undated, DAH [TG]), (1978, DAH [DAH])

Huffman makes 'Cone, reflected seven times' in vinyl and documents the entire construction process exhaustively via photographs (Fig 4.2.40). The 8 truncated cones rotate in one direction similar to the previous examples. It is curious that this example is from 1978, 20 years before Huffman investigates the previously presented designs. The salient feature of this version consists of the last vertically rotated reflection plane in the folded state (Fig 4.2.42).

Section analysis and crease pattern

The rotated reflections consist of 3 horizontal and 4 inclined cuts at a slope of $\pi/14$ (Fig 4.2.41). Huffman carefully calibrates the reflections such that no truncated cones pass through the base

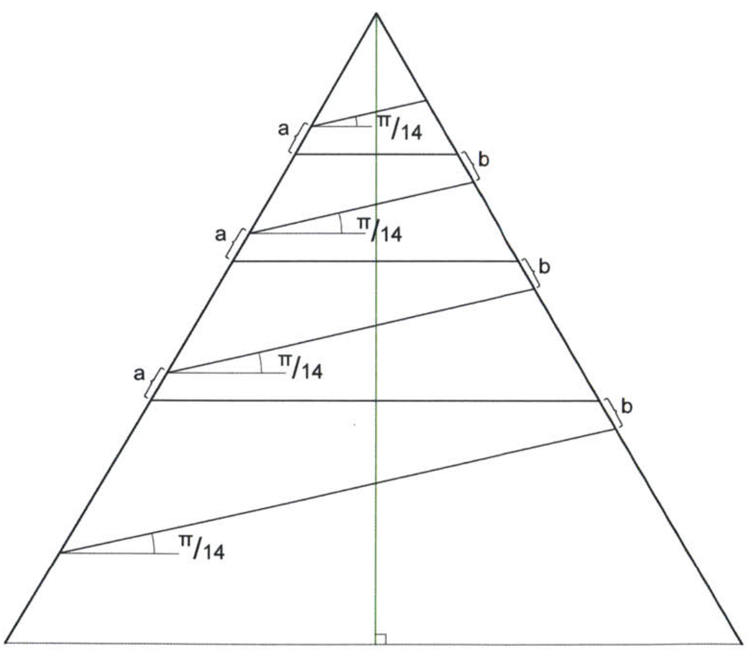


Fig 4.2.41 Section [DK]

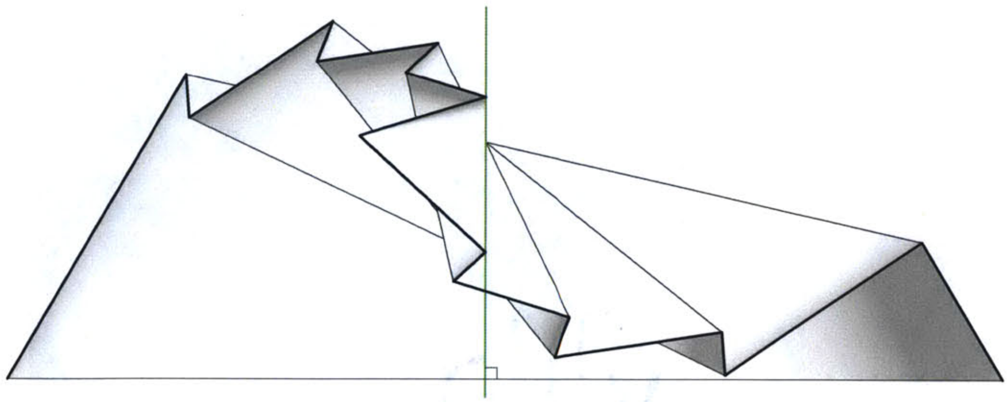


Fig 4.2.42 Folded section [UP]

and appears to use equal distances from crease to crease indicated as *a* and *b*. The algorithm would consist of an uneven number of reflections with the first, third, fifth and last reflection as sloped planes.

The crease pattern of the equilateral cone consists of 4 mountain folds and 3 arcs as valley creases (Fig 4.2.44). The distances *a* and *b* give the crease pattern a very regular appearance.

Notes

In Huffman's documentation we can see a straight tab-like extension (Fig 4.2.45 left). Since it does not follow the corresponding arc, Huffman completes the model without it (Fig 4.2.45 center). He appears to not be content with the quality of his execution and mentions in one of his inventories that this model should not be exhibited because the seam is not crafted well enough .

This is the only reflected cone with a gradually rotating axis that Huffman makes in vinyl. The model was hung with a different orientation, which can be seen in Matthew Mulbry's images,

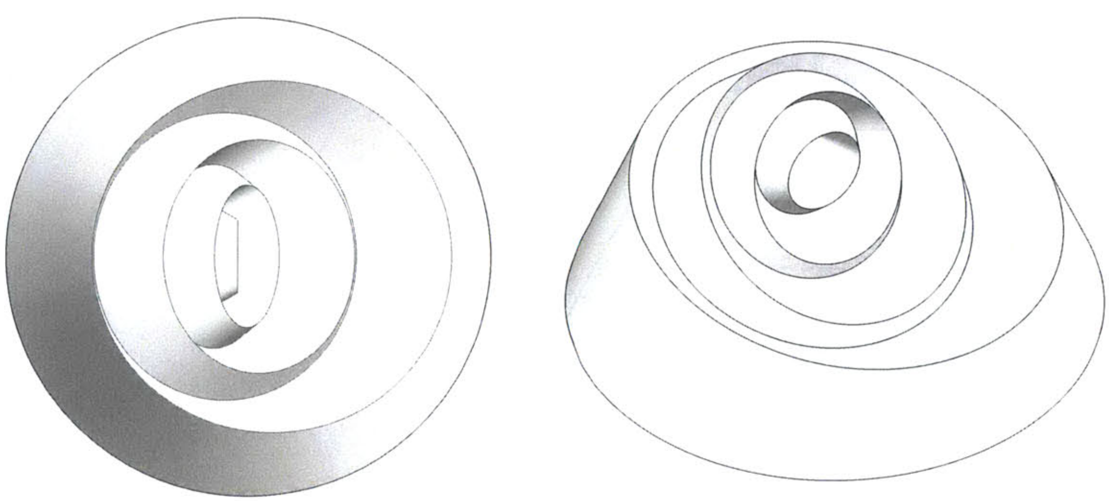


Fig 4.2.43 3d model [UP]

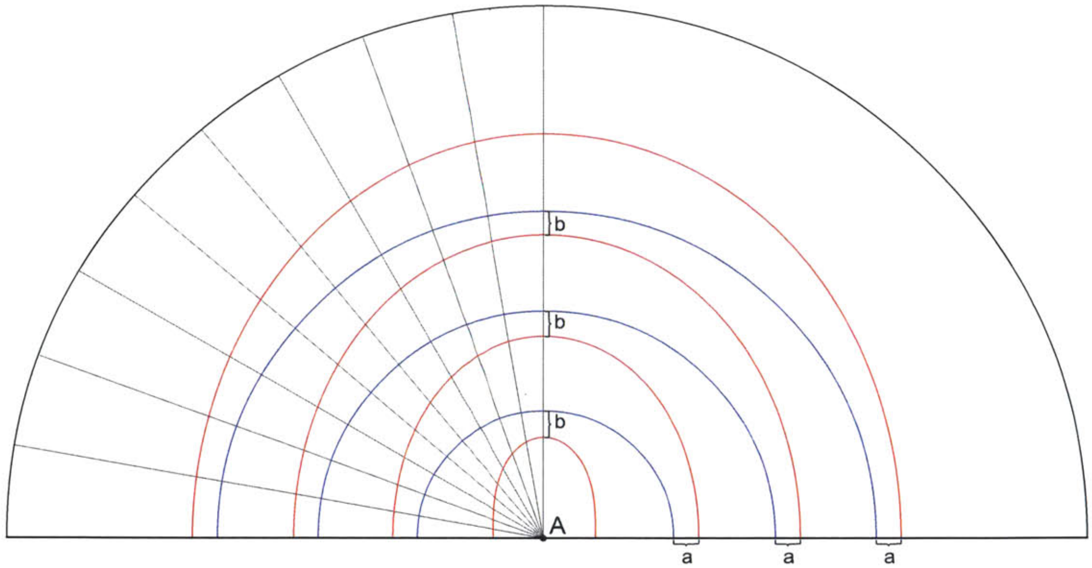


Fig 4.2.44 Crease pattern [DK]

who photographs Huffman for an article in the Scientific American (Fig 4.2.45 right). This is one of the models published by Margaret Wertheim in her New York Times article [Wer 04].

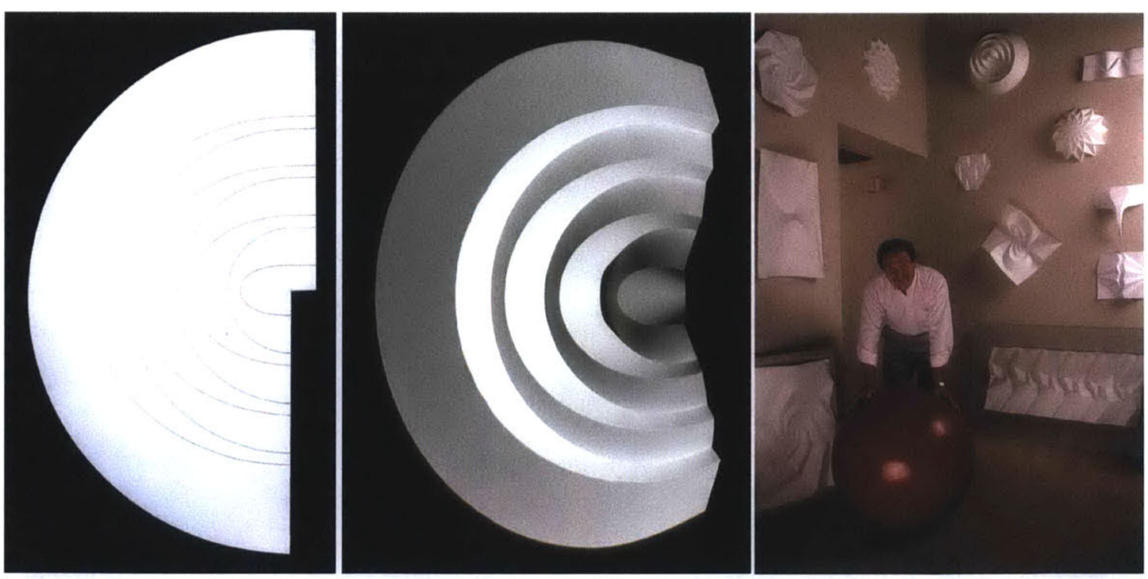


Fig 4.2.45 Vinyl model (1978, DAH [DAH]), Identical model (1978, DAH [DAH]), Photo (1991, [MM])

Cone reflection parallel to axis

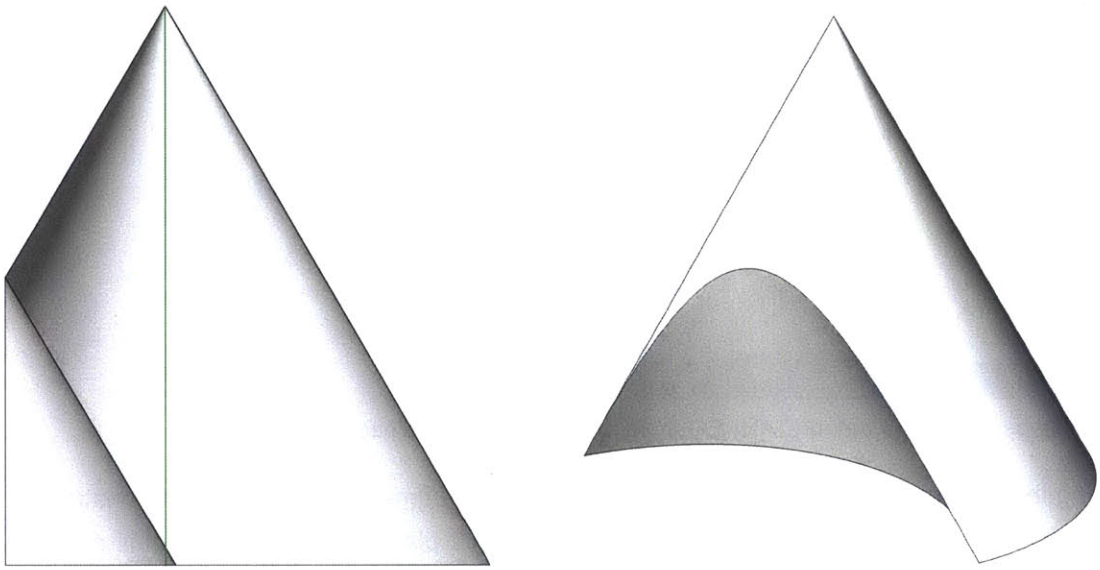


Fig 4.2.46 Folded section and 3d model [UP]

This section presents Huffman's cone reflections that occur parallel to the main axis of the cone. Only 2 examples exist and both appears to be sketch models as he does not make any vinyl versions of the designs (Fig 4.2.47).

Section analysis and crease pattern

Huffman probably uses this model to derive the formula for a hyperbolic section of a cone in its developed state. He writes the result on the back of the crease pattern (Fig 4.2.47 right).

The crease pattern consists of a 180° developed cone with only 1 mountain crease and the distance from the apex to the crease along the vertical ruling is not equal to the distance from the crease to the base. The discrepancy can be seen in the folded section at the base where the axis intersects the left edge of the reflected cone (Fig 4.2.46 left).

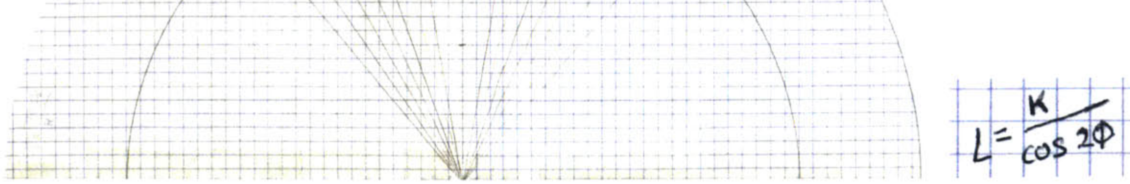


Fig 4.2.47 Paper model (undated DAH [DK]), Detail of identical model (undated DAH [DK])

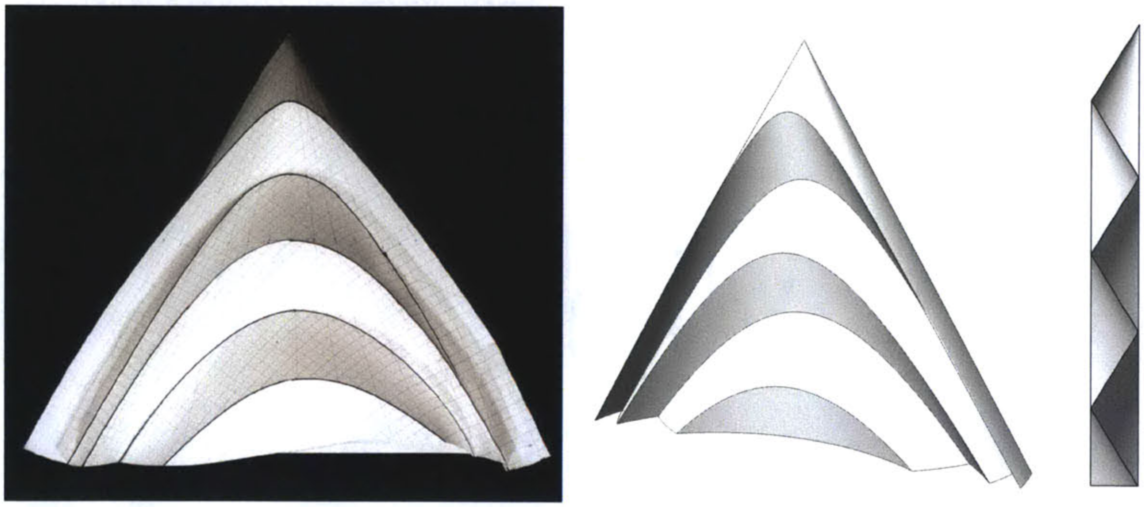


Fig 4.2.48 Paper model (undated [DAH] EAH), 3d model [UP], Folded section [UP]

Regarding the above design with vertical cone reflections, Huffman sets up a similar constraint to previous examples, where all creases need to stay within 2 planes (Fig 4.2.48). In this case the reflections occur along a plane through the main axis and another plane parallel to it (Fig 4.2.48 right).

Section analysis and crease pattern

Using the previously derived formula, Huffman distributes 5 curves equidistant from the apex of the cone. He only constructs half a cone as the crease pattern would have to continue and complete 180° (Fig 4.2.49). This is also evident in his daughter's photograph (Fig 4.2.48 left).

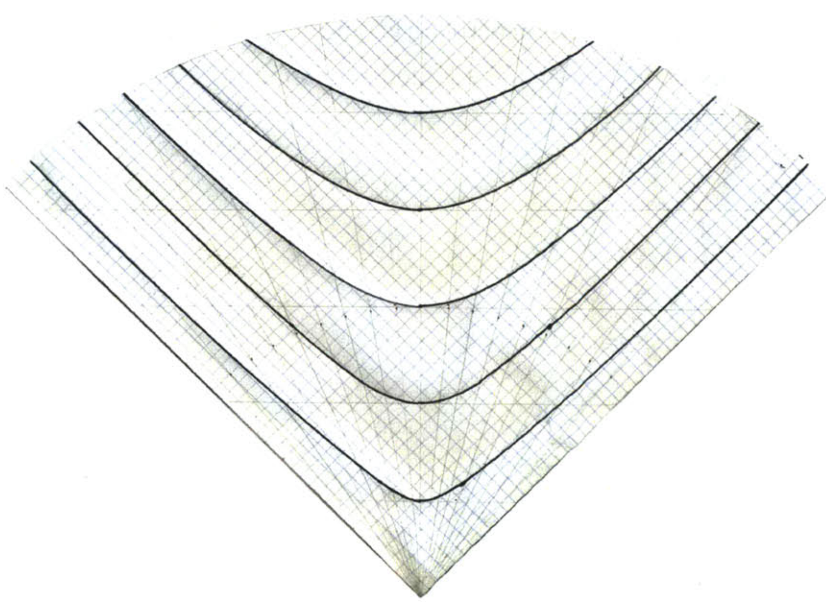


Fig 4.2.49 Paper model (undated, DAH [DK])

Cone reflection and tucking

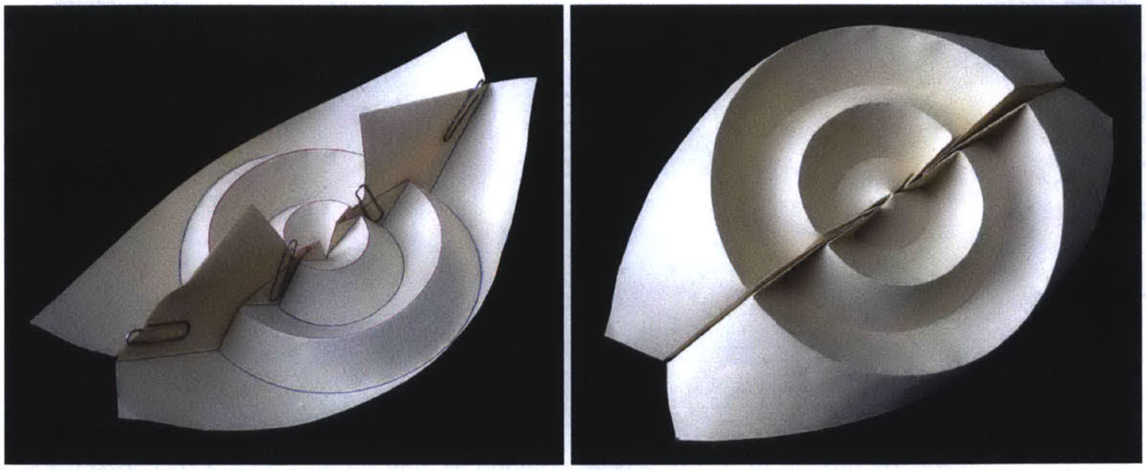


Fig 4.2.50 Paper model (undated, DAH [EAH]), (undated, DAH [EAH])

This subsection focuses on cone reflections in combination with tucking. The cone reflections appear simple, as they are parallel to the base. Remarkably, the models can be made with a single sheet of paper without cutting. Huffman only makes 3 paper models and one can think of each of these designs as their own gadget, as they tuck away paper in a specific way.

Crease pattern and ruling analysis

The prototile consists of 1/4 of the crease pattern. A triangle with 30° angle at the apex and aligned quadrilaterals form the tuck fold. The 4 arcs reach the vertical edge of the prototile (Fig 4.2.51). All vertices of the tuck fold are flat foldable and satisfy Huffman's π-condition .

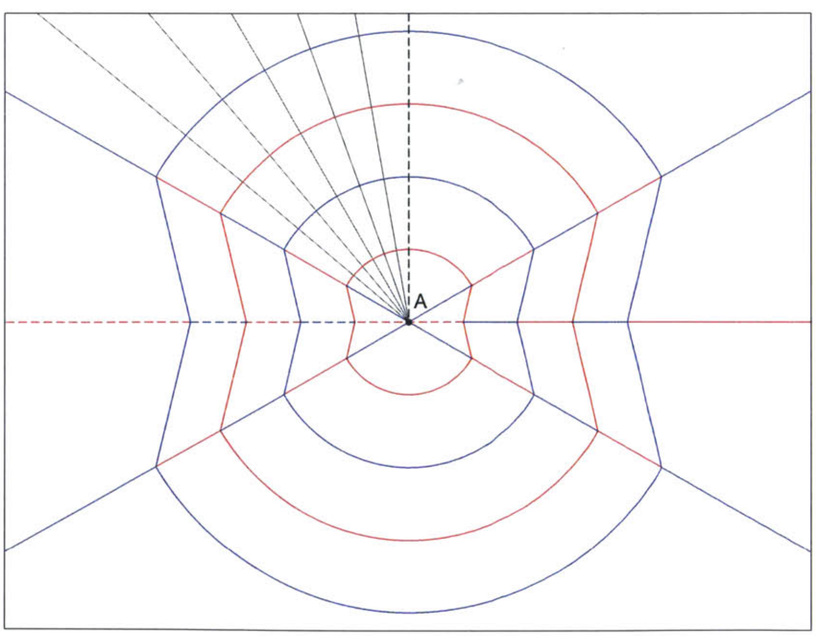


Fig 4.2.51 Crease pattern [DK]

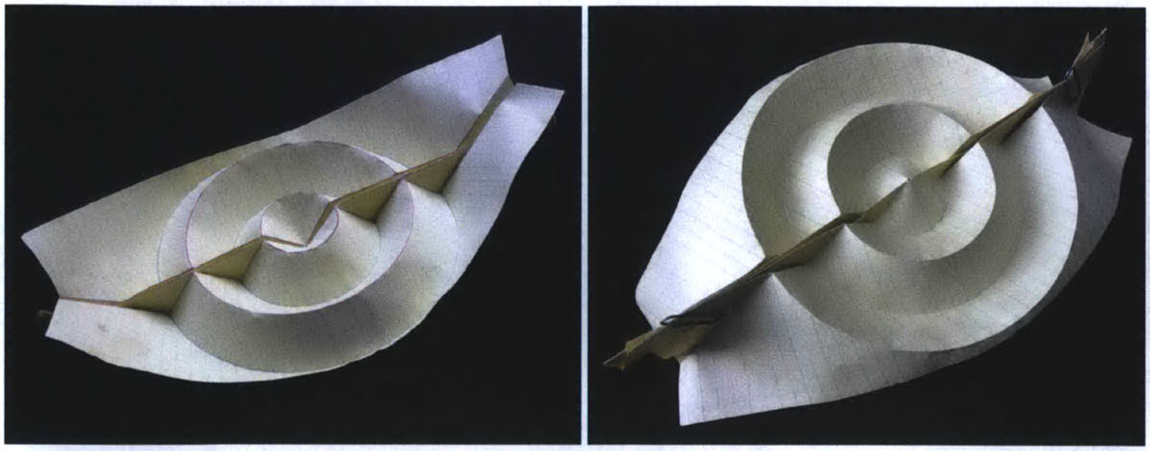


Fig 4.2.52 Paper model (undated DAH [EAH]), (undated DAH [EAH])

A variation of the previous design provides a neat solution to hide paper (Fig 4.2.52). The upper and lower edges of the tuck folds lie in the reflection planes, which give the design a clean appearance.

Crease pattern and ruling analysis

The prototile consists again of 2 main parts, the tuck folds and the cone reflections. The flat parts consist of a triangle and aligned quadrilaterals. In this case the triangle has the same 30° angle at the bottom right but an obtuse angle at the top. The arcs reach the vertical edge of the prototile in a similar way (Fig 4.2.53).

The mountain and valley assignments alternate just like in the previous example and all vertices of the tuck fold have to be flat foldable.

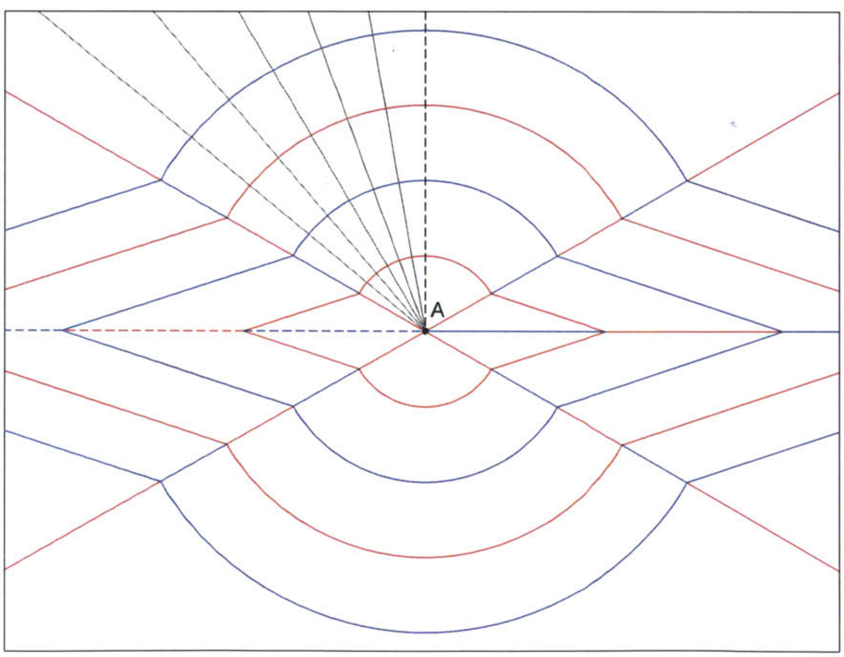


Fig 4.2.53 Crease pattern [DK]

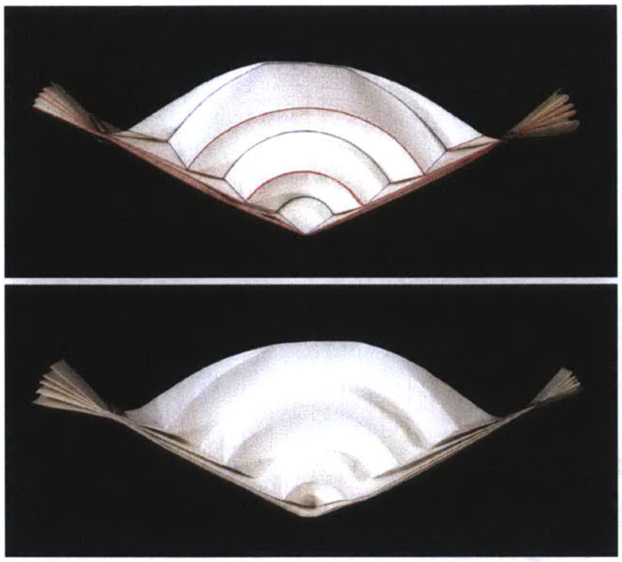


Fig 4.2.54 Paper models (undated DAH [DK])

The 3rd and last example in this series resembles the previous examples in terms of its solution for the tuck fold (Fig 4.2.54). All upper and lower creases of the tuck folds lie in both reflection planes in the folded state.

Crease pattern and ruling analysis

The prototile, one half of the crease pattern in this case, consists of half of the reflected cones and flat quadrilaterals. Some of the polygons have parallel pleated creases at the bottom of the crease pattern.

Mountain and valley assignments alternate and all vertices of the tuck folds must be flat foldable.

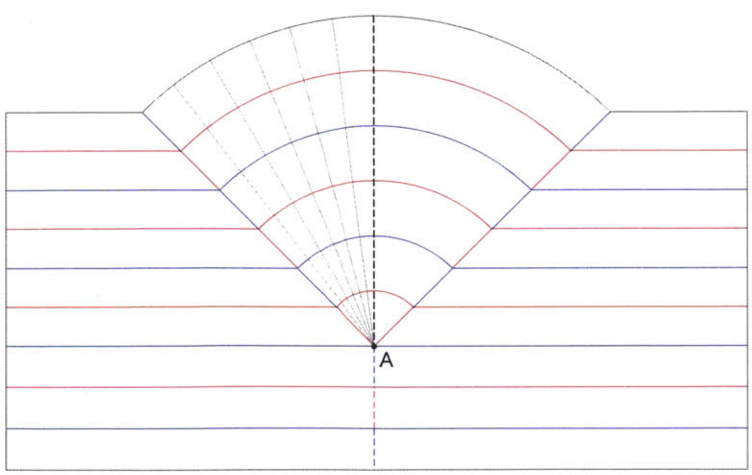


Fig 4.2.55 Crease pattern [DK]

Cone reflection of general and partial cones

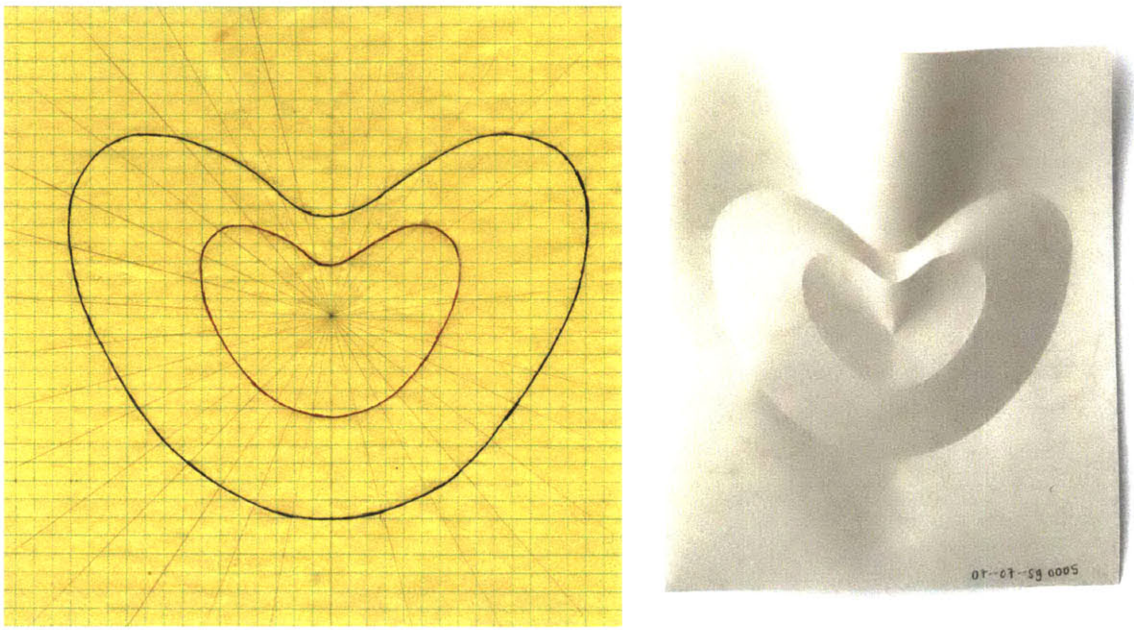


Fig 4.2.56 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The 2 last designs in this chapter show explorations that relate to cone reflection, but do so as solitary examples. Huffman folds them into their 3d configuration as the paper shows traces of having been folded.

Crease patterns and ruling analyses

The above design has 2 prototiles with scaled freeform splines the have concave and convex parts. The resulting reflected convex and concave general cone is difficult to fold as a paper model (Fig 4.2.56).

The below design consists of arcs at unequal intervals that form 6 prototiles with alternating mountain and valley creases (Fig 4.2.57). This reconstruction is also difficult to fold.

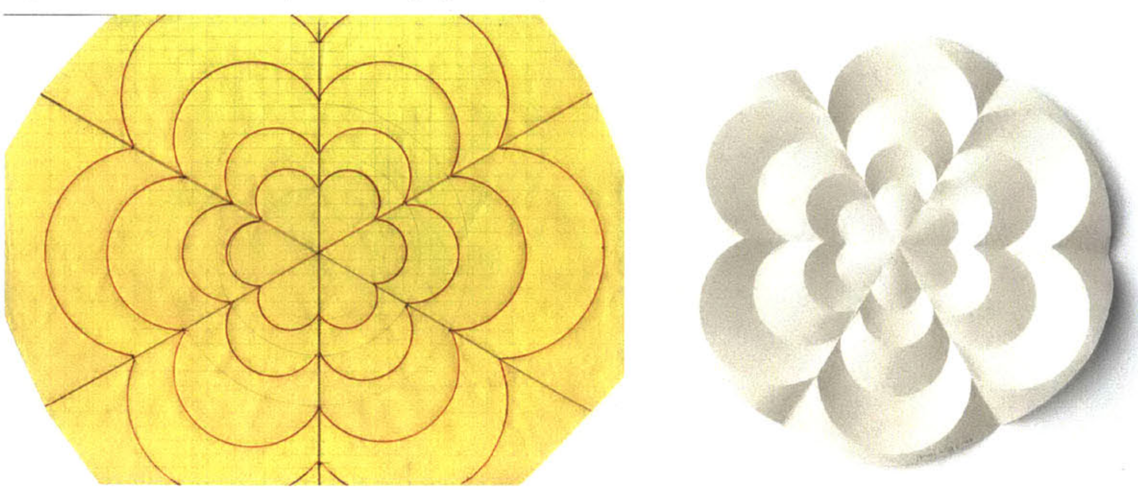


Fig 4.2.57 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

4.3 Gadgets with ellipses

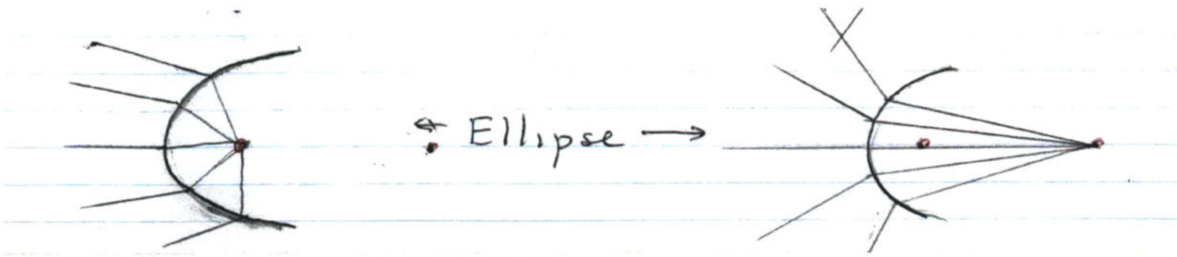


Fig 4.3.1 Index card (undated, DAH [DK])

This section delineates the beginning of a technique Huffman invents for himself, namely 'refraction gadgets'. The 2 previous sections in the taxonomy rely on rulings that are predetermined by the definition of a cone or a cylinder. Here, we can study an alternate way via his interpretation of the phenomenon in optics.

He lets the ruling attain the role of a ray and utilizes a special property of conics as a way to predict the directional change of the ruling, once it passes through a crease. Huffman keeps a set of diagrams of the ray refraction on conics on the above index card, which we will see again in the introductions of new refraction gadgets (Fig 4.3.1).

A ray that starts outside of the ellipse on the left aims at the right focus and changes course when it crosses the ellipse such that it will arrive at the left focus. He provides the inverse case on the right.

The Gadget

The gadget relates to Huffman's diagram on the left above (Fig 4.3.1). The partial ellipse on the left intersects with 3 possible edges for a prototile. The case on the right assumes different edges for a prototile and has a flat triangle that Huffman uses as a tuck fold in some cases.

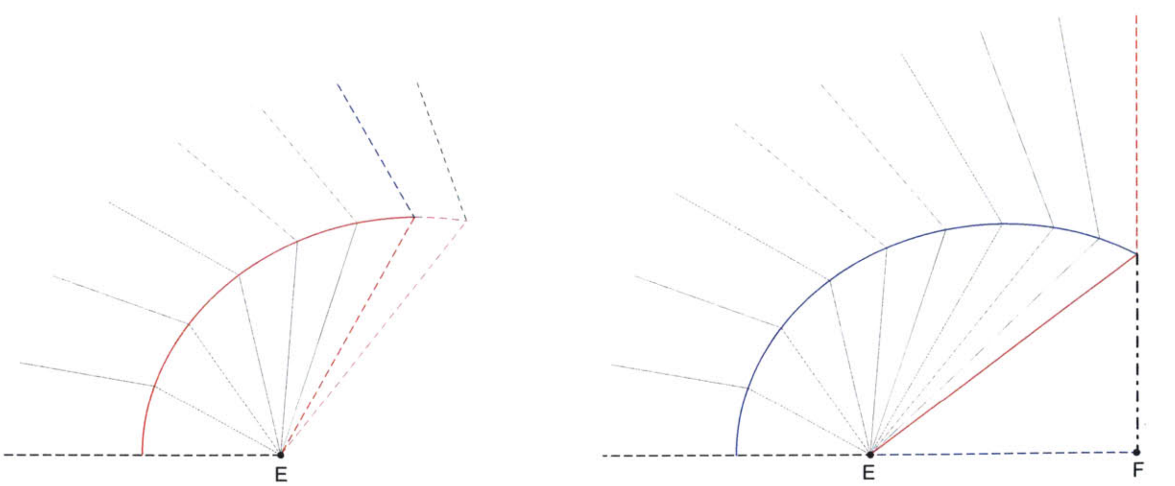


Fig 4.3.2 Gadget with ellipses [DK]

Single ellipse

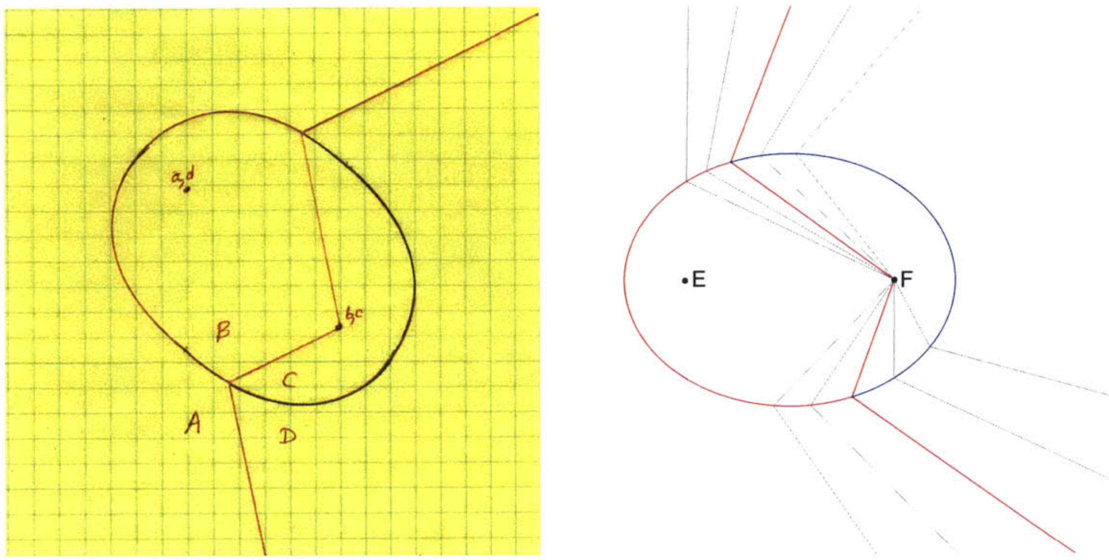


Fig 4.3.3 Paper model (undated, [DAH] [DK]), Crease pattern [DK]

Huffman uses partial ellipses throughout this section, but investigates the possibility of a single ellipse in a crease pattern in the above example. Since it is difficult to fold the interior of the ellipse as a single cone, he splits the curve into a mountain and a valley fold (Fig 4.3.3).

Crease pattern and ruling analysis

The 2 straight mountain creases in the center follow the rules of ray refraction by starting in *F* and changing direction once they pass through the ellipse. The 2 partial ellipses have alternating mountain and valley assignments.

The convex conical surfaces within the ellipse turns into a larger concave cone and the inverse is the case for the second partial cone (Fig 4.3.4). It is a rare example of an asymetrical design.

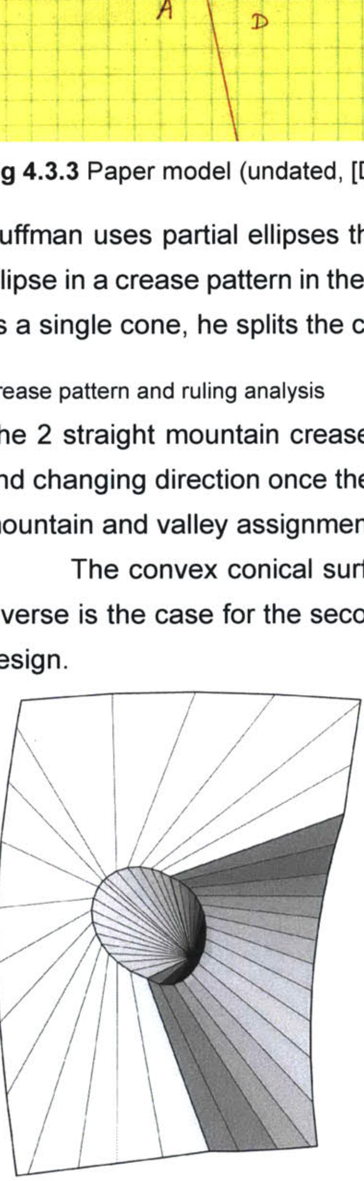


Fig 4.3.4 Simulated model [AH]

Gadgets with ellipses and tucking

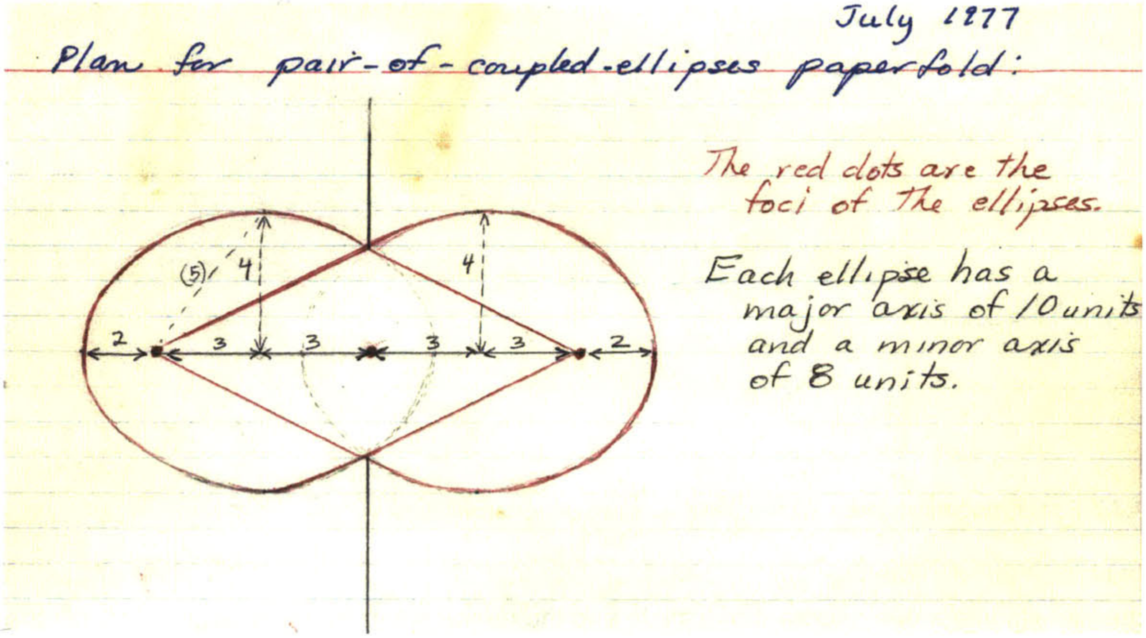


Fig 4.3.5 Index card (1977, DAH [DK])

Huffman investigates designs with the ellipse gadget in a series of tilings. He calls the design 'Pair-of-coupled ellipses' in 1977 on an index card that explains the crease pattern (Fig 4.3.5). The below images show the first vinyl model in this section, in which Huffman uses the gadget to create 2 large cones outside of the ellipse separated by a straight mountain crease (Fig 4.3.6). The edges of the gadget at the center are configured such that 2 tuck folds can hide the diamond shape of the crease pattern. The photographs show the tuck fold half open.

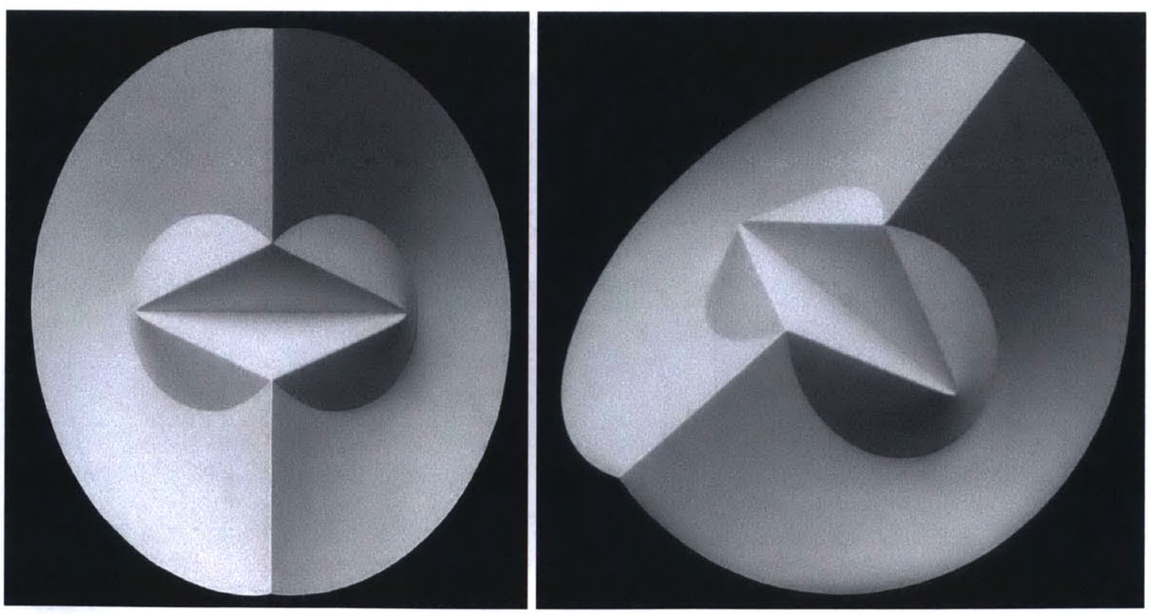


Fig 4.3.6 'Pair-of-coupled ellipses' Identical models (undated, [DAH] [EAH])

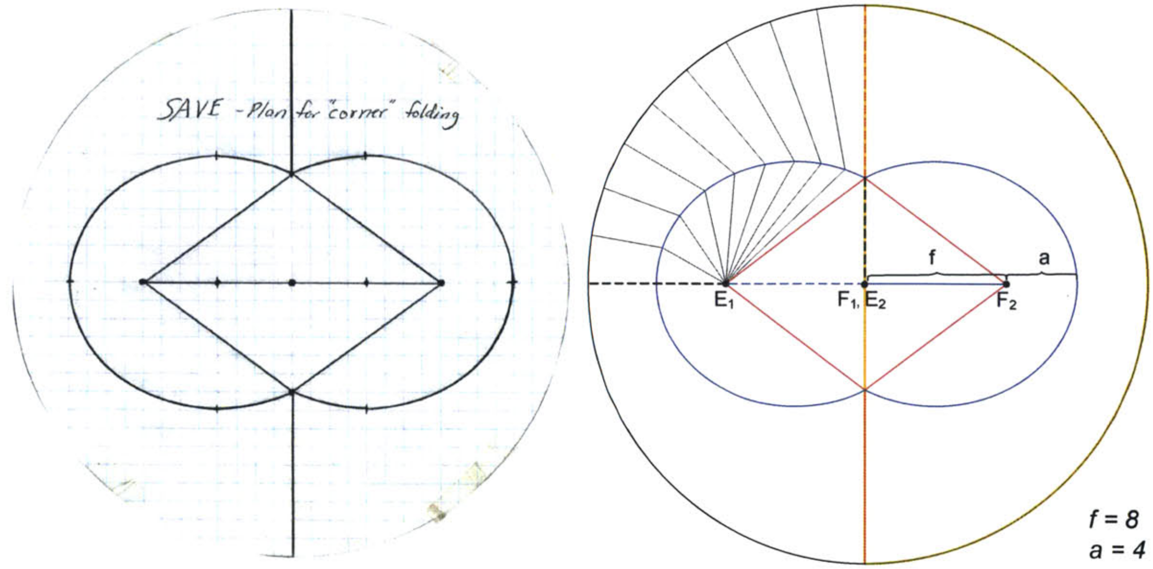


Fig 4.3.7 Paper model (undated, [DAH] [DK]), Crease pattern [DK]

Crease pattern and ruling analysis

The gadget fits into a one quadrant of a circle and the design tile in this case consists of half the crease pattern. Huffman uses the gadget in this configuration for a few similar designs and it is helpful to use a relative definition of the proportions of an ellipse (Fig 4.3.7). The distance between the 2 foci of the ellipses is called *f* and the distance from either focus to the closest boundary of the ellipse on the major axis is called *a*. The proportion Huffman chooses for 'Pair-of-coupled ellipses' on the index card is *f* = *6* and *a* = *2*. However, the vinyl model is based on a sketch called 'corner folding' with *f* = *8* and *a* = *4* in graph paper increments (Fig 4.3.7 left). The straight edges form the tuck fold with a valley fold in the center and end in the outer foci of the 2 ellipses. It is unclear, if the vertical mountain crease is necessary.

A reconstructed model in paper can fold further than Huffman's vinyl model such that the

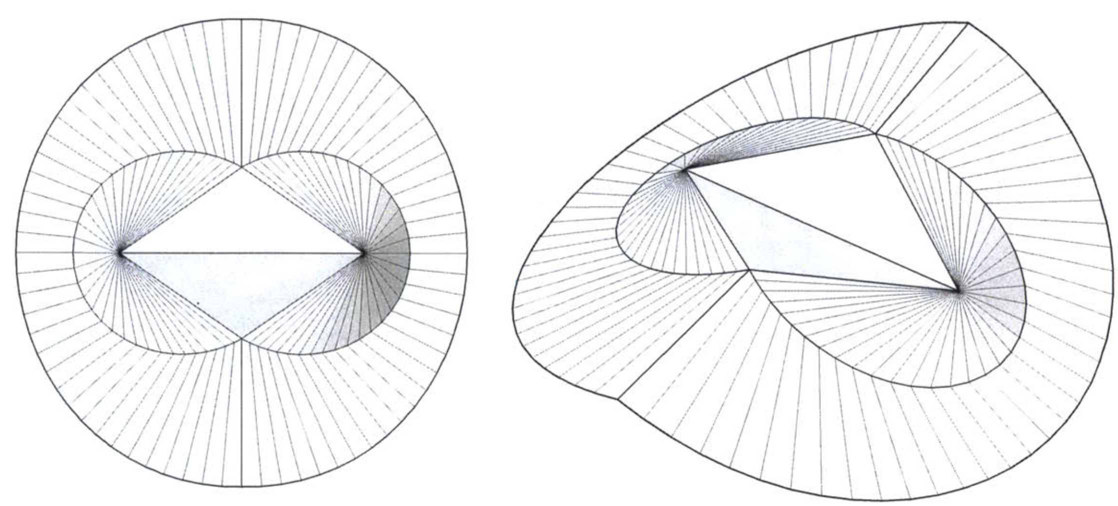


Fig 4.3.8 Simulated model [UP]

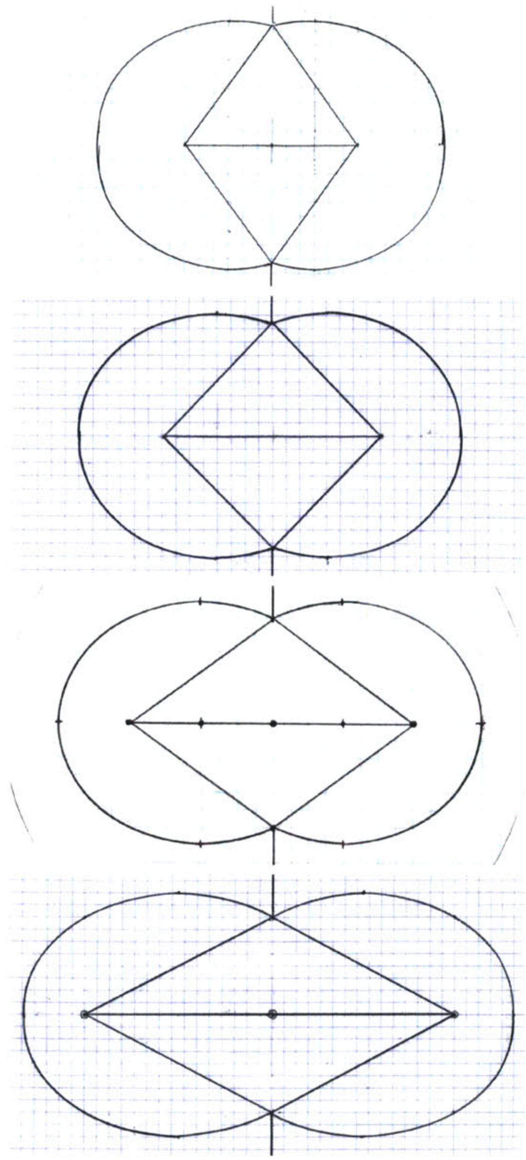


Fig 4.3.9 Paper models (undated, [DAH] [DK])

2 central triangles become co-planar to form a complete tuck, but one has to hold it in place as the paper will not remain in that configuration (Fig 4.3.6). However, the simulation does not fold as much as the vinyl model and suggests that the rulings might have to change during the folding motion (Fig 4.3.8).

The previously defined parametric or proportional definition of the ellipses, based on counting increments on graph paper, describes the above variations (Fig 4.3.9). The first example uses a ratio of 1/1, which creates an area similar to an equilateral triangle. The second example with a ratio of 4/3 starts to widen the triangle. The third is the vinyl model and the last example which uses a ratio of 3/1 which is similar to the design on the index card. Huffman defines ratios and proportions in similar ways for other designs and often relies on graph paper units as they are easy to use.

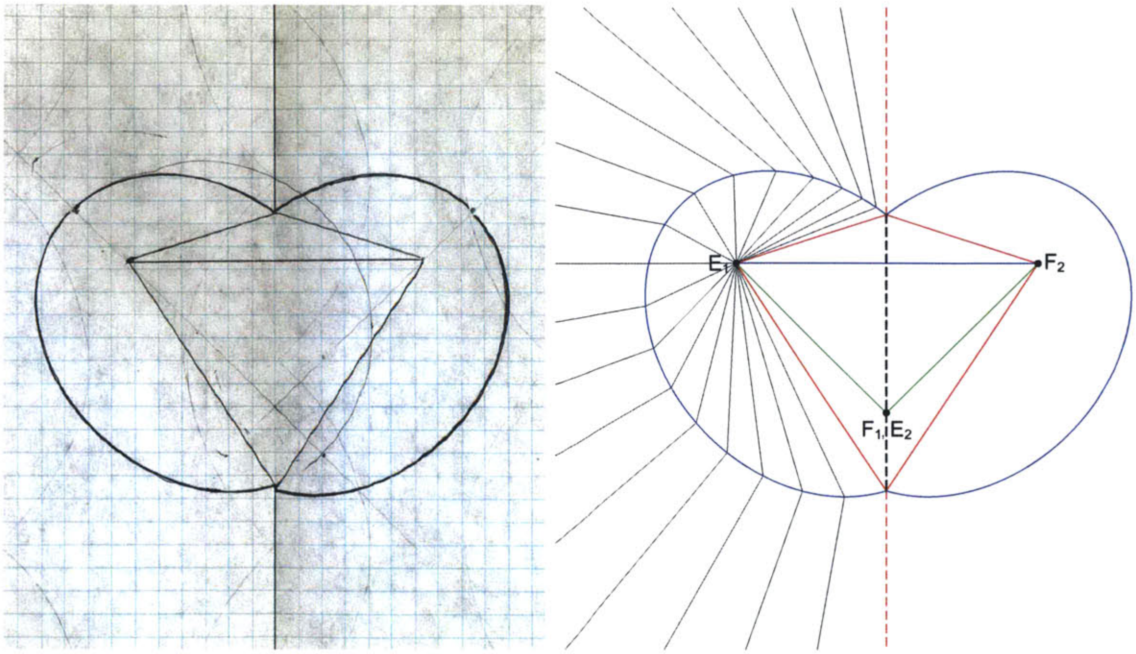


Fig 4.3.10 Paper model (undated, [DAH] [DK]), Crease pattern [DK]

Crease pattern and ruling analysis

Huffman explores an iteration with 2 ellipses in crease pattern that is only symmetrical along the vertical axis (Fig 4.3.10). He rotates both ellipses to form the lower equilateral triangle in the center. The gadget operates in the same way as before, but the main axis of the ellipse on the left rotates clockwise and the main axis of the right ellipse rotates counterclockwise. The moved foci remain confocal.

The resulting simulation folds well and raises the upper half of the model quickly. The upper mountain crease is far more pronounced than its lower counterpart and the previous tuck folds do not fold flat (Fig 4.3.11).

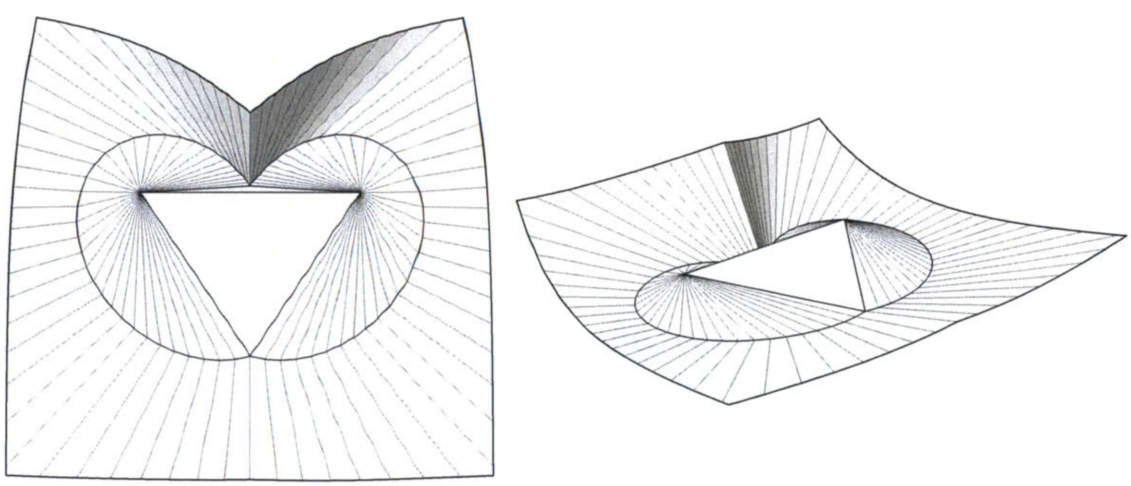


Fig 4.3.11 3d model [UP], same model

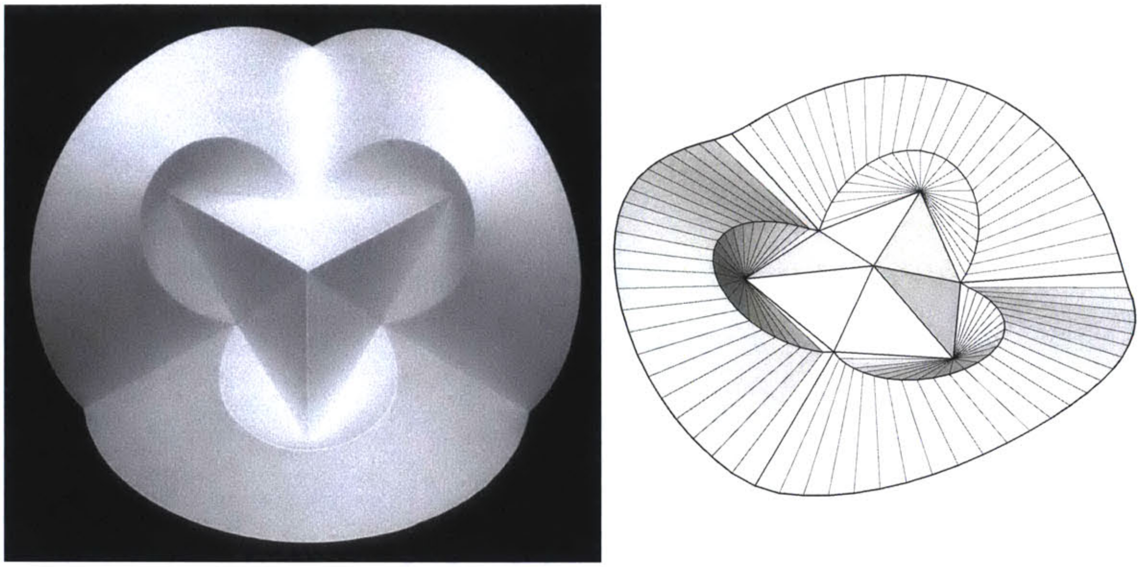


Fig 4.3.12 Vinyl model (1997, [DAH] [EAH]), Simulated model [UP]

The next series shows designs with 3 ellipses such as the above '3-wall, corner folding' (Fig 4.3.12). Huffman dates a drawing of the design in August 1997. The photograph of the vinyl version shows the central polyhedron pointing toward the camera, which Huffman uses as tuck folds in the opposite direction in the other examples of this series.

Crease pattern and tiling analysis

The design consists of 6 ellipse gadgets that each fit into a 60° angle, which are also the prototiles. Huffman explores several ellipses in the series. Establishing relational definitions similar to previous examples will facilitate a better comparison. The distances *a* and *f* on the main axis

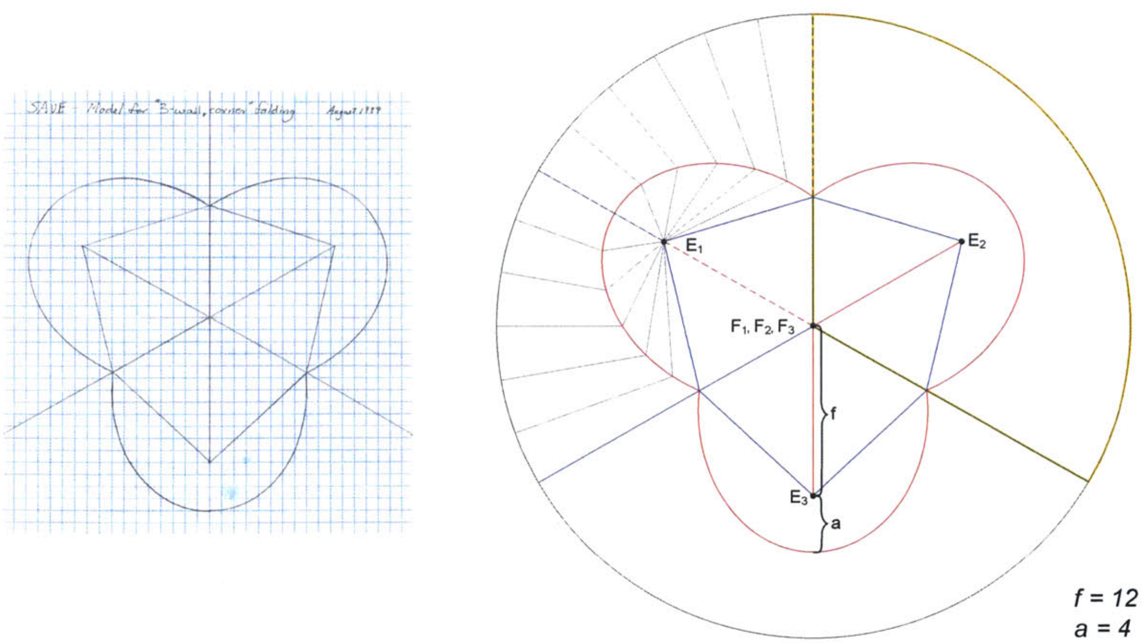
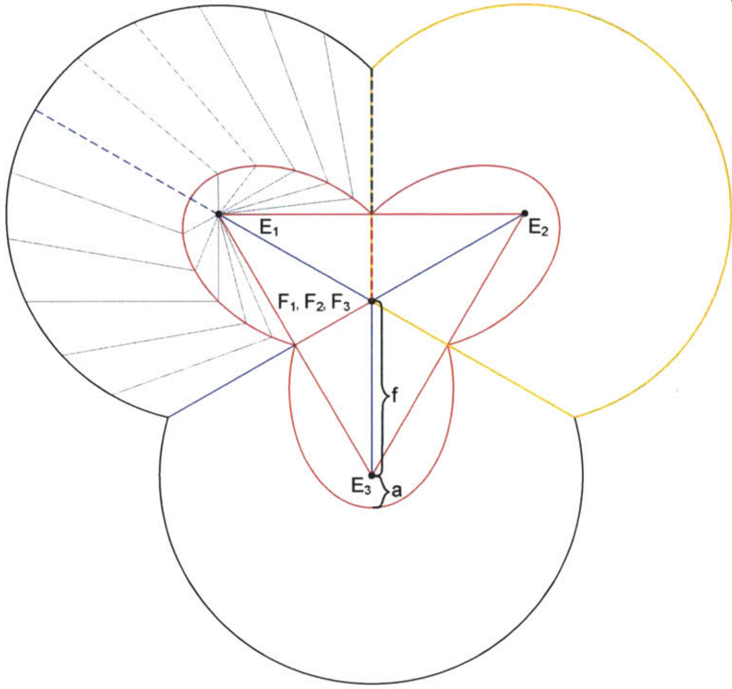
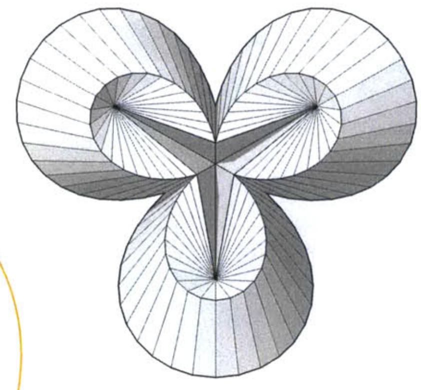


Fig 4.3.13 Paper model (1997, DAH [DK]), Crease pattern [DK]





f = 10
a = 1.84

Fig 4.3.14 Crease pattern [DK], Simulated model [UP]

have a 3/1 ratio for the first design (Fig 4.3.13). The mountain and valley assignments of his paper model (Fig 4.3.13 left) are consistent with the photo (Fig 4.3.12 left).

The simulated model appears to folds less than the vinyl version and has less pronounced valley folds between the 3 large conical surfaces (Fig 4.3.12 right).

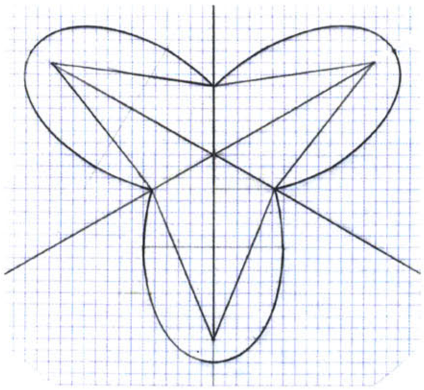
Crease pattern and tiling analysis

The next example is shown as crease pattern and simulation above (Fig 4.3.14). The crease pattern consists of the same gadget that creates an equilateral triangle of mountain creases at the center. This demands an *a* to *b* ratio of 10/1.83, the values are based on the correct mathematical definition and not on integer steps. The tuck folds are pushed in, similar to the previous examples with 2 ellipses.

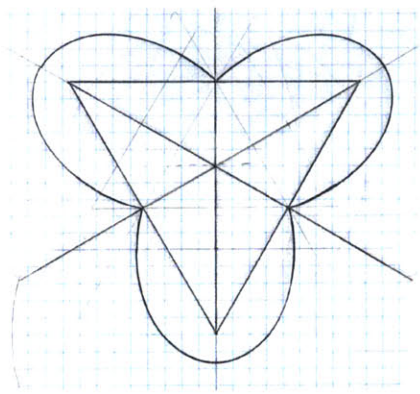
The simulation folds well and can be forced to complete the tuck folds (Fig 4.3.14 right).

Comparison table

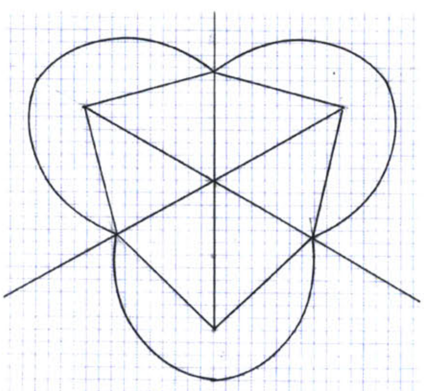
Both designs are represented in the table to show the set of all variations Huffman is interested in (Fig 4.3.15). The example at the top uses an 8/1 ratio, which results in an inward pointing triangle a the center. The next one down has the equilateral triangle at the center. The third example is the vinyl model and in the last version the center area is a hexagon.



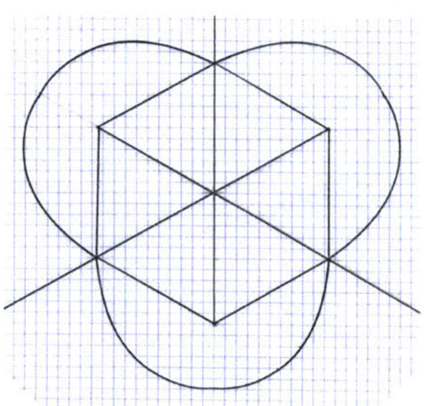
f = 16
a = 2



f = 10
a = 1.84 (Fig 4.3.14)



f = 12
a = 4 (similar to to Fig 4.3.5)



f = 12
a = 6

Fig 4.3.15 Paper models (undated, DAH [DK])

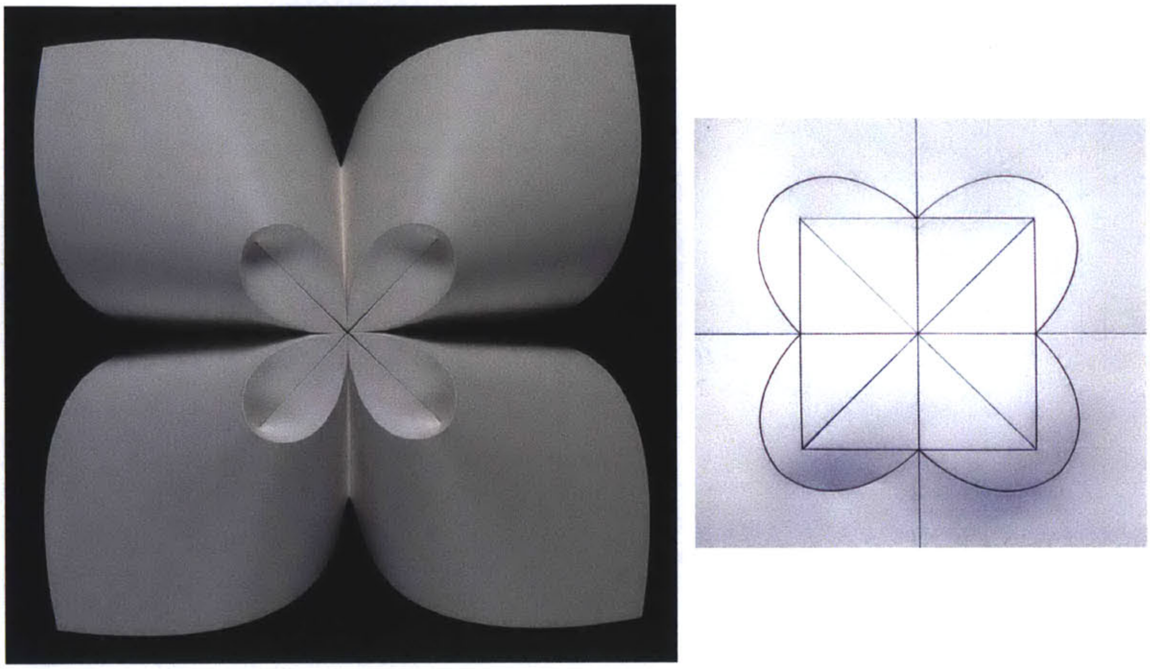


Fig 4.3.16 Vinyl model (1977, DAH [DAH]), Identical model, back side (1977, DAH [DAH]

Huffman expands the gadget to 4 ellipses. '4-lobed, cloverleaf design' from 1977 is the first of a pair of 2 (Fig 4.3.16). He decides to make two vinyl models with almost identical crease patterns that differ only in the shape of their boundary. The above design has a square outline and the second design features a circular perimeter in the flat state (Fig 4.2.20).

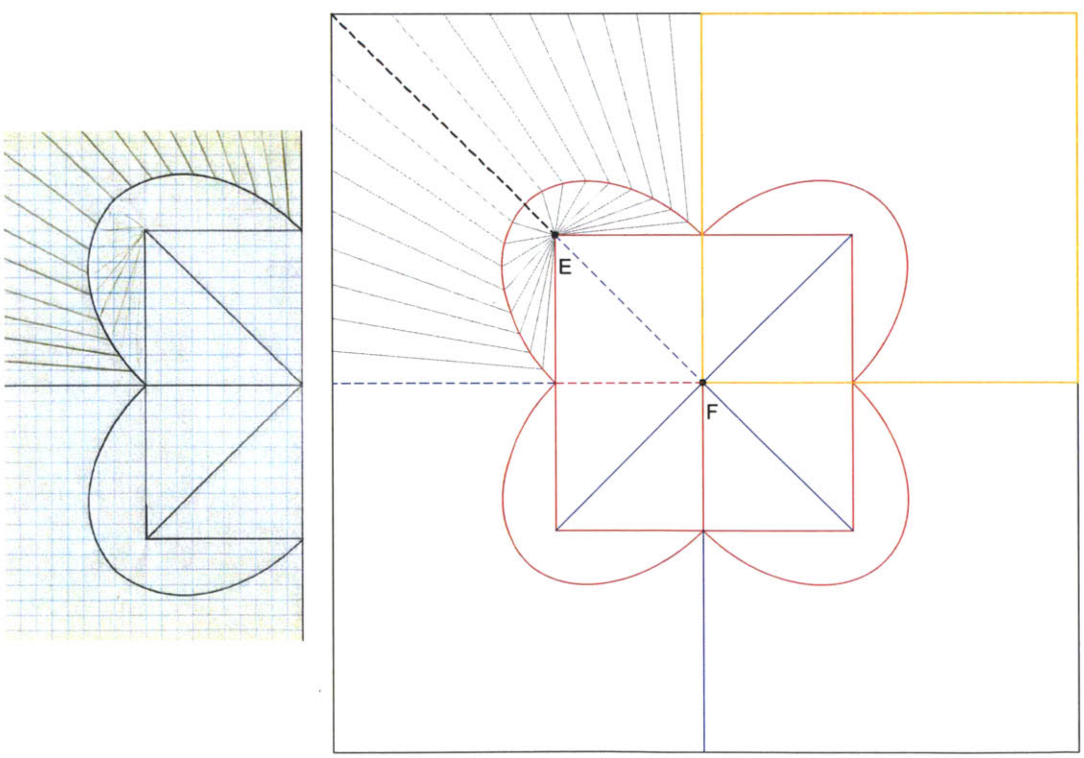


Fig 4.3.17 Drawing (undated, DAH [DK]), Crease pattern [DK]

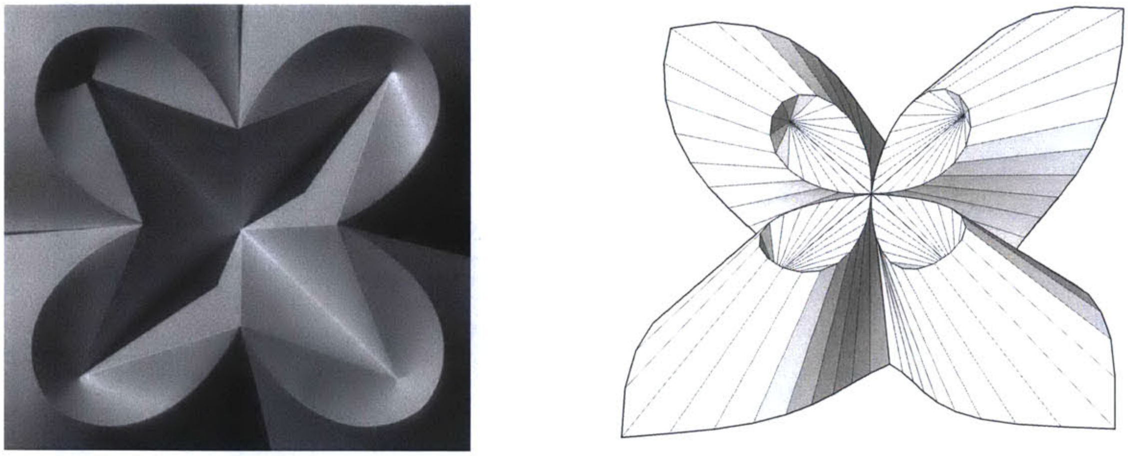


Fig 4.3.18 Vinyl model (1977, DAH [DK]), Simulation [UP]

Huffman also alters the orientation of the second design from straight to diagonal in terms of how the model should be presented on the wall.

Crease patterns

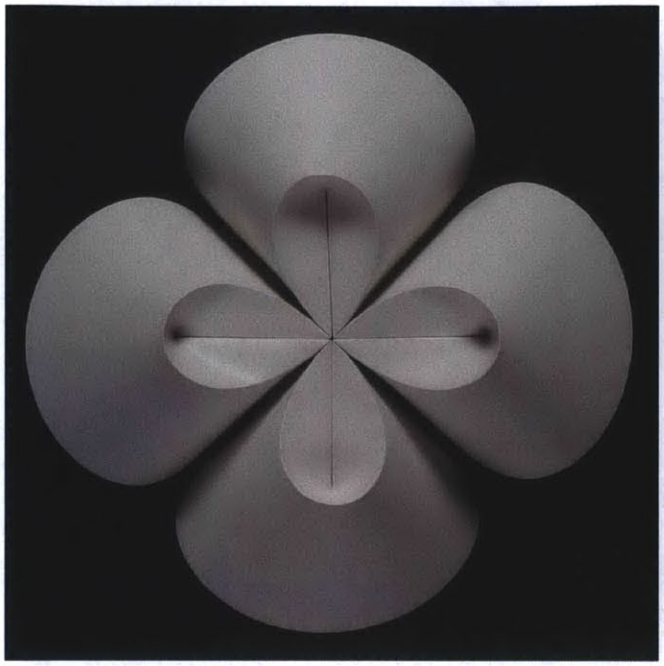
Huffman uses the identical gadget for both designs. The crease pattern consists of 8 prototiles, which are also its gadget. The tuck fold in the center consists of 4 mountain and 4 valley creases that form 8 triangles (Fig 4.3.17). The ellipses have tangents that intersect the main vertical and horizontal axes of the crease pattern at a 45° angle.

The design tile consists of 2 mirrored prototiles. As mentioned, the shapes of the outlines are a square and a circle.

Ruling analyses

The drawing Huffman makes for the design includes lines for the rulings, which is something he rarely does (Fig 4.3.17 left). He uses a pencil and only draws them in one quadrant of the crease pattern. The model appears to be stable in a state that is less folded than seen in photographs of

Fig 4.2.19 Vinyl model (1977, DAH [DK]), Paper model (1977, DAH [DAH])



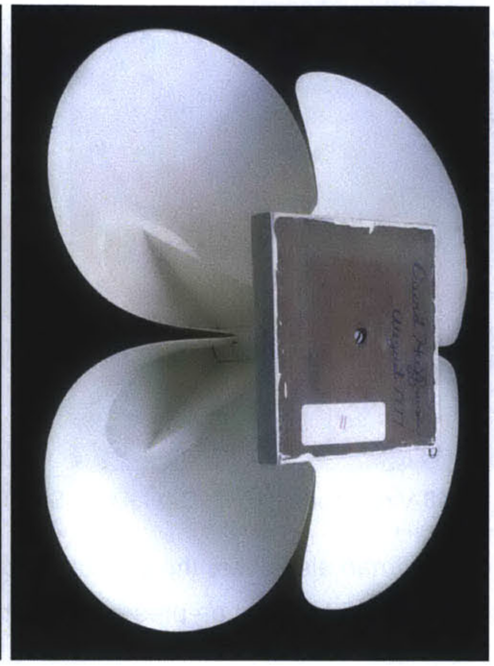


Fig 4.3.20 Vinyl model (1977, DAH [TG]), Identical model (DAH, [EAH])

the final model (Fig 4.3.16 and 20). He takes a photograph of one of the models in that state, which requires no pressure to stay in place (Fig 4.3.18 left).

The creases fold well in simulation and the tuck fold can be completed such that all faces meet (Fig 4.3.18 right).

Notes

A wooden dowel with 1" deep cuts holds the tuck folds in place (Fig 4.3.19 left). Huffman uses the same technique for both examples, but only the second design is mounted to a plate (Fig 4.3.20 right).

A humorous version of the same design with a square cut-out consists of the cartoon page of a newspaper (Fig 4.3.19 right).

4.4 Gadgets with Parabolas

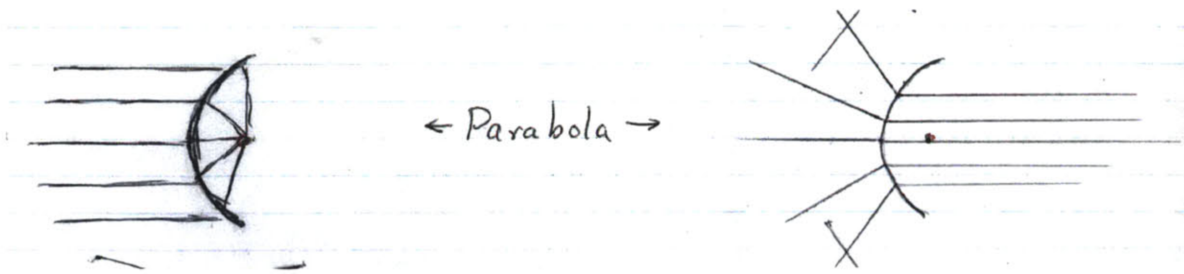


Fig 4.4.1 Index card (1977, DAH [DK])

Ray refraction through a parabola allows Huffman to control parallel and converging rule lines, which means he can work with cones and cylinders at the same time within one gadget. He appears to be fond of this discovery and uses the gadget many times in a large variety of configurations and tilings. Two ways exist of refracting rays and this section starts with the version on the right in Huffman's diagram (Fig 4.4.1). The gadget on the left occurs frequently in general tilings with grid-like formations.

The Gadgets

The parabola usually starts at the main axis and often ends at the latus rectum, the horizontal through the focus indicated in green (Fig 4.4.2 left). Alternatively, the curve continues on either side of the axis and ends at equal distances from the axis. This is often the result of a mirrored gadget. Huffman uses available alignments and works with the logics given by the parabola and is aware of the 'design potential' of a gadget.

The parallel rulings are on the concave side of the parabola and the edges of tilings usually consist of a ruling before and after refraction.

The second gadget consists of the inverse case, at which the rulings intersect in the focus of the parabola and become parallel on the concave side of the curve (Fig 4.4.2 right).

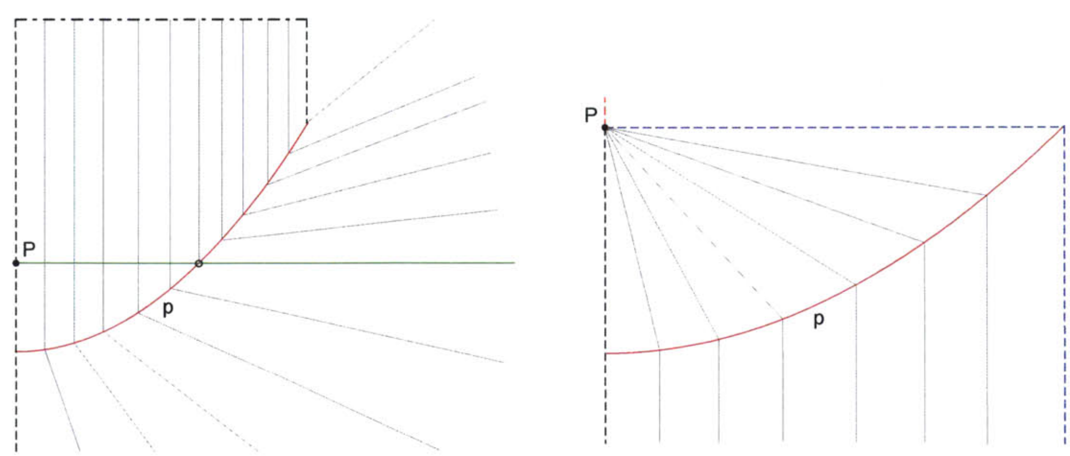
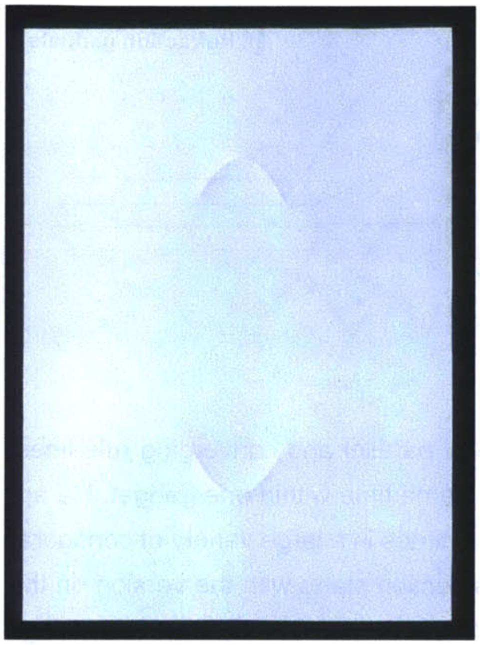


Fig 4.4.2 Gadgets with parabolas [DK]



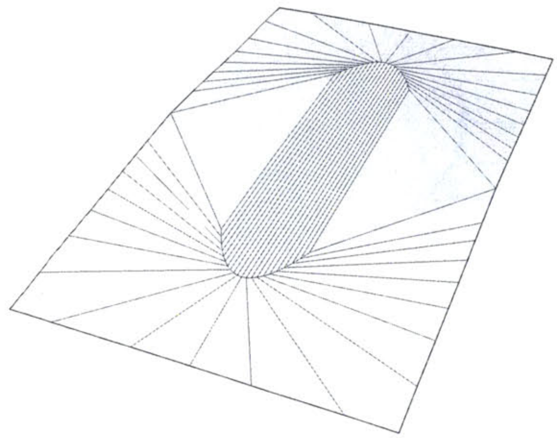
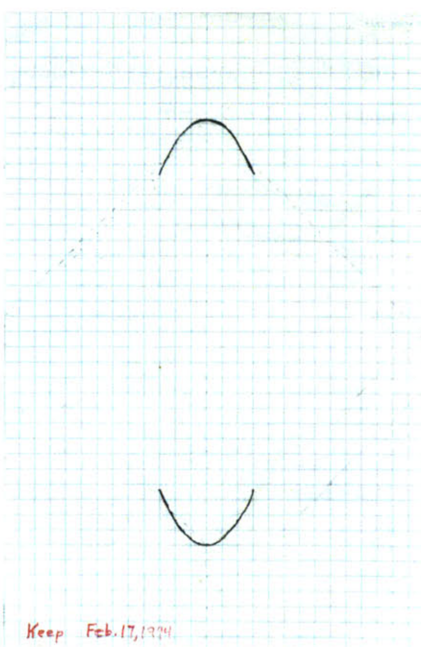


Fig 4.4.3 Vinyl model (undated, DAH [EAH]), Simulated model [JH]

Huffman makes 2 versions of the above design and dates one of them 1994. They appear to be very simple, but prompt Huffman to make one in vinyl (Fig 4.4.3). His second paper model displays the formula for the used scaled parabola (Fig 4.4.5).

Crease patterns and ruling analyses

Both designs use the gadget 4 times. The design tile might be considered to be half the crease pattern in both cases. Faint pencil marks in his paper model indicate that he thinks of the flat tri-



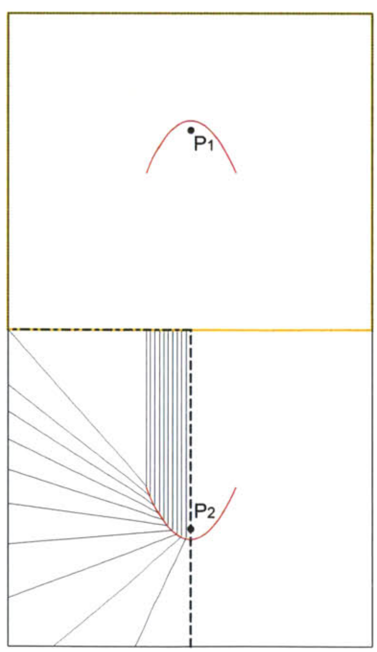


Fig 4.4.4 Paper model (1994, DAH [DK]), Crease pattern [DK]

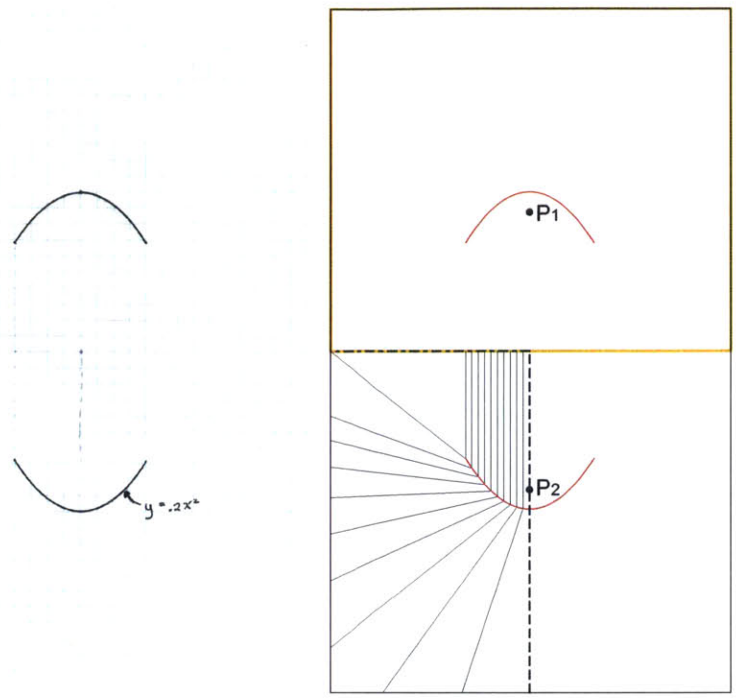


Fig 4.4.5 Paper model (undated, DAH [DK]), Crease pattern [DK]

angle that forms the transition between the conical and the cylindrical part within the prototile (Fig 4.4.4 left).

Both models behave in similar ways during simulation and fold very little (Fig 4.4.3 right, Fig 4.4.6). This is consistent with the appearance of the design in the photograph (Fig 4.4.3 left).

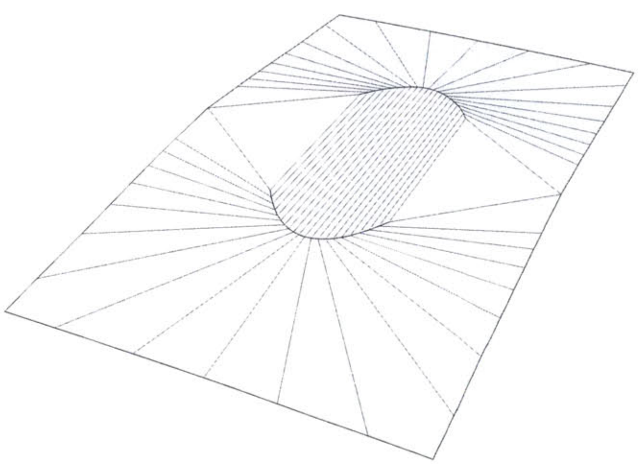


Fig 4.4.6 Simulated model [JH]

Huffman explores the maximum degree to which a parabola can be folded in the next design (Fig 4.4.7). He makes a paper model that creates a closed volume.

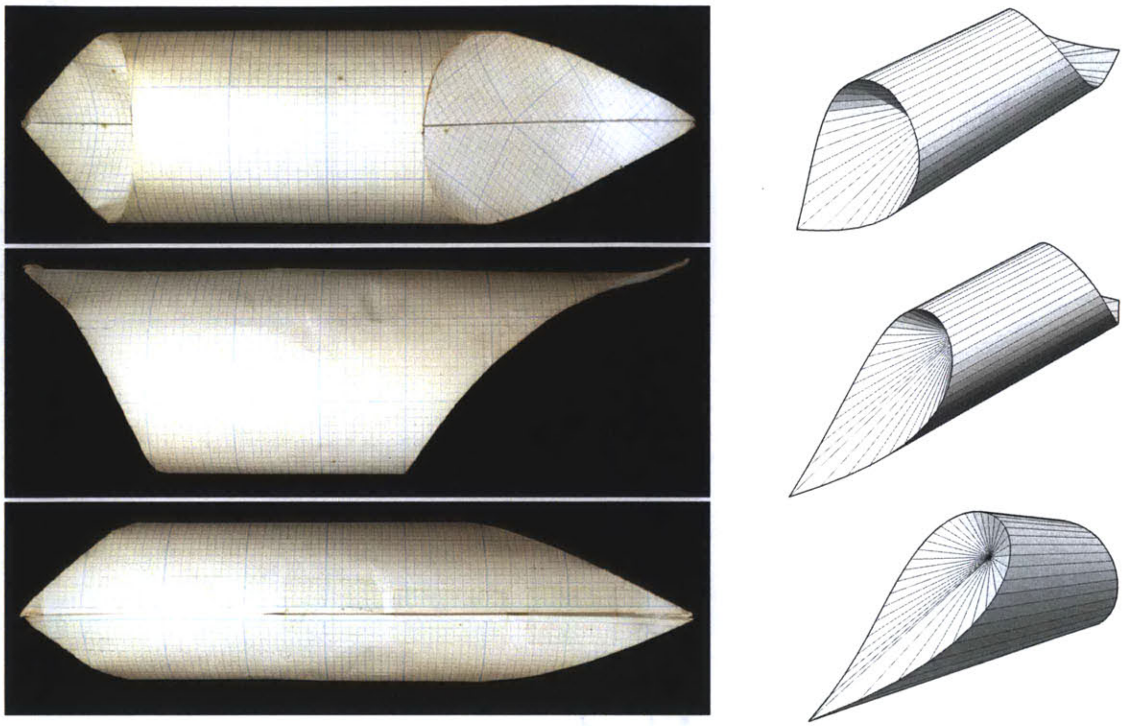


Fig 4.4.7 Paper model (undated, DAH [EAH]), Simulated model [JH]

Crease pattern and ruling analysis

The short parabola at the top has a single horizontal edge across the crease pattern. The long parabola requires the triangular cut out at the bottom (Fig 4.4.8).

It is possible to create an enclosure during simulation, but the seam forms a mountain fold in the conical area of the long parabola (Fig 4.4.7 bottom right).

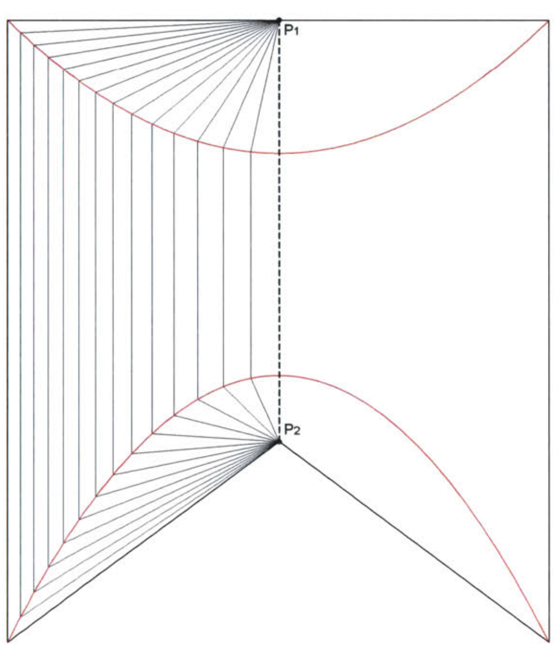


Fig 4.4.8 Crease pattern [DK]

Gadgets with parabolas and pleating

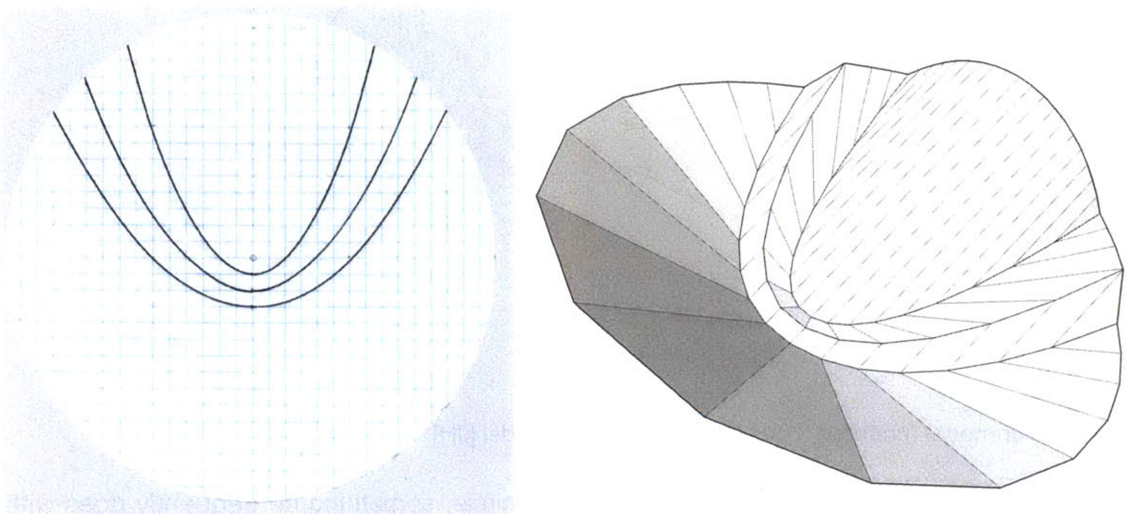


Fig 4.4.9 Paper model (undated, DAH [DK]), Simulated model [AH]

Huffman pleats parabolas by folding them with alternating mountain and valley assignments and this section presents all designs he makes using this technique. He often uses the distance from the focus to the first parabola along its axis as the dimension to distribute further curves (Fig 4.4.9). The pleating results in alternating cylindrical and conical surfaces and the cones share an identical apex. Huffman uses the gadget with parallel rulings at the top of the crease patterns in this series. I generally omit the design tile for this series as they would consist of the entire crease pattern.

Crease pattern and ruling analysis

The gadget occupies half of the crease pattern (Fig 4.4.10). No other tiling would work with

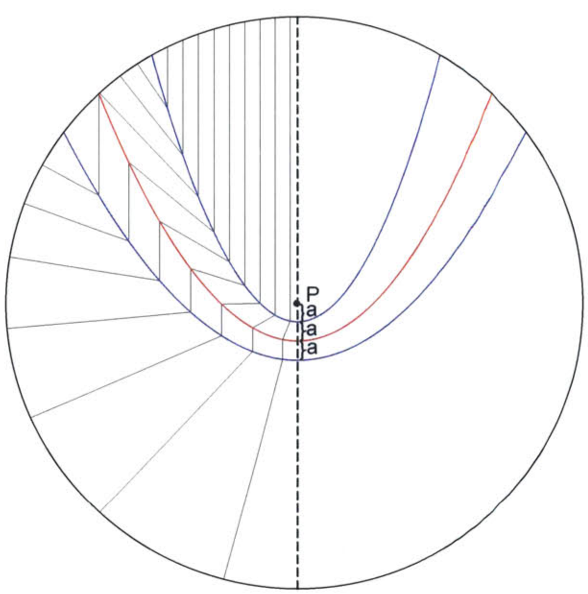


Fig 4.4.10 Crease pattern [DK]

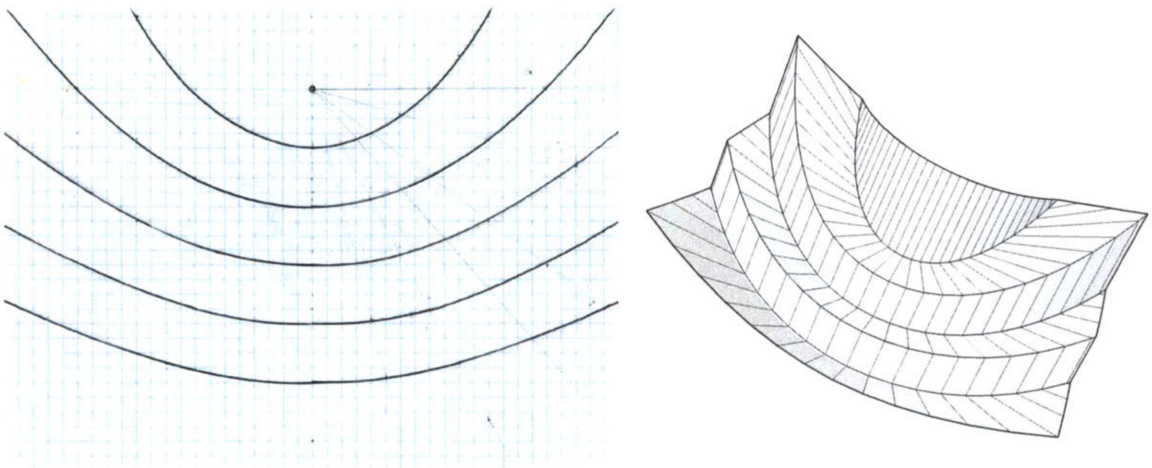


Fig 4.4.11 Paper model (undated, DAH [DK]), Simulated model [JH]

an odd number of creases. He cuts the model with a circle, something he frequently does with rotational tilings.

The simulation folds very well and we can see the conical surface at the bottom, a result of the odd number of creases (Fig 4.4.9 right).

Crease pattern and ruling analysis

The above design also consists of an odd number of parabolas with constant distances between them along their axes (Fig 4.4.11). He does not modify the 8 1/2″ by 11″ boundary of the sheet as in most of his designs. The cylinder at the top of the crease pattern determines the path for the rulings as they cascade down the creases pattern (Fig 4.4.12).

The simulation behaves in similar ways to the previous example (Fig 4.4.11 right).

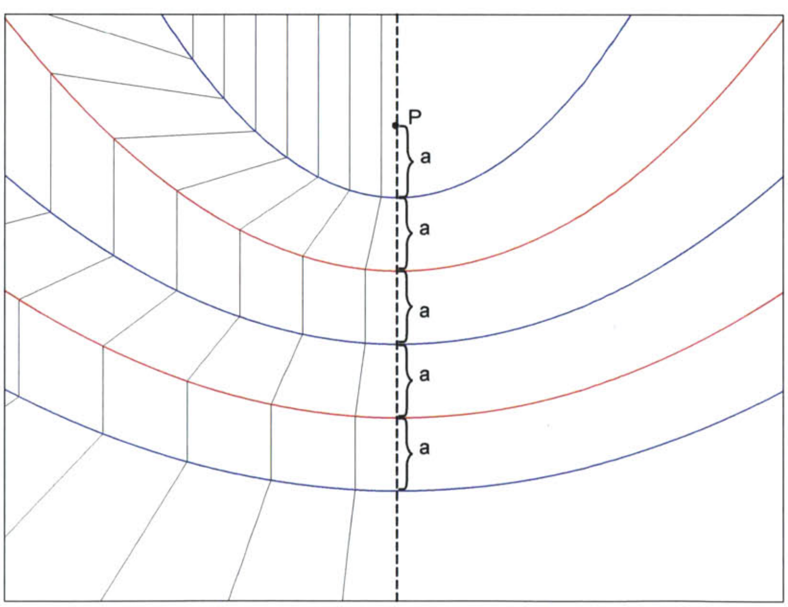


Fig 4.4.12 Crease Pattern [DK]

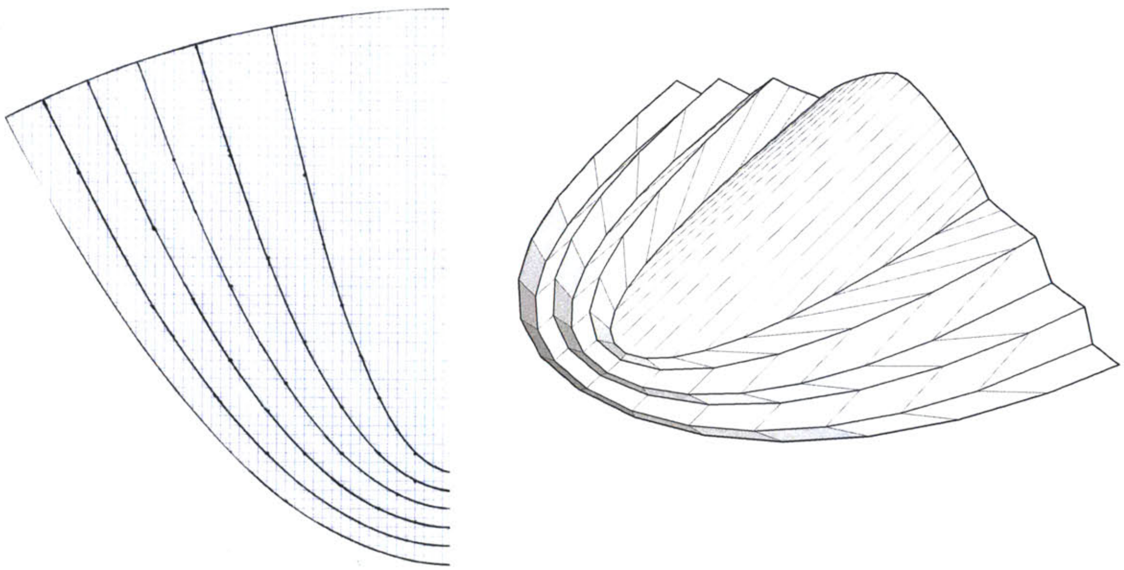


Fig 4.4.13 Paper model (undated, DAH [DK]), Simulated model [JH]

Huffman trims the above pleated model in a very particular way (Fig 4.4.13 left). The upper circular edge has its center at the very bottom of the crease pattern and the bottom edge follows the last parabola. He uses a similar cut-out for a pleated model that me makes with hyperbolas (Fig 4.5.5).

Crease pattern and ruling analysis

The design consists of 5 parabolas with constant distances between them on the main axis (Fig 4.4.14). Apart from the boundary, the design has very similar characteristics to the previous examples.

The model behaves as expected during simulation (Fig 4.4.13 right).

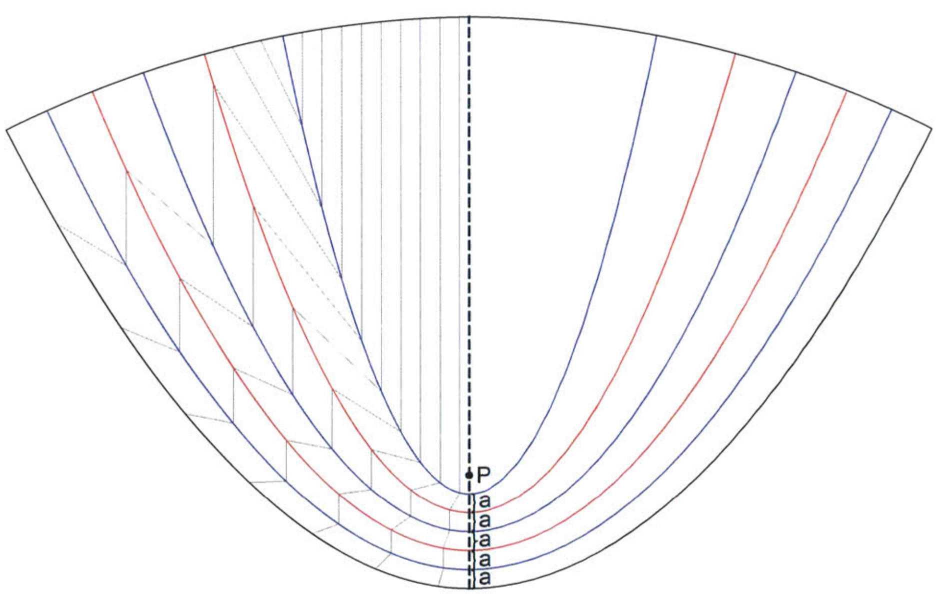


Fig 4.4.14 Crease pattern [DK]

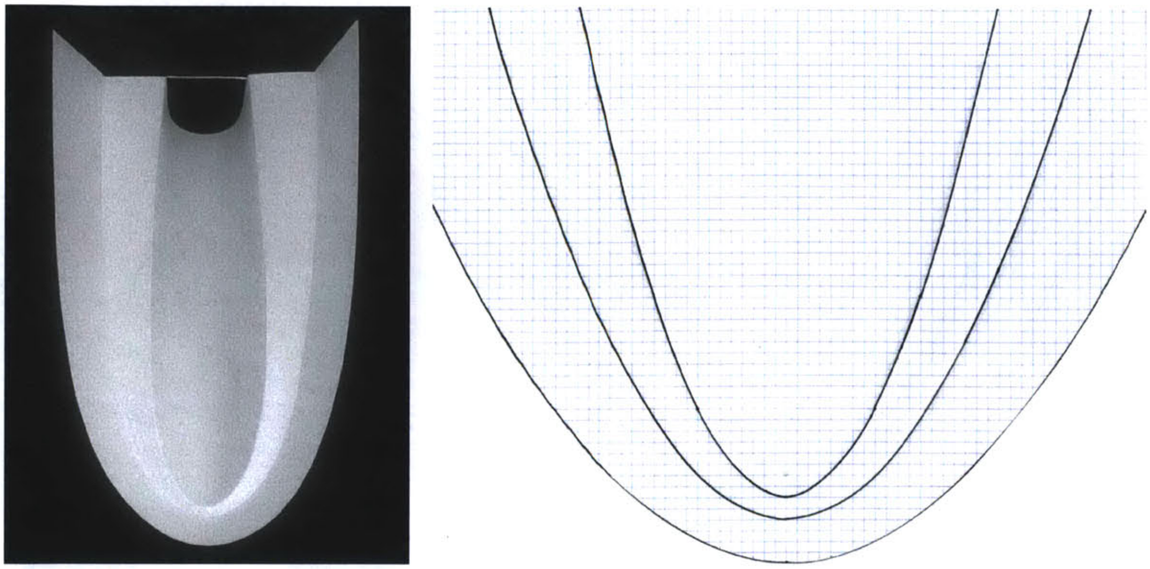


Fig 4.4.15 Vinyl model (1998, DAH [EAH]), Paper model (undated, DAH [DK])

The above vinyl model, photographed by Elise Huffman, uses only 2 creases and is cut with a parabola (Fig 4.4.15). It also appears to be held in place with thread or a thin piece of wire in the image.

Crease pattern and ruling analysis

The mountain and the valley are placed equidistantly from the focus, but Huffman cuts the model with a parabola that is placed further away. The crease pattern shows the 2 distances and also completes the boundary to the correct shape (Fig 4.4.16). Huffman crops his paper model that most probably serves as sketch model for the vinyl version (Fig 4.4.15 right).

The bottom surface in the design results in a general cylindrical surface as expected.

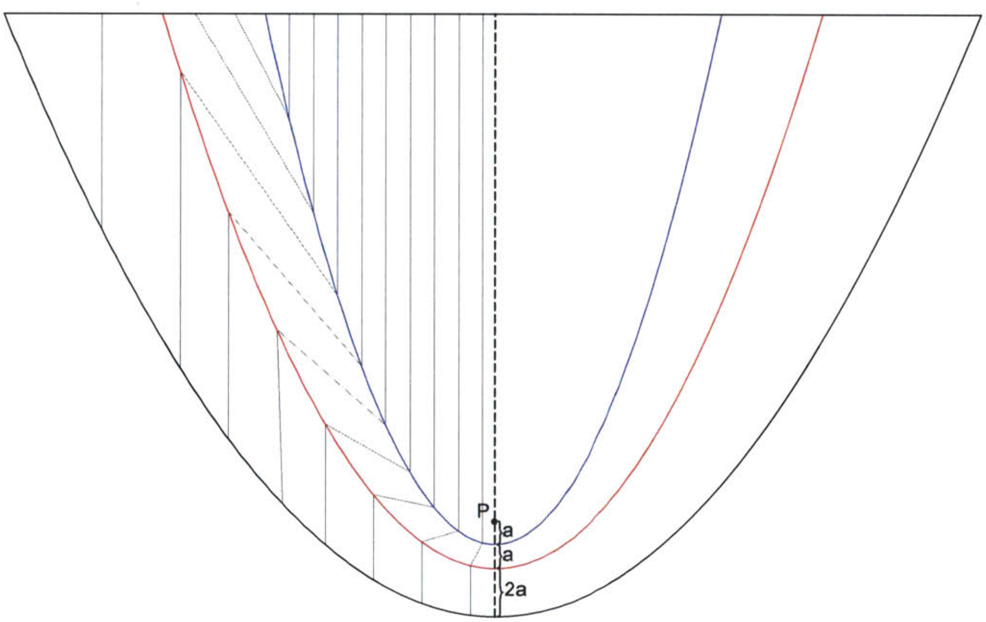
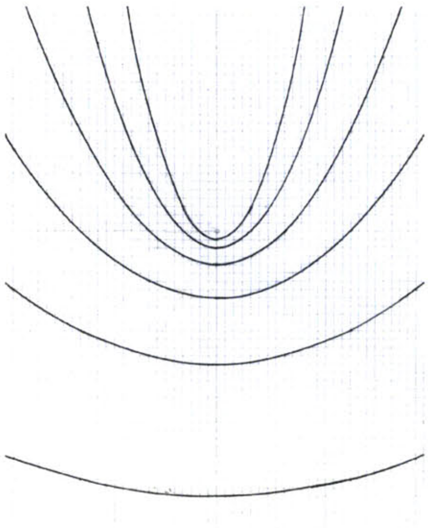


Fig 4.4.16 Crease pattern [DK]



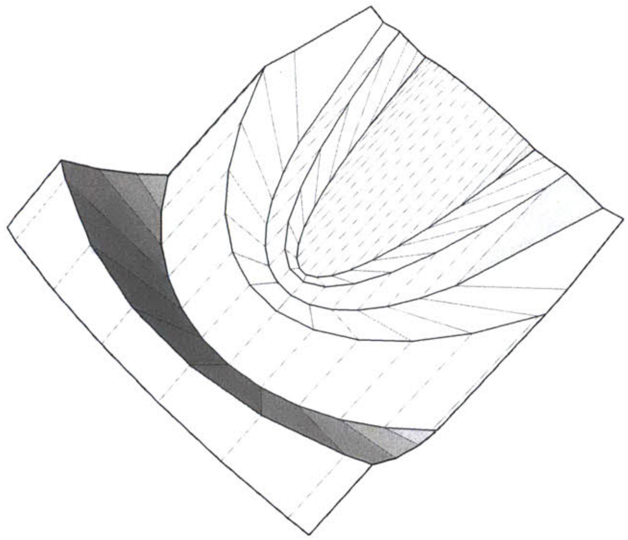


Fig 4.4.17 Vinyl model (1998, DAH [EAH]), Simulated model [JH]

The distances between the parabolas in the above design vary as a quadratic series (Fig 4.4.17). Huffman folds, but does not cut the boundary of the paper model.

Crease pattern and ruling analysis

Huffman places the creases, 3 mountains and 3 valleys, using the square of *a* in the crease pattern (Fig 4.4.18).

The model simulates as expected (Fig 4.4.17).

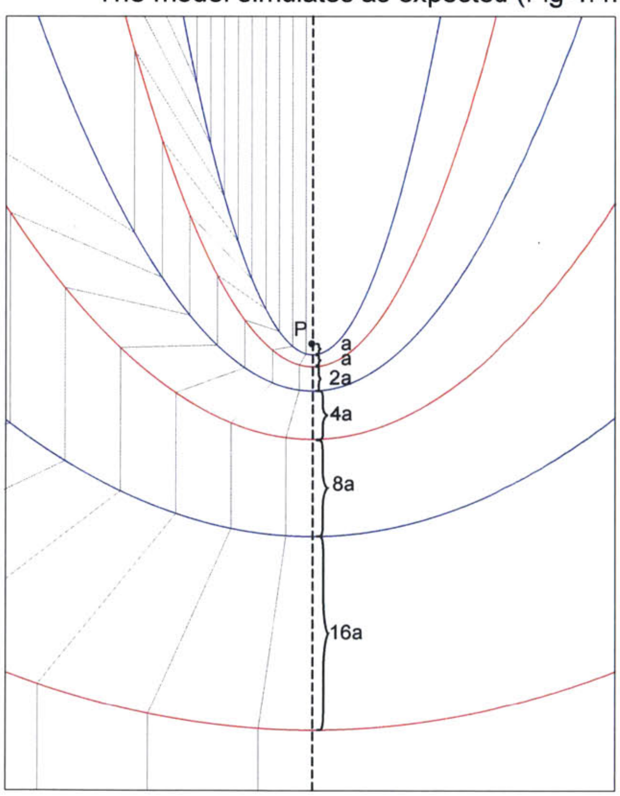


Fig 4.4.18 Crease pattern [DK]

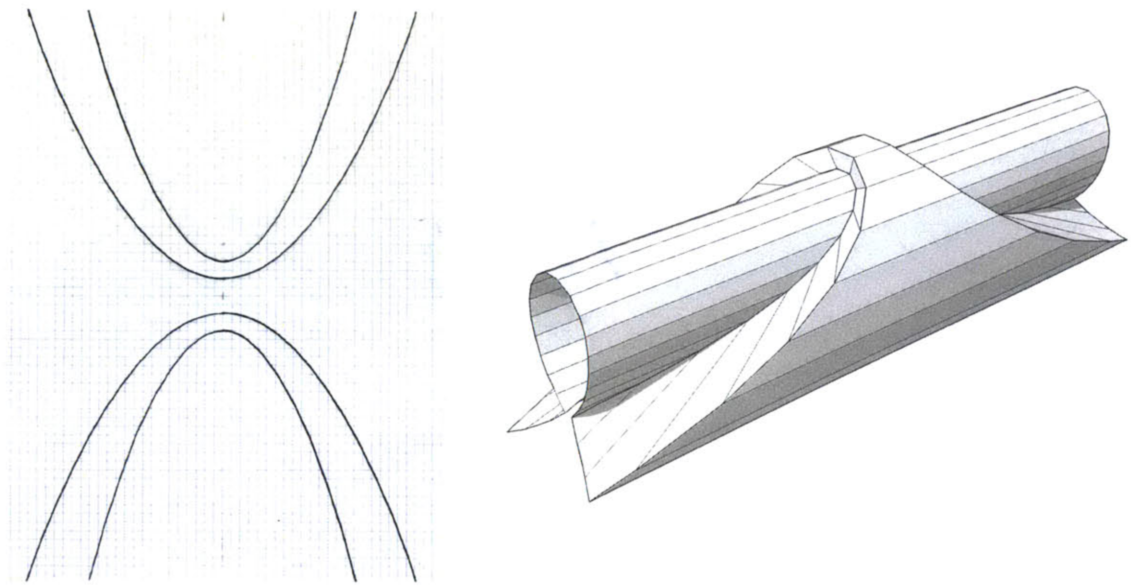


Fig 4.4.19 Paper model (undated, DAH [DK]), Crease pattern [DK]

Huffman tiles a regular variation of the previous designs with an even number of parabolas by mirroring it (Fig 4.4.19 left).

Crease pattern and ruling analysis

Huffman uses constant distances between *P* and the first parabola and between the first and the second parabola. He mirrors the upper half, the design tile, 4 graph paper units downward (Fig 4.4.20).

The model simulates well and can be folded until the paper starts to touch itself on the opposite side of the 3d configuration (Fig 4.4.19 right).

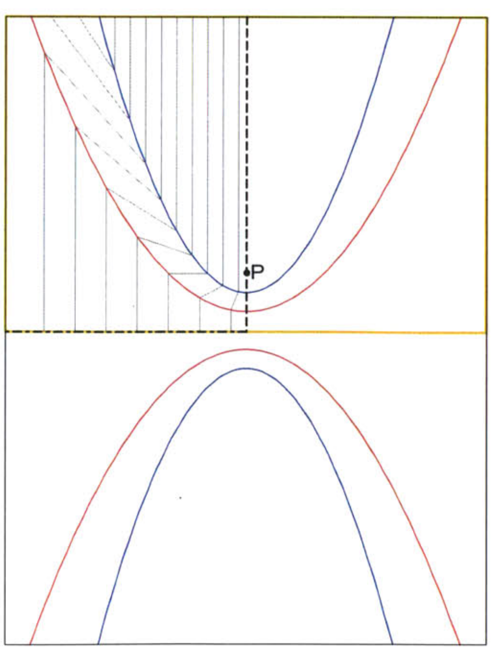


Fig 4.4.20 Crease pattern [DK]

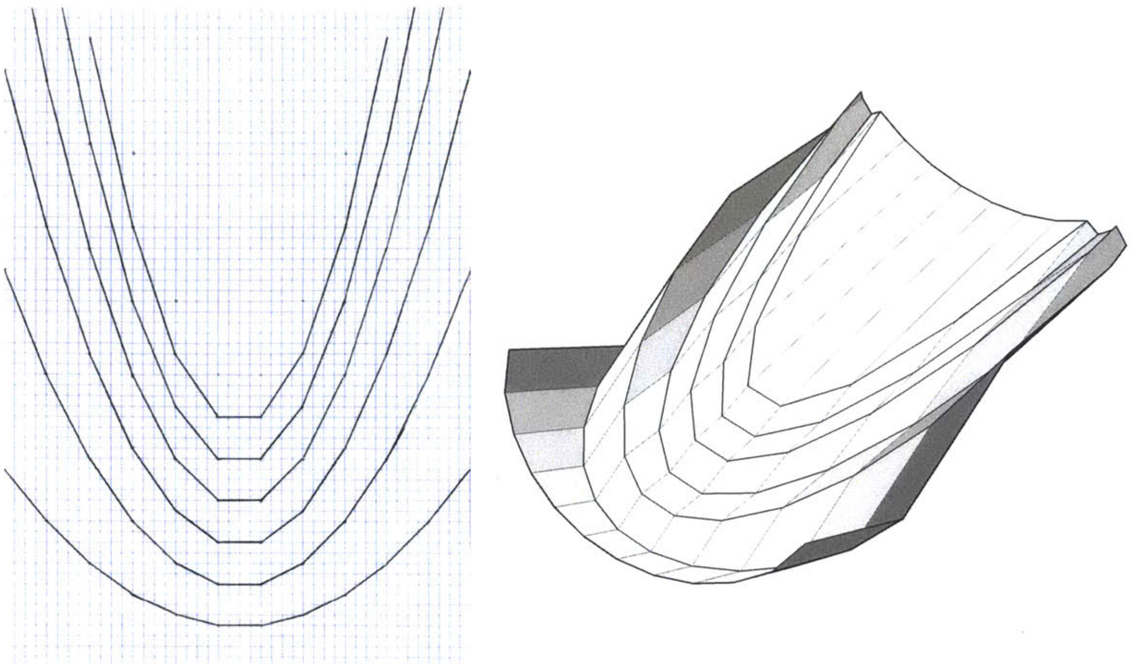


Fig 4.4.21 Paper model (undated, DAH [DK]), Simulated model [AH]

The section concludes with a discrete version of pleated parabolas. The model is reminiscent of examples in the cylinder reflection chapter as the rulings are parallel, but it is included here as the parabolas are confocal here (Fig 4.1.13).

Crease pattern and ruling analysis

Huffman uses 6 approximated confocal parabolas and places them apart from each other with equal distances similar to the previous designs in this section (Fig 4.4.21).

All surfaces are approximated with parallel rulings and the model simulates well as expected. It has a less dramatic appearance than other examples in this series as it is missing the conical surfaces.

Gadgets with parabolas and line segments

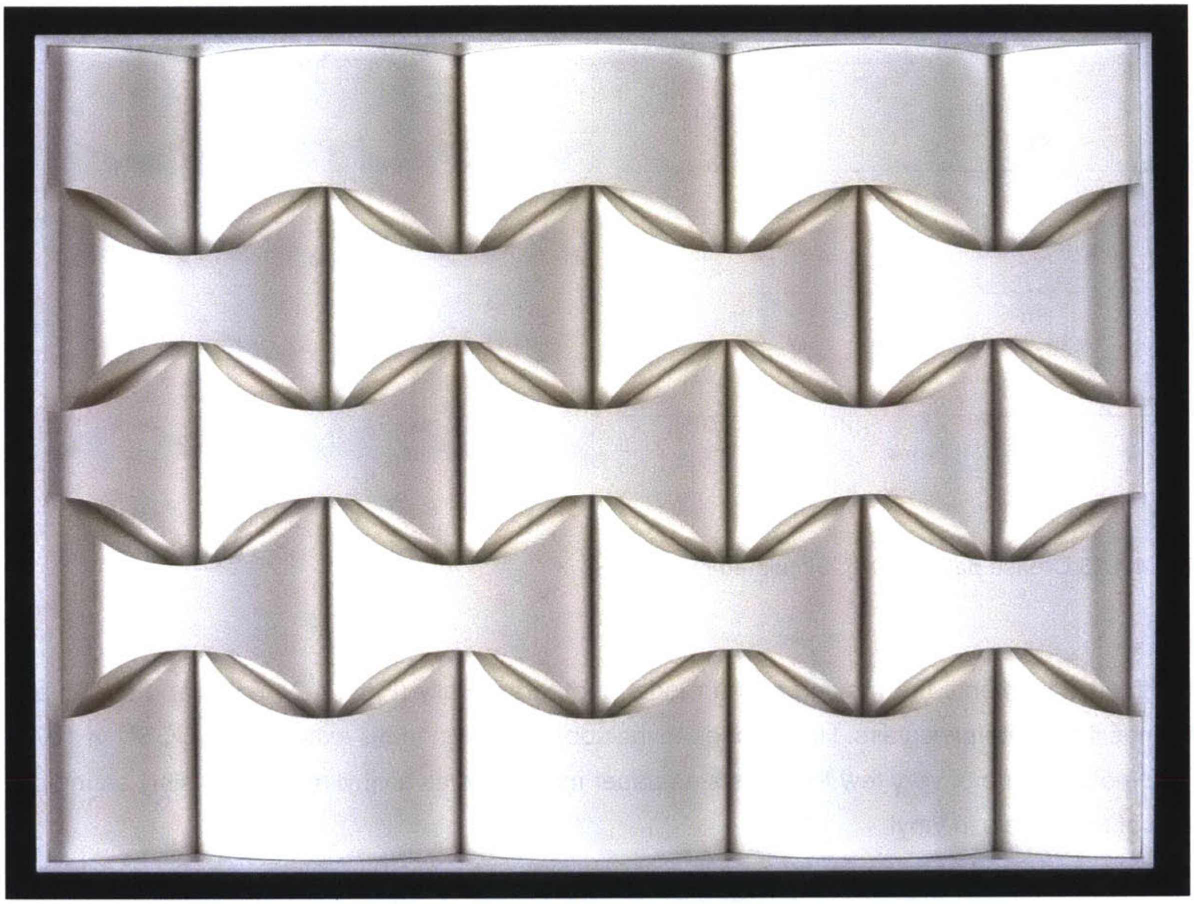


Fig 4.4.22 'Arches' (1978, DAH [TG])

Huffman makes the above vinyl model he calls 'Arches' in 1978 (Fig 4.4.22). He uses the second parabola gadget in conjunction with folded line segments. The main difference to the first parabola gadget consists of its tiling abilities, which appears to be what Huffman is interested in exploring.

Crease pattern and ruling analysis

Huffman uses all 4 sides of the second parabola gadget to create a design tile that consists of 2 parabolic mountain creases and 2 valley folds (Fig 4.4.23 left). I define the design tile here as a group of 4 prototiles, drawn in yellow, which are arranged in a staggered grid.

The rulings of the design tile fold into a cylinder in the center and an upper and lower cone. Huffman has to stagger the design tile, because every apex of the cones aligns with the straight crease between the cylinders. The 2 edges of the cone form the zigzag crease on the back of the model (Fig 4.4.24 left). All straight creases remain in a plane, which makes the design appear very regular. The model folds reasonably well during simulation (Fig 4.4.23 right).

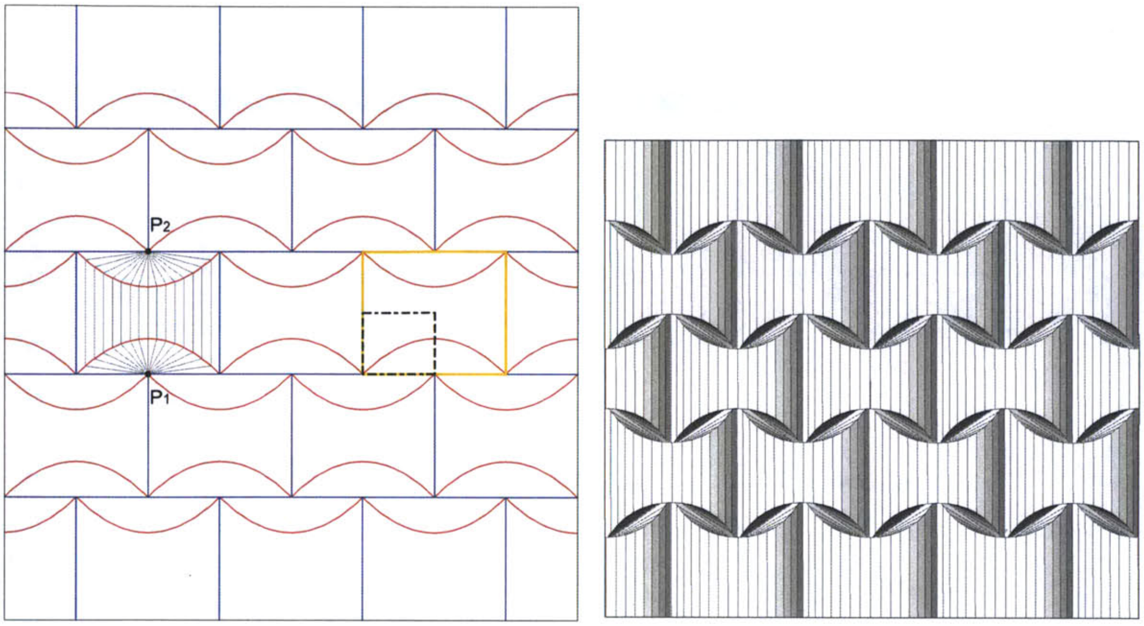


Fig 4.4.23 Crease pattern [DK], Simulated model [AH]

Notes

Huffman documents the making of the model photographically (Fig 4.4.24 right) and eventually frames it in white plexiglass. He chooses to include the design in his exhibition at UCSC in 1978. The model is one of very few that exists as paper model and as a vinyl model, possibly an indication of early trials in vinyl.

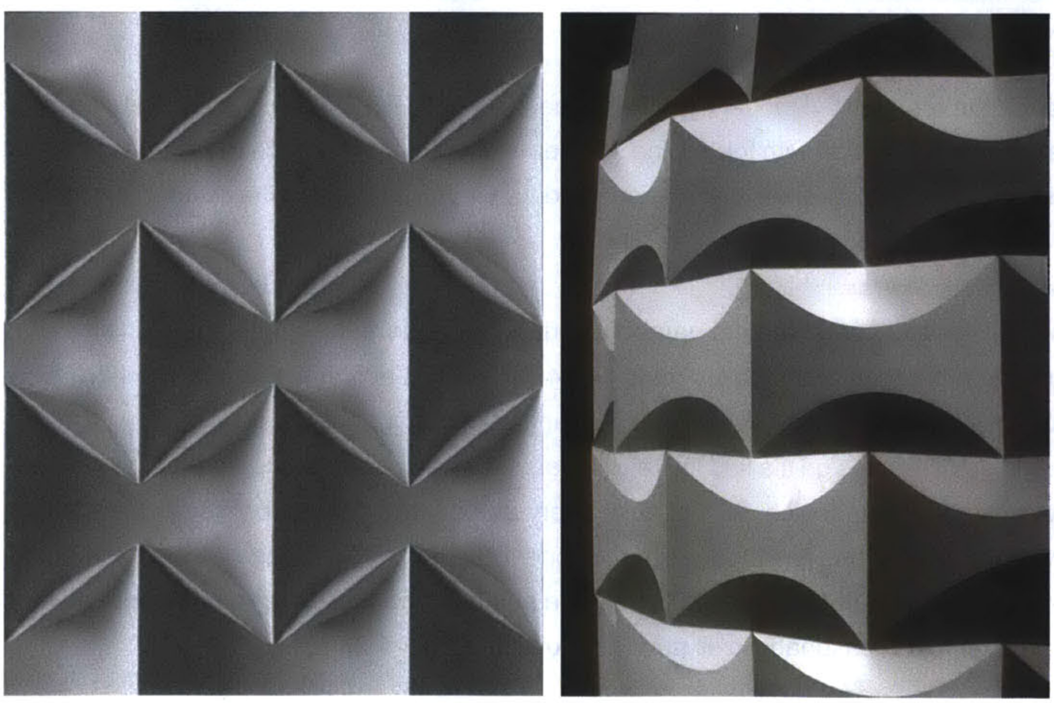
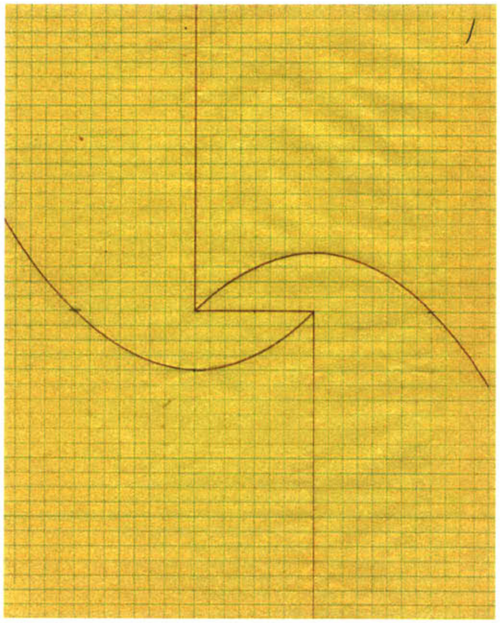


Fig 4.4.24 Vinyl model (1978, DAH [EAH]), Same model (1978, DAH [DAH])



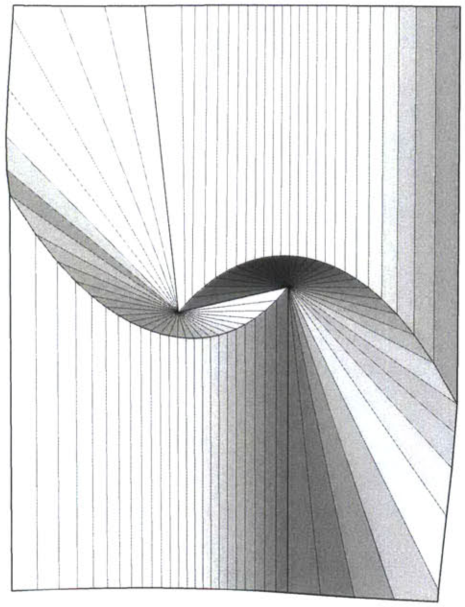
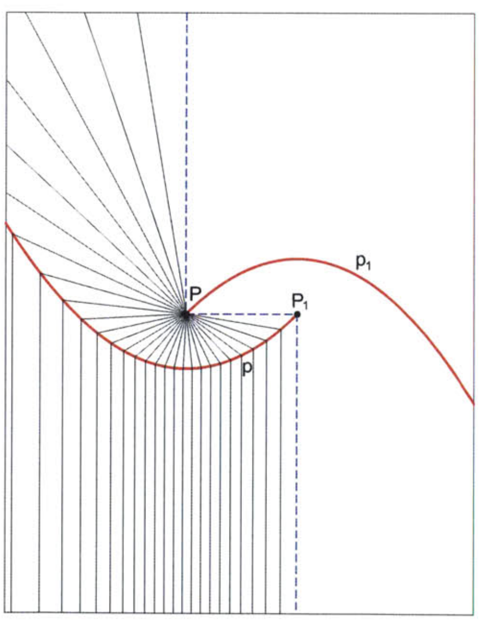


Fig 4.4.25 Paper model (undated, DAH [DK]), Simulated model [PC]

The above design uses the second parabola gadget, but Huffman extends the parabola toward the left and right edge of the paper (Fig 4.4.25 left).

Crease pattern and ruling analysis

The parabola gadget is not used in a regular tiling, but as asymmetrical tile rotated about the center of the crease pattern(Fig 4.4.25 left). The straight creases along the edges of the tiles are used as valleys. The rulings start at the foci of the parabolas and are refracted to a general cylinder. The simulation folds well (Fig 4.4.25 right).



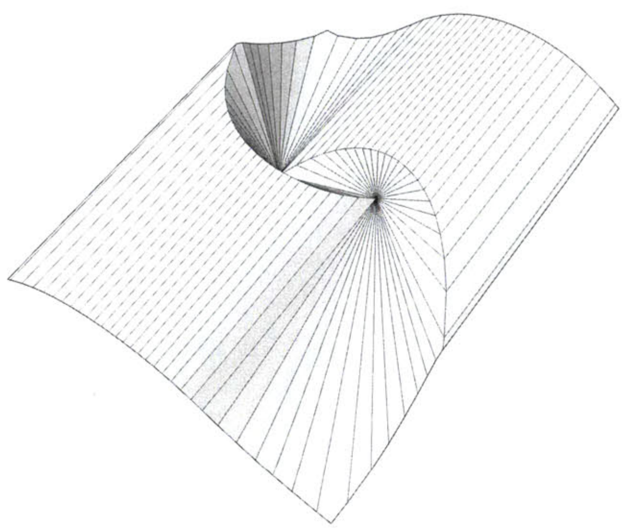


Fig 4.4.26 Crease pattern [DK], Simulated model [PC]

Gadgets with parabolas and line segments with a smooth transition

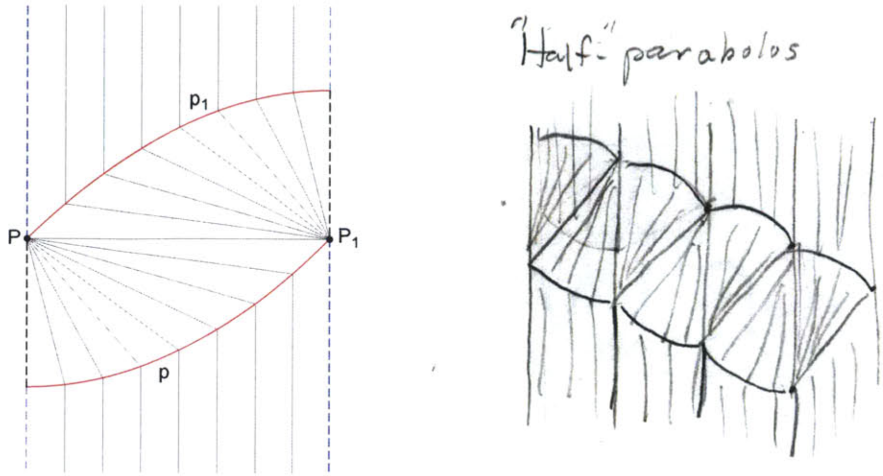


Fig 4.4.27 Gadget [DK], Sketch (undated, DAH [DK])

Huffman rotates and copies the second parabola gadget in such a way that it turns into to a new gadget with a smooth transition between *p* and *p'* (Fig 4.4.27 left). The sketch titled 'Half'-parabolas conveys the idea and shows cones and cylinders neatly packed next to each other (Fig 4.4.27 right). He explores this smooth transition in the next few designs and struggles with the pressure that builds up in the paper within the smooth area. The following design consists of a tiling of the new gadget (Fig 4.4.28).

Crease pattern and ruling analysis

Huffman extends the upper and lower half of the gadget with straight creases as valleys. The tiling connects the gadget 5 times and ends with horizontal line segments (Fig 4.4.28).

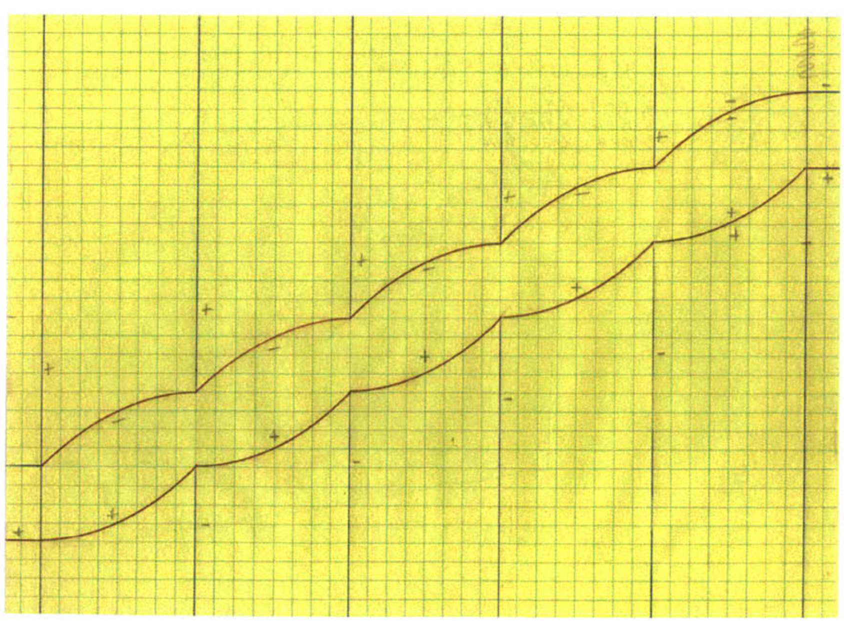


Fig 4.4.28 Paper model (undated, DAH [DK])

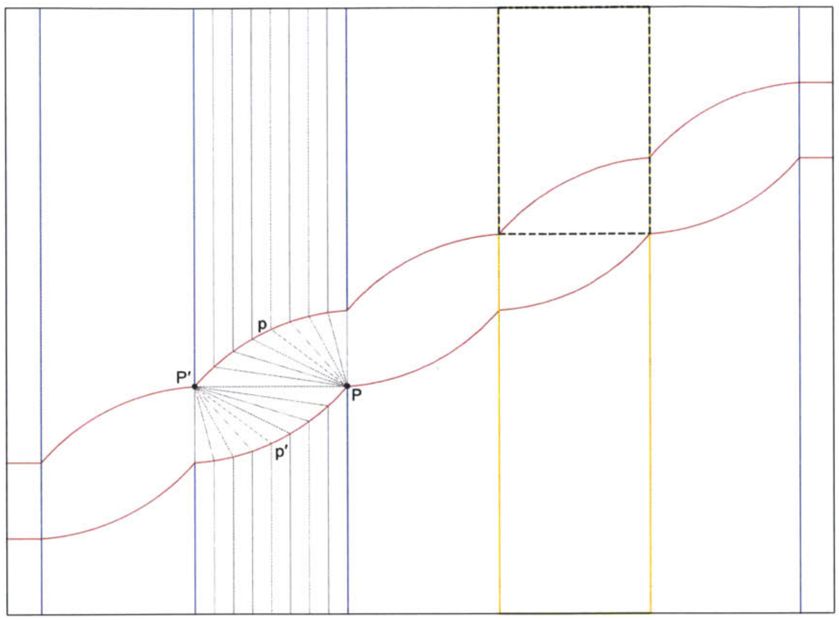


Fig 4.4.29 Crease pattern [DK]

The folded state results in long cylinders and the cones create a continuous concave transition in a diagonal formation. The model simulates well, but pushes the cylindrical surfaces away from the conical surfaces quickly (Fig 4.4.30).

Notes

Huffman marks the paper model with '+' and '-' , which may indicate that he is re-assigning mountain and valley creases. The second simulation shows this edited iteration (Fig 4.4.30 right).

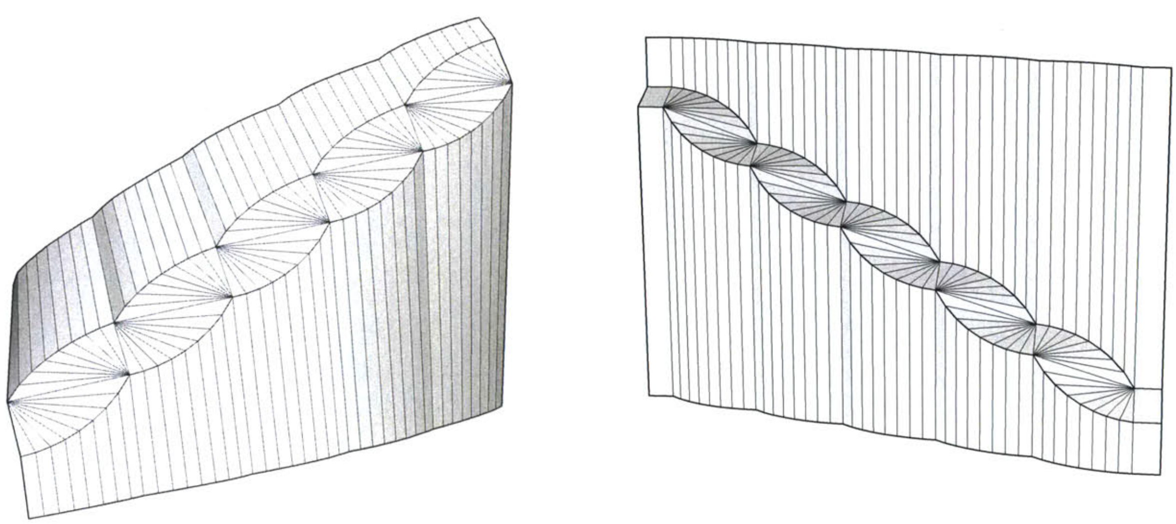


Fig 4.4.30 Simulated models [AH]

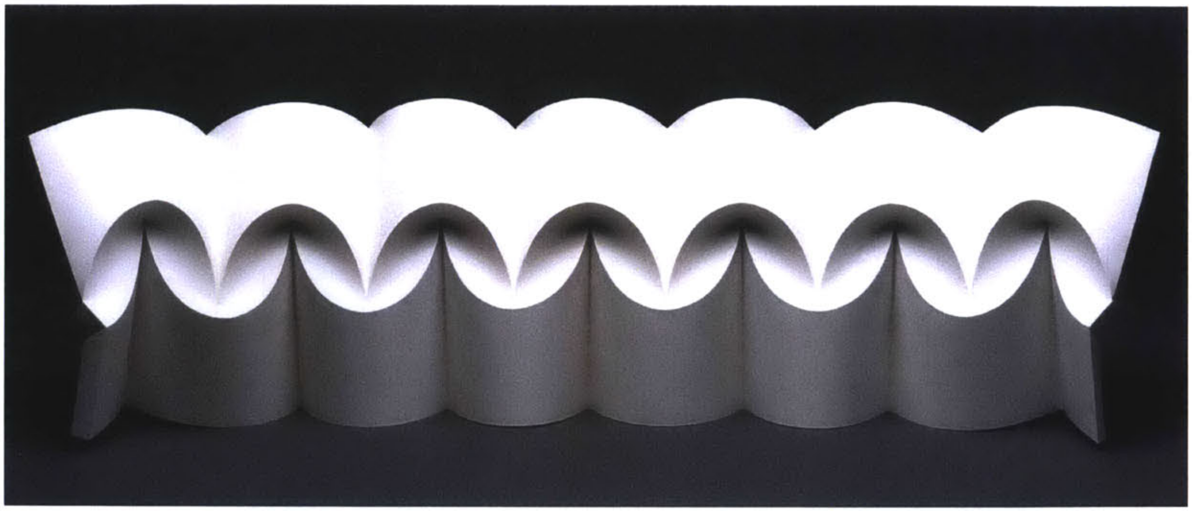


Fig 4.4.31 Vinyl model (1977, DAH [TG])

The above design appears in a research report and continuation proposal to NSF, number DCR75-12814, in 1977 (Fig 4.4.31). It is one of Huffman's attempts to combine his curved crease paperfolding work with his prior work on machine vision. The previous combined smooth gadget is used to form a symmetrical partial parabola that ends at the latus rectum. His photograph of the partial crease pattern shows the horizontal alignment of all foci (Fig 4.4.32).

It is unclear, whether this model can fold such that the general cylinders remain parallel to each other. One needs to use a lot of force to bend the material in the convex area of the partial cones between the parabolas in order to achieve the alignment.

Crease pattern and ruling analysis

Huffman creates a staggered regular tiling, similarly to the previous 'Arches' design, but only creates 1 row with the new smooth transition (Fig 4.4.33).The crease pattern consists of 14 prototiles.

Simulation is possible, but the crease Huffman eliminates is very pronounced and does

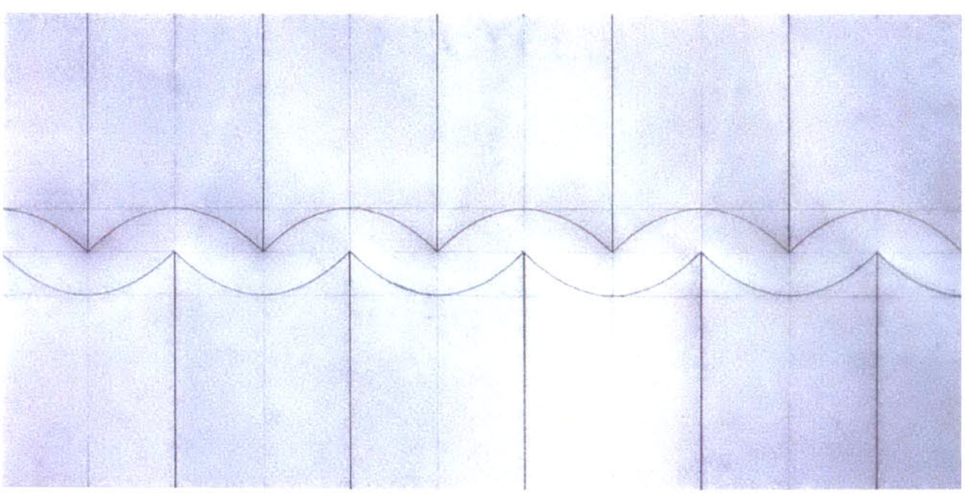


Fig 4.4.32 Vinyl model (1977, DAH [DAH])

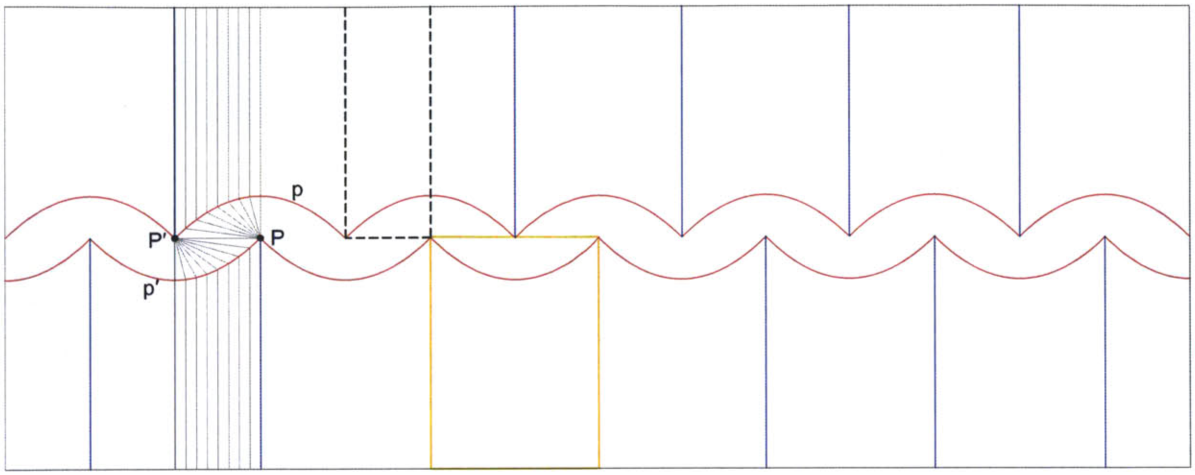


Fig 4.4.33 Crease pattern [DK]

not resemble the vinyl model.

Notes

Huffman appears to have planned to make a final model as he adds tabs, which he typically attaches to an acrylic frame. He might not have been content with the result and decides to use the model for demonstration purposes instead. It shows signs of having been handled many times.

He photographs the model with dramatic shadows in the conical transition (Fig 4.4.34).

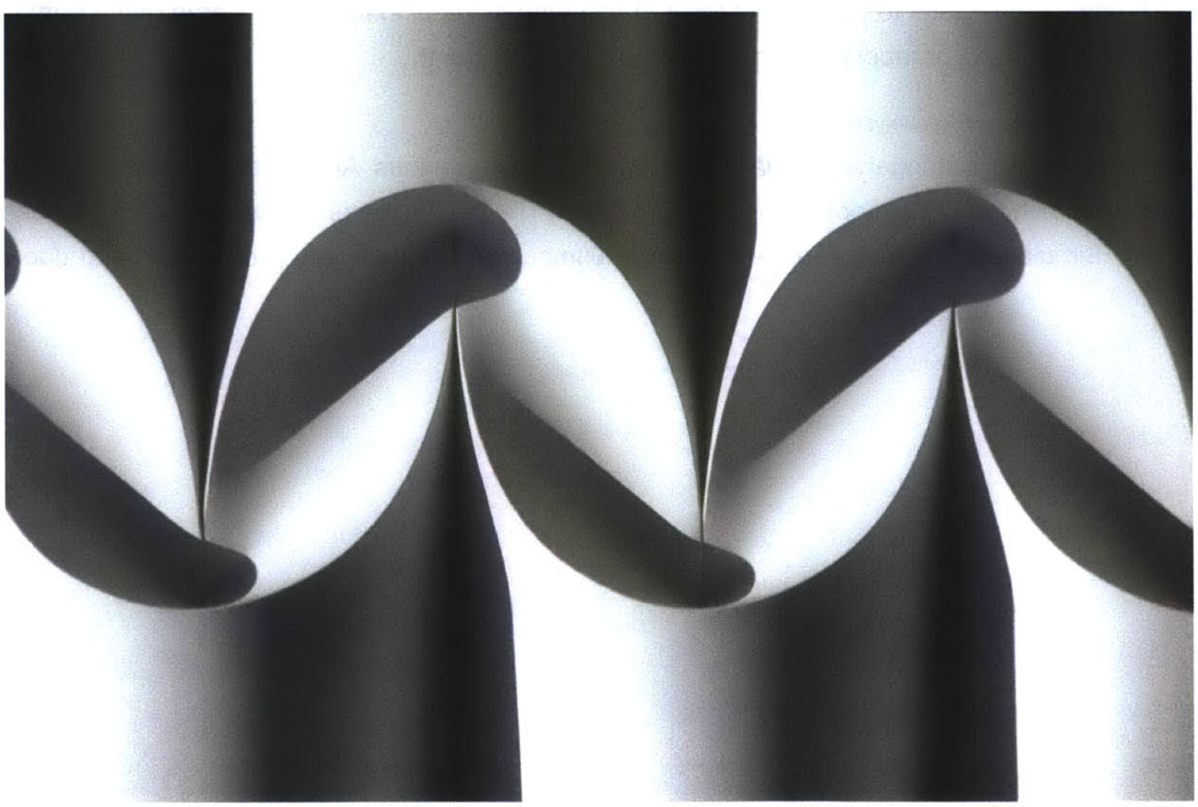


Fig 4.4.34 Vinyl model (1977, DAH [DAH])

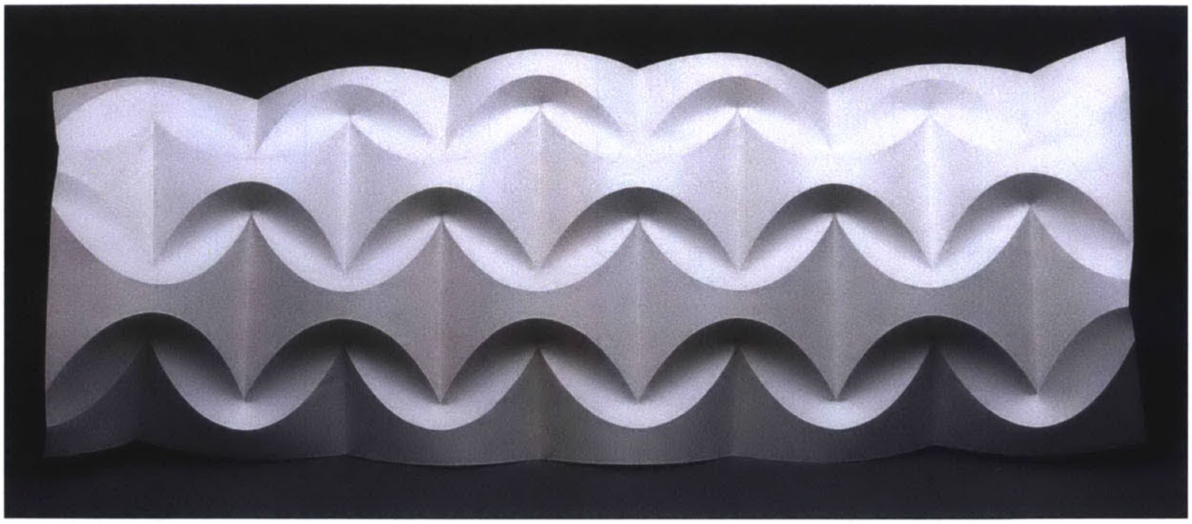


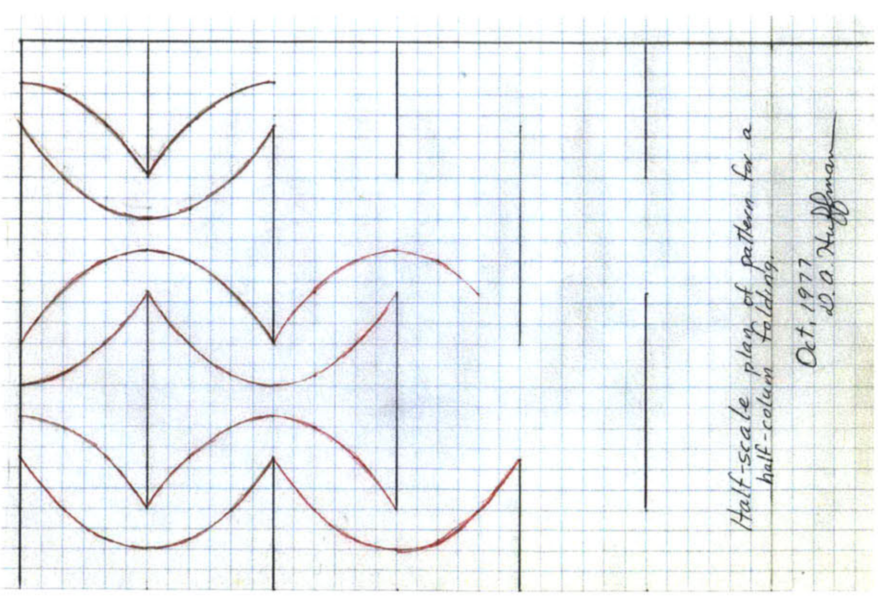
Fig 4.4.35 'Half-column folding' (1977, DAH [TG])

In the above example Huffman uses the previous design and tiles it both directions (Fig 4.4.35). He does not make a frame for this iteration either as the design has a tendency to form a cylindrical configuration in 3d. He instead keeps this example as a loose model.

Crease pattern and ruling analysis

The parabolas extend further than in the previous designs (Fig 4.4.36). The prototile consists of the gadget itself (Fig 4.4.37).

The simulated model shows the cylindrical tendency of the entire tiling in 3d (Fig 4.4.38). This characteristic appears to be the reason for the name 'half-column folding' which Huffman uses in his drawing (Fig 4.4.36). The connecting edge between tiles in the conical area is more



$a = 2.5$
$d = 3$
$a_1 = 2.5$
$l = 10$ (Fig 4.4.137)

Fig 4.4.36 Drawing (1977, DAH [DK])

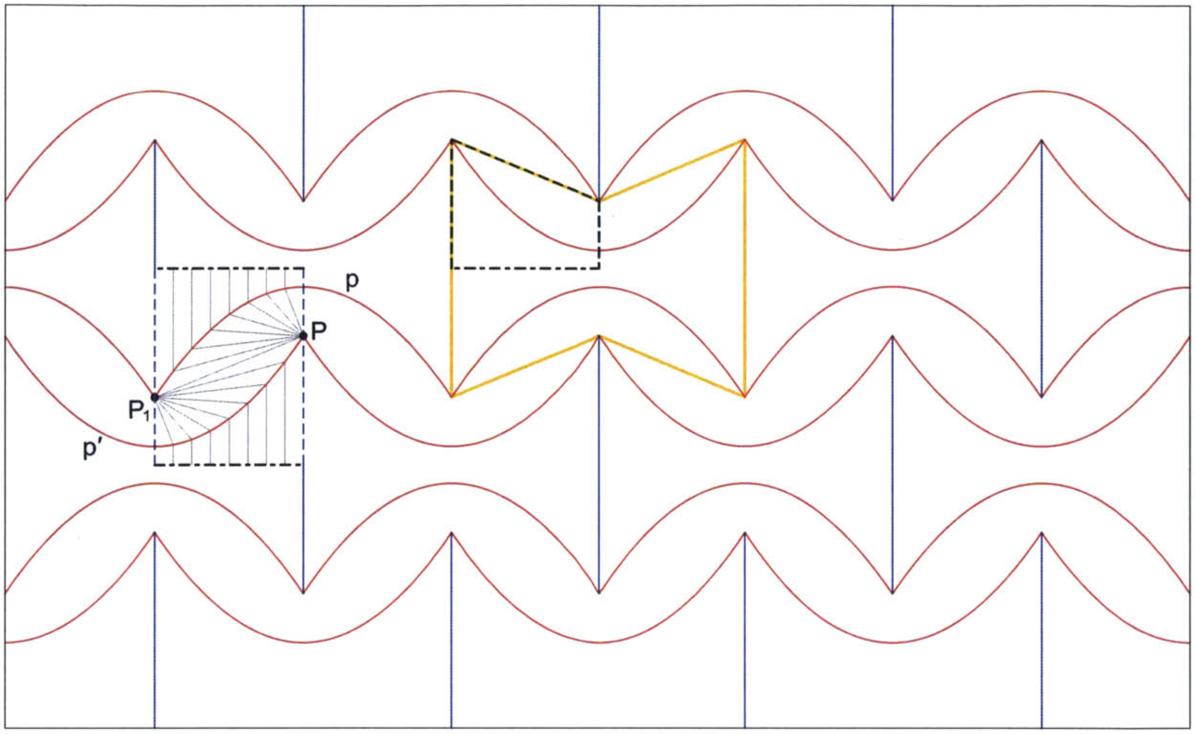


Fig 4.4.37 Crease pattern [DK]

pronounced as the folding angles between rulings in that area. This is the same area with high pressure in the vinyl model.

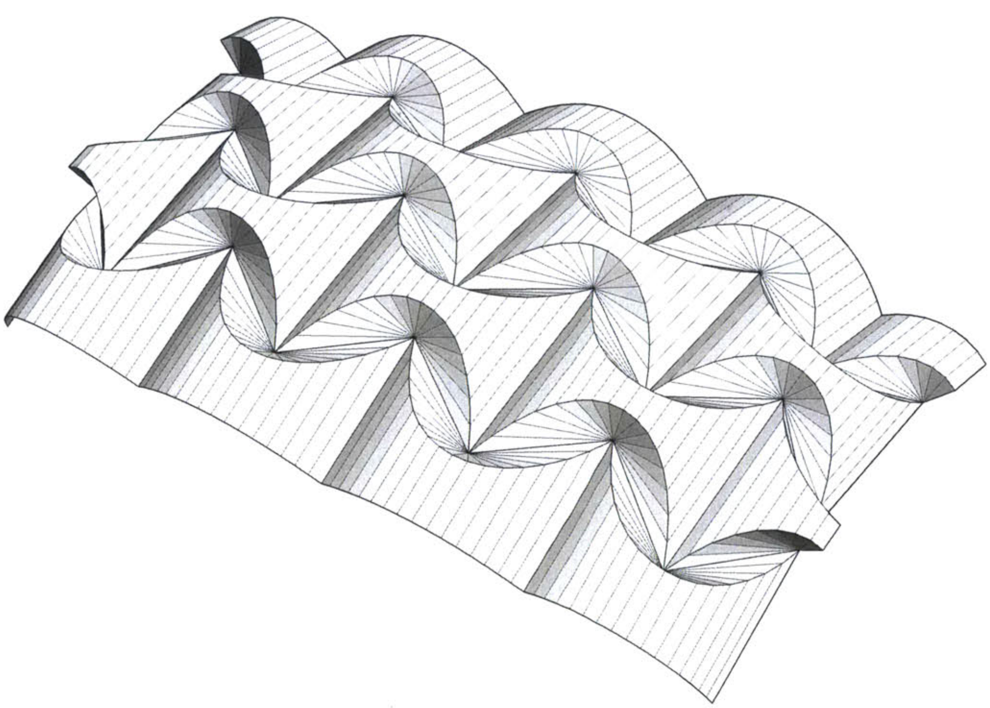


Fig 4.4.38 Simulated model [JH]

Gadgets with parabolas, line segments and circles

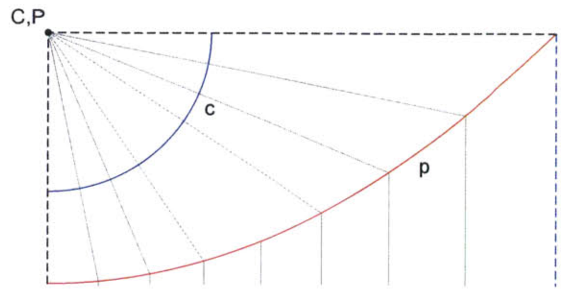


Fig 4.4.39 Gadget [DK]

Huffman incorporates an additional arc in this gadget, but the curve has little impact. The gadget operates in similar ways as it did previously as the parabola refracts rulings in the same way and the circle arc does not alter the ruling direction. The non-dominant arc gets assigned an alternating folding direction. It is folded as valley if the parabola is a mountain, for example (Fig 4.4.40).

The gadget

The parabola can extend to any height less than the latus rectum, the horizontal through *P*. The same 3 edges as in the last gadget can act as tile boundaries. The rulings continue straight through the arc.

Huffman makes 2 examples with this gadget that look similar, but have a significantly different crease patterns.

Crease pattern and ruling analysis

The gadget in the example below appears 12 times mirrored along the horizontal (Fig 4.4.40). The design tile consists of two prototiles, similarly to the previous examples. The simulation folds well and can fold beyond what is shown in the figure (Fig 4.4.40 right).

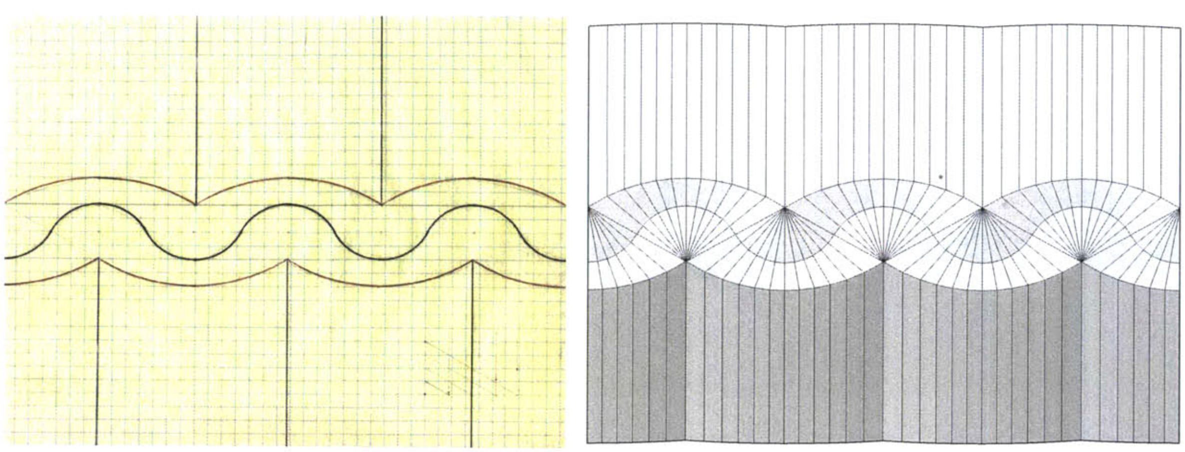


Fig 4.4.40 Paper model (undated, DAH [DK]), Simulated model [JH]

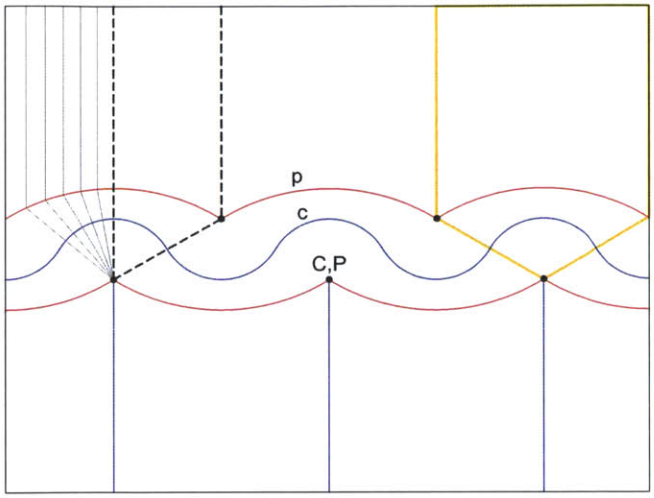


Fig 4.4.41 Crease pattern [DK]

Crease pattern and ruling analysis

The gadget of the second iteration repeats 11 times and all foci are aligned on a horizontal (Fig 4.4.42). Here the arc touches the parabola, which has little impact on the folding behavior in the same figure on the right.

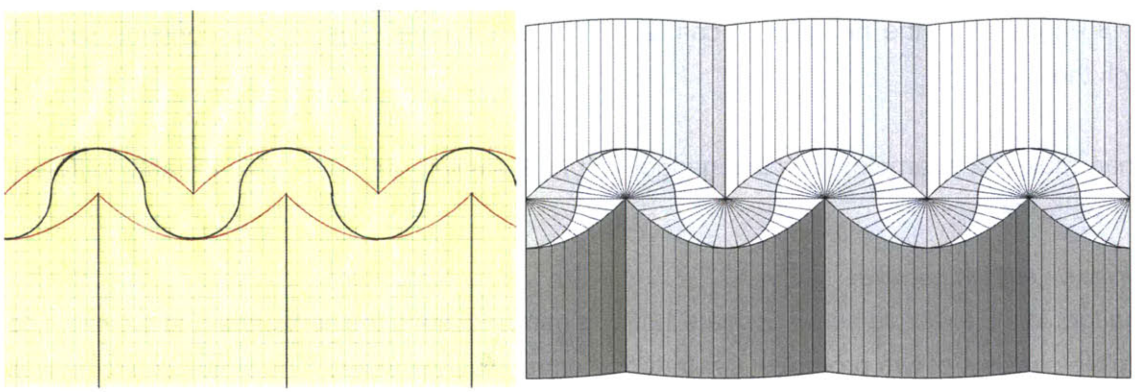


Fig 4.4.42 Paper model (undated, DAH [DK]), Simulated model [JH]

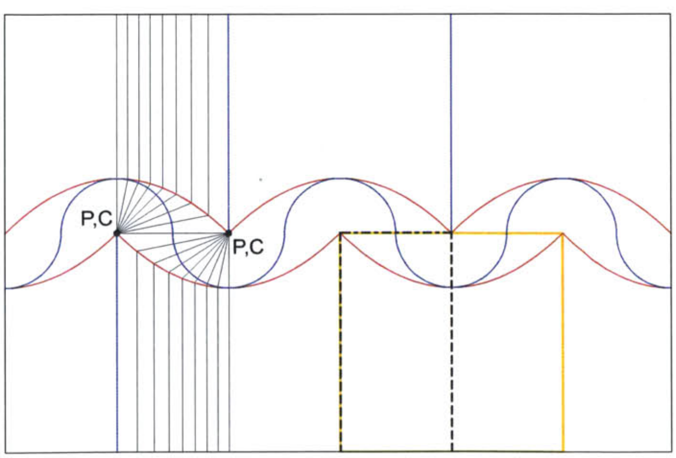


Fig 4.4.43 Crease pattern [DK]

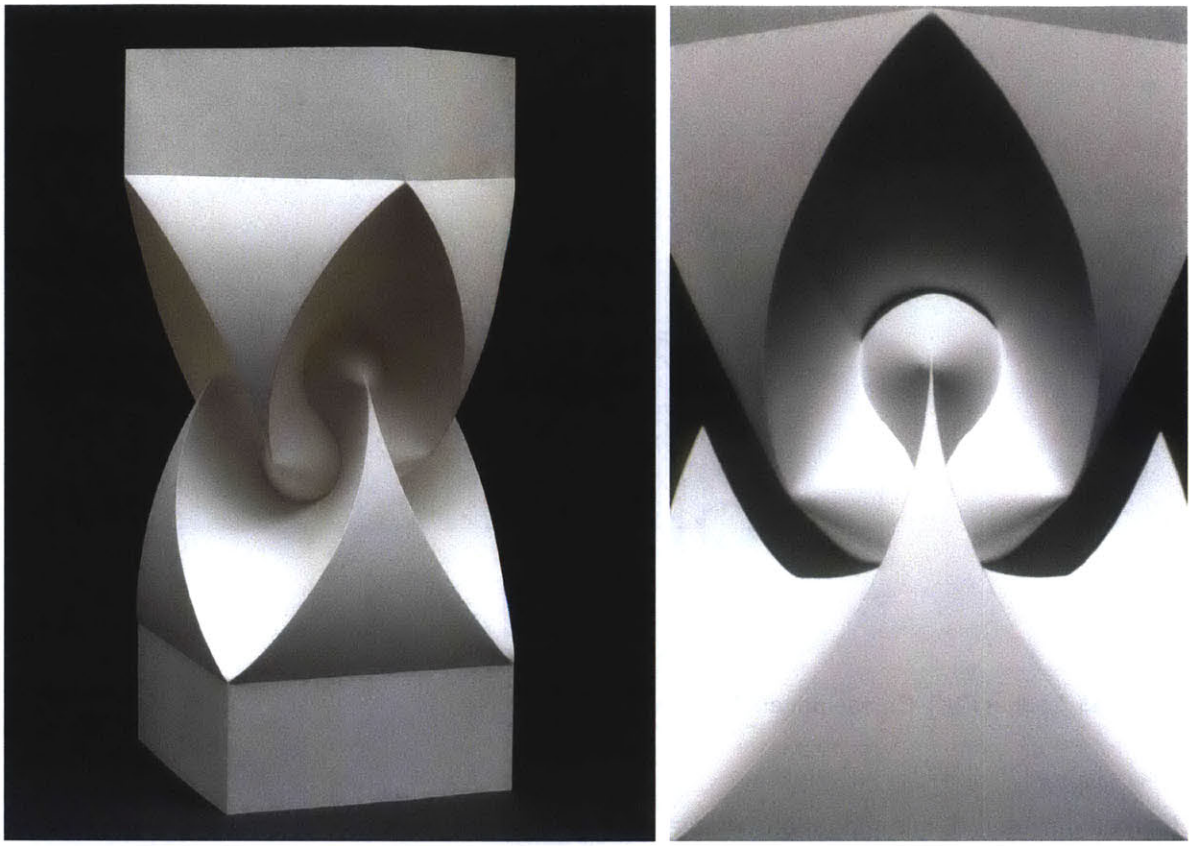


Fig 4.4.44 Vinyl model (1978, DAH [TG]), Identical model (1978, DAH [DAH])

This part of the section presents several designs that have characteristics common to the above model (Fig 4.4.44). They form cylindrical enclosures that meet on the back side of the model. They do so with a rotated gadget such that it has a horizontal parabola axis. Huffman works on this series extensively and we can study several different sketches, drawings and sketch models that elucidate his thinking.

The gadget

The gadget consists of the same curves, which share *C* and *P*. The parabola changes the direction of the rulings and the arc has no impact on the ruling direction (Fig 4.4.45 left). The edges of the tile us that the gadget is rotated as the dash-dotted line crosses through rulings of a horizontal cylinder that was oriented vertically in previous designs. The prototile forms a rectangle.

Crease pattern

The parabola extends to the intersection of the horizontal line through *P*, the latus rectum, when looked at in its default position with a vertical axis. Once rotated, the tiling is constructed by vertically mirroring the prototile into a pair, which is then mirrored along the horizontal and moved by the width of the gadget (Fig 4.4.45 right).

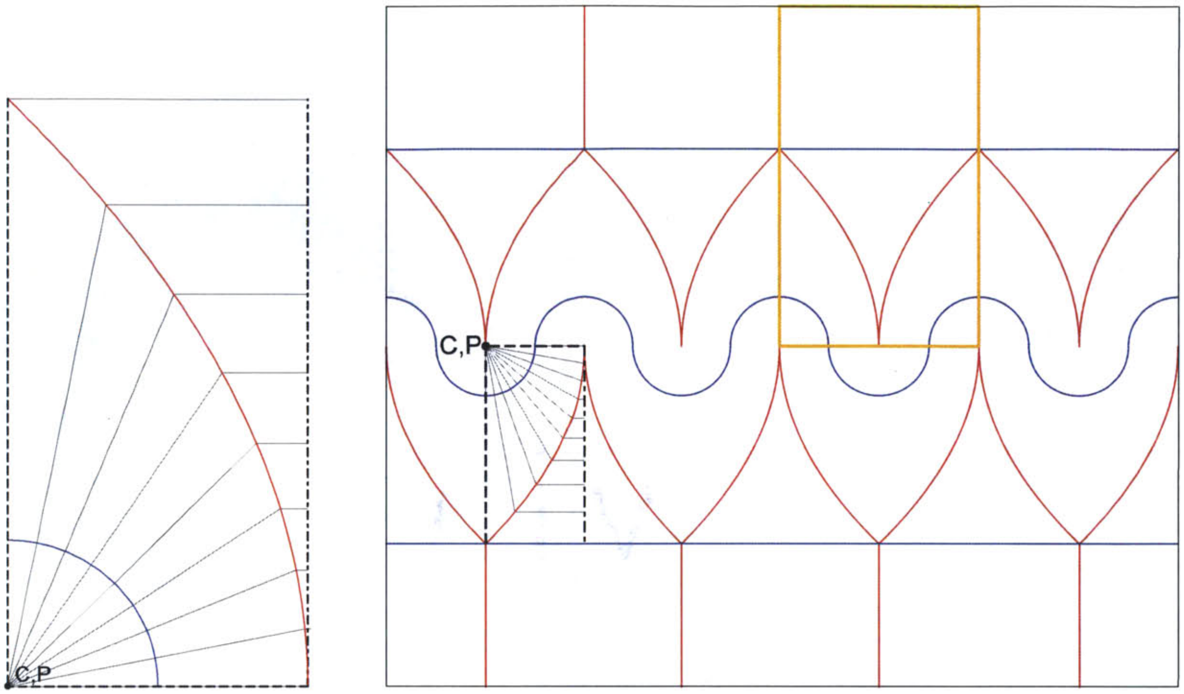


Fig 4.4.45 Gadget [DK], Crease pattern [DK]

Ruling analysis

Huffman's sketch of this model provides a rare glimpse into his thoughts on the direction of the rulings (Fig 4.4.46 left). The circle clearly has no impact on the ruling direction. The model does not fold during simulation and has to be modified in order to achieve the 3d configuration of Huffman's vinyl model. The circle arcs have to remain in a plane during folding which results in a similar configuration to the first example in his primer on paper (Fig 2.3.6 left).

The only reasonable solution Tomohiro Tachi and I could obtain during simulation conforms with Huffman's in-progress model in the below figure (Fig 4.4.46 right). It is likely that the

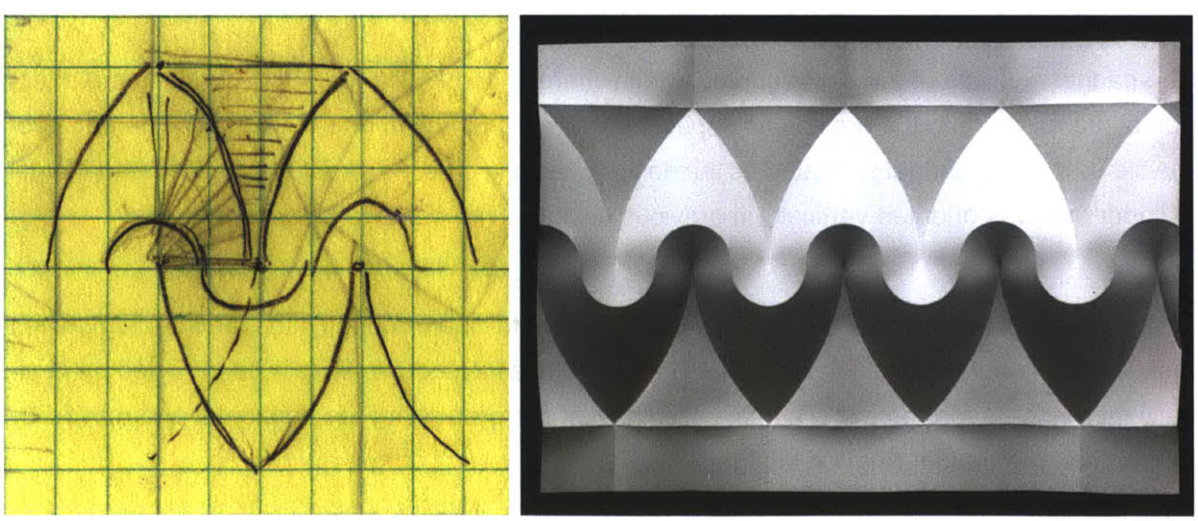


Fig 4.4.46 Sketch (undated, DAH [DK]), Vinyl model (1977, DAH [DAH])

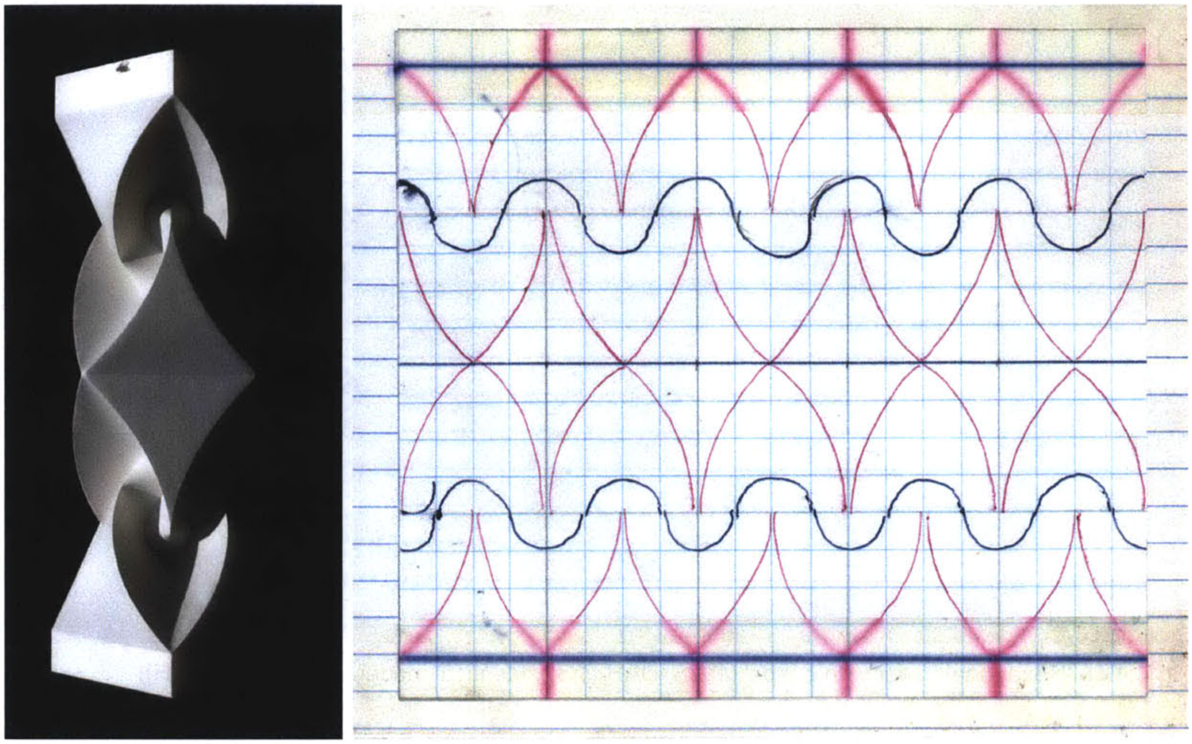
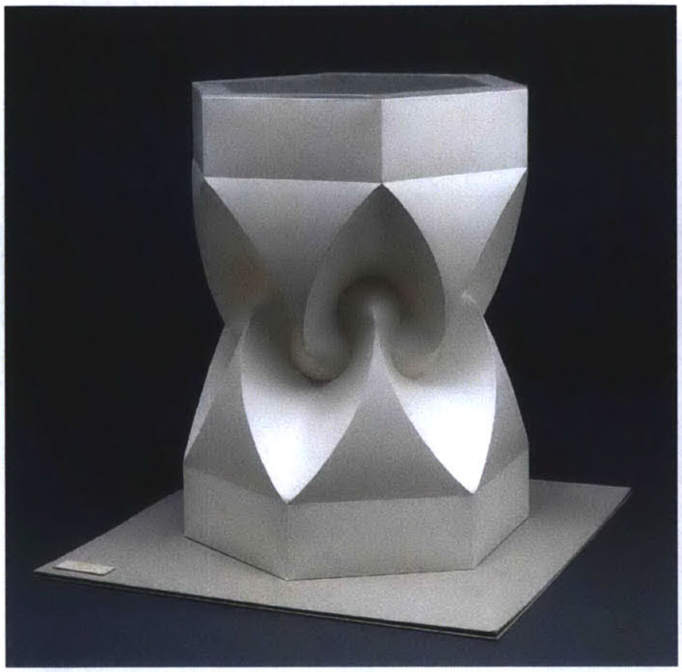


Fig 4.4.47 Paper model (1978, DAH [DAH])

model in its 3d configuration does not exist, mathematically speaking.

Notes

Huffman develops a tower made of two copies of the design (Fig 4.4.47 left). He makes a paper model that he subsequently photographs with dramatic shadows. The sketch on the right is taped to an index card and reveals the proportional relationships within the gadget.



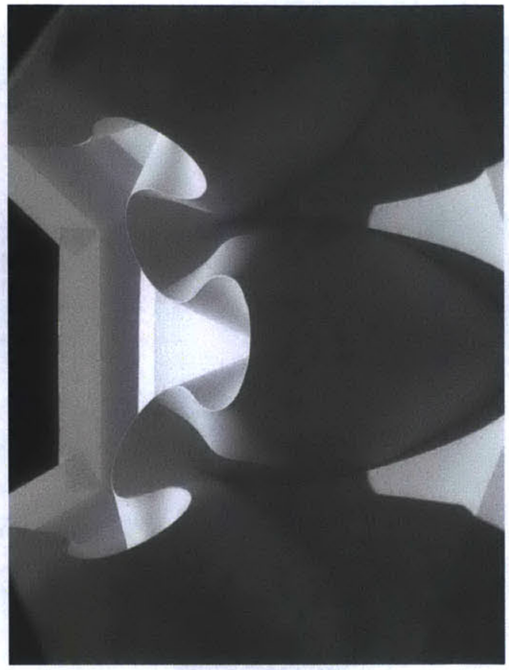


Fig 4.4.48 'Hexagonal tower with cusps' (1978, DAH [TG]), Identical model (1978, DAH [DAH])

The above design (Fig 4.4.48), has more facets than the previous 2 examples, hence the name 'Hexagonal tower with cusps'. Huffman includes it in his 1978 exhibition and also has a photograph taken of himself holding it for a UCSC news bulletin (Fig 1.1). It is probably the most published model by Huffman. The model has been investigated and remade many times and is also part of a paper by Erik Demaine, Martin Demaine, Tomohiro Tachi and myself.

Crease pattern and ruling analysis

The parabola gadget with a horizontal main axis from the previous examples is also the prototile in this example. It has a *1:2* proportion, which means it is the typical parabola Huffman uses (Fig

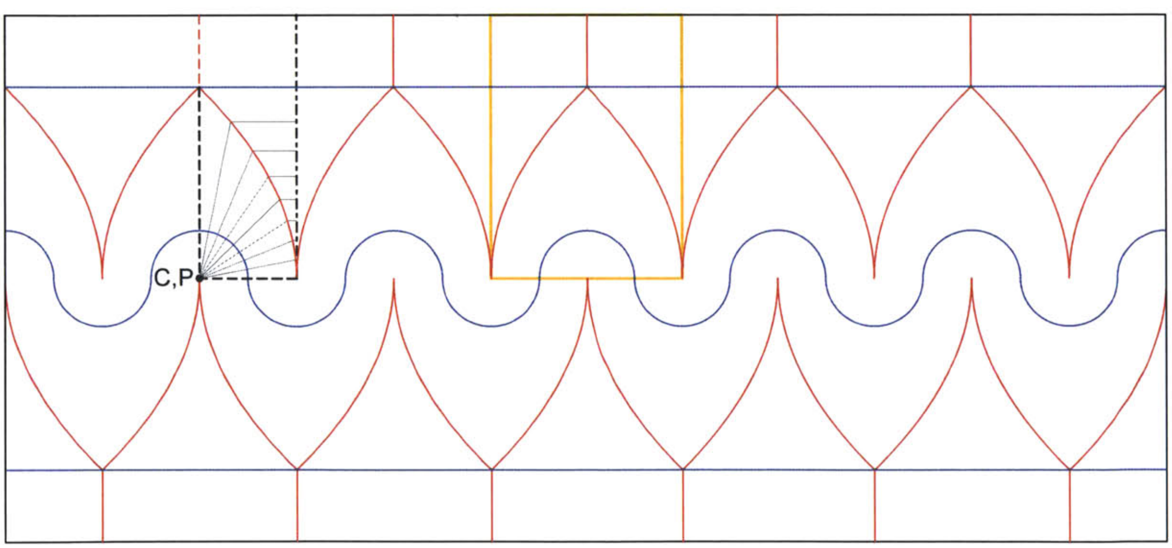
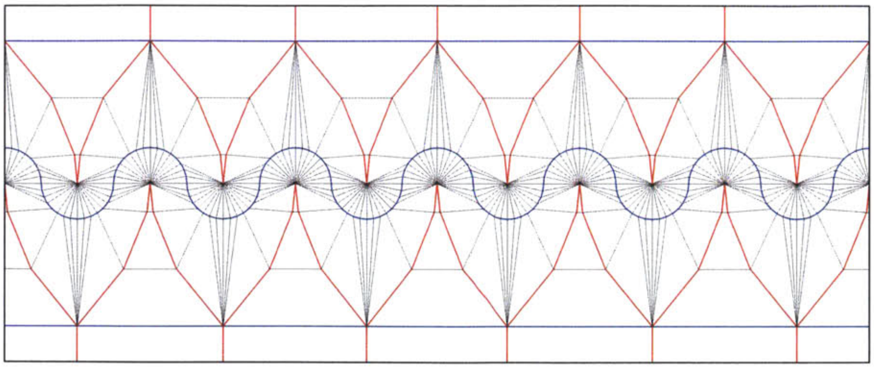


Fig 4.4.49 Crease pattern [DK]



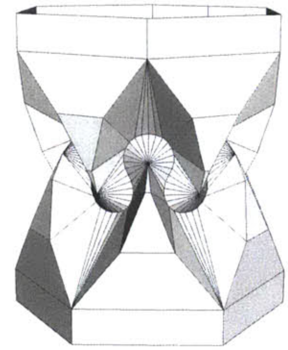
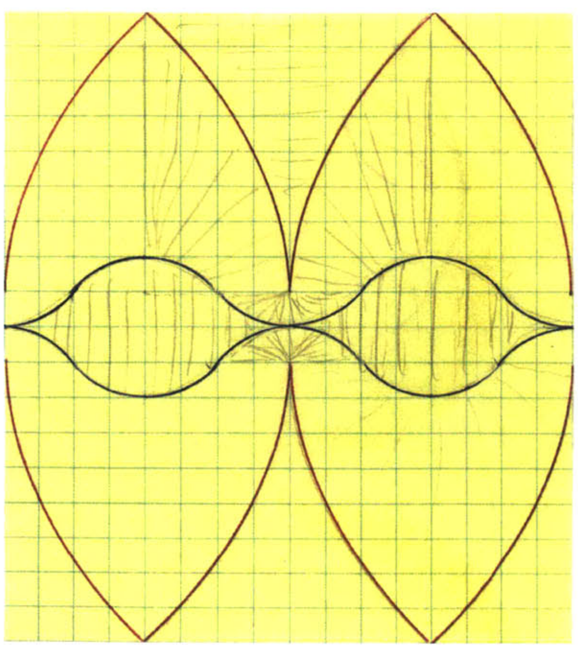


Fig 4.4.50 Discrete crease pattern [DK], Simulated model [AH]

4.4.49). All observations for the previous example apply here as well. This includes a failed simulation based on Huffman's suggested ruling pattern.

Tomohiro Tachi and myself have worked on a discrete version of the design that can fold (Fig 4.4.50), which consist of a partial cone at the intersection of the parabolas on the outer edges at the top an bottom of the crease pattern. A joint authored paper with Erik and Martin Demaine with further explanations is pending.

The iteration below is included in this section for comparison even though the curves are different (Fig 4.4.51). The valley curves consist of parabolic splines, which I discuss in a later section. It might have been an early attempt to designing the final model, but its folding behavior displays very different results. The top and bottom parts move away from the central area rapidly. The simulated model shows the folded design about half-way through the possible folding motion (Fig 4.4.53 right).



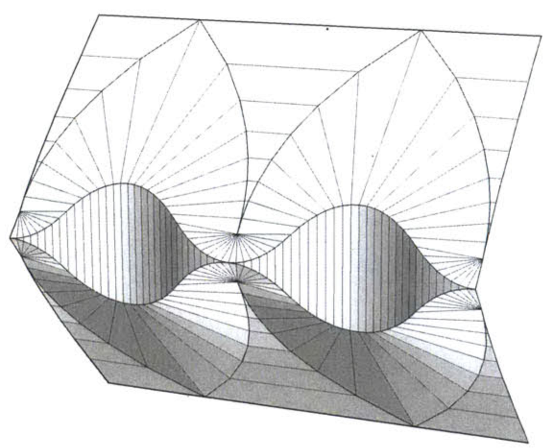


Fig 4.4.51 Sketch (undated, DAH [DK]), Simulated model [AH]

Gadgets with parabolas and line segments with inverted smooth transition

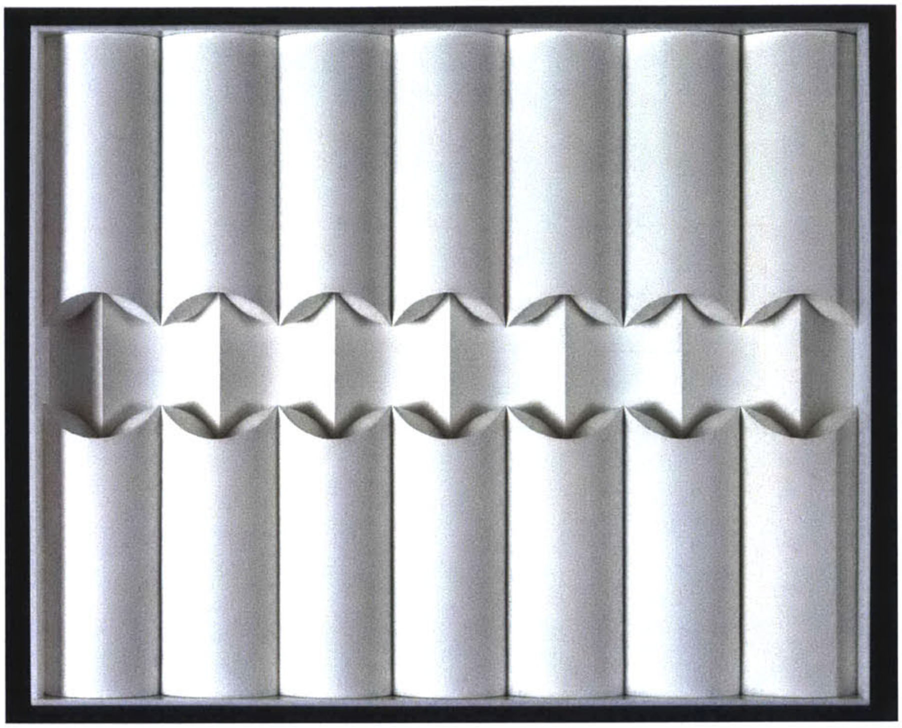
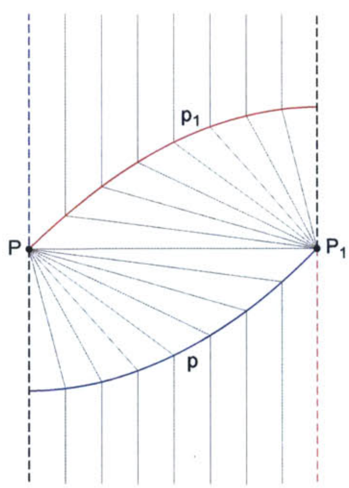


Fig 4.4.52 'Seven interrupted semi cylinders' (1977, DAH [TG])

Huffman calls the above design 'Seven interrupted semi cylinders' and includes it in his exhibition at UCSC in 1978 (Fig 4.4.52). The model introduces a new use of the parabola gadget that results in concave cylinders in the transition area between the tall cylinders at the top and bottom.

The gadget

The previous gadget is modified here by assigning the bottom parabola as a valley and changing the lower straight crease to a mountain (Fig 4.4.53).



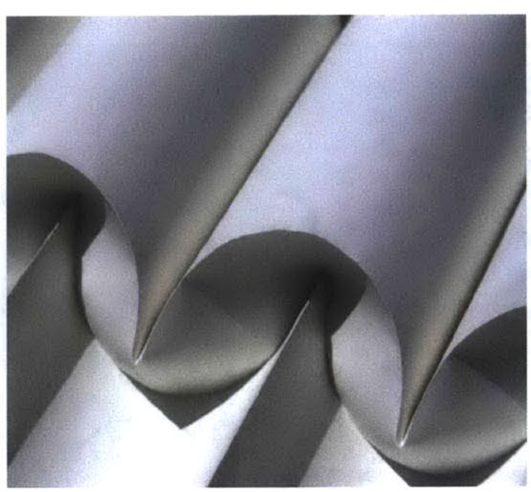


Fig 4.4.53 Gadget [DK], Paper model (1977, DAH [DAH])

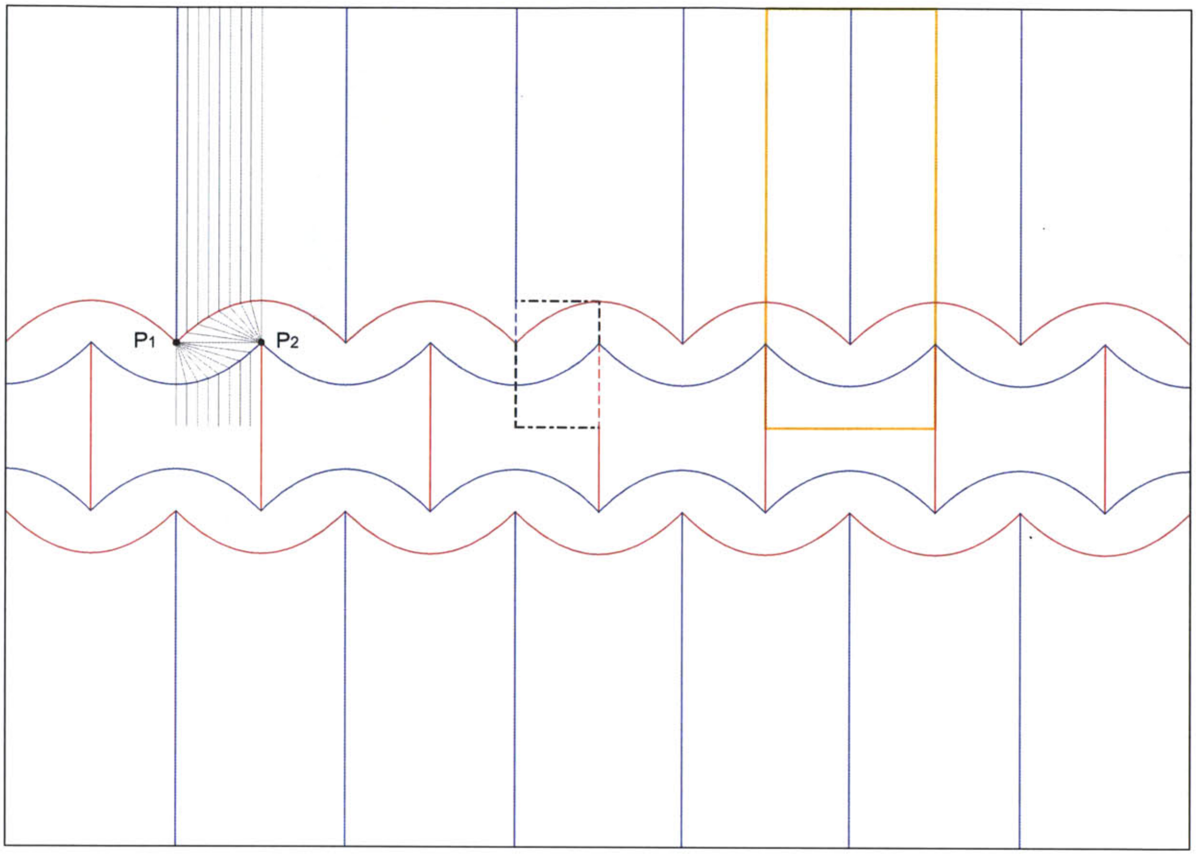
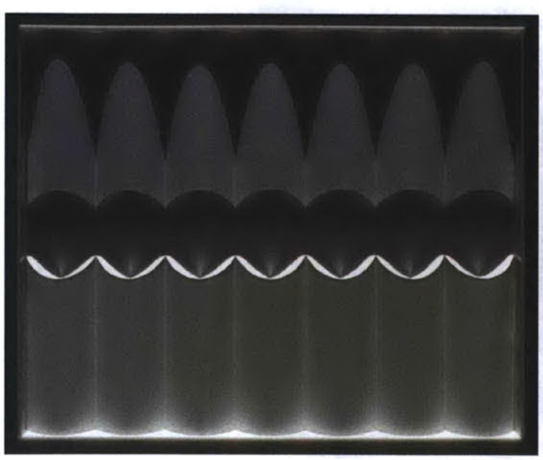


Fig 4.4.54 Paper model (undated, DAH [DK]), Crease pattern [DK]

Crease pattern and ruling analysis

The similarities to the previous designs consist of 2 the mirrored parabolas of the gadget and one half of the crease pattern looks very similar (Fig 4.4.54).

The rulings of half the crease pattern are also similar to the previous versions in the flat case. The difference consists of the valley crease in this gadget that creates a different conical transition between the 2 parabolas (Fig 4.4.53 right). The configuration resolves the previous is-



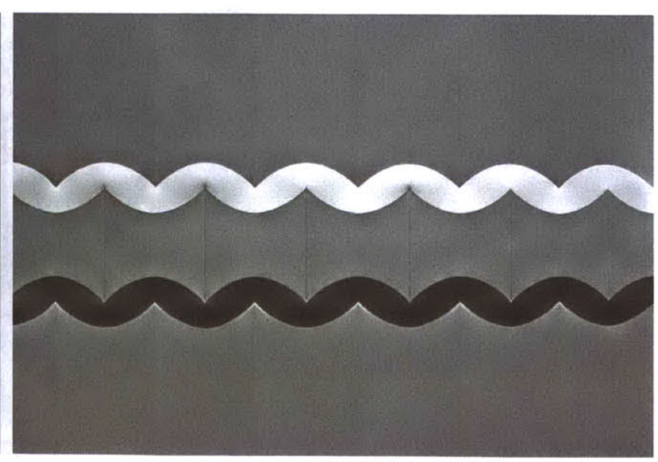


Fig 4.4.55 Vinyl model (1977, DAH [DAH]), Vinyl model (1977, DAH [DAH])

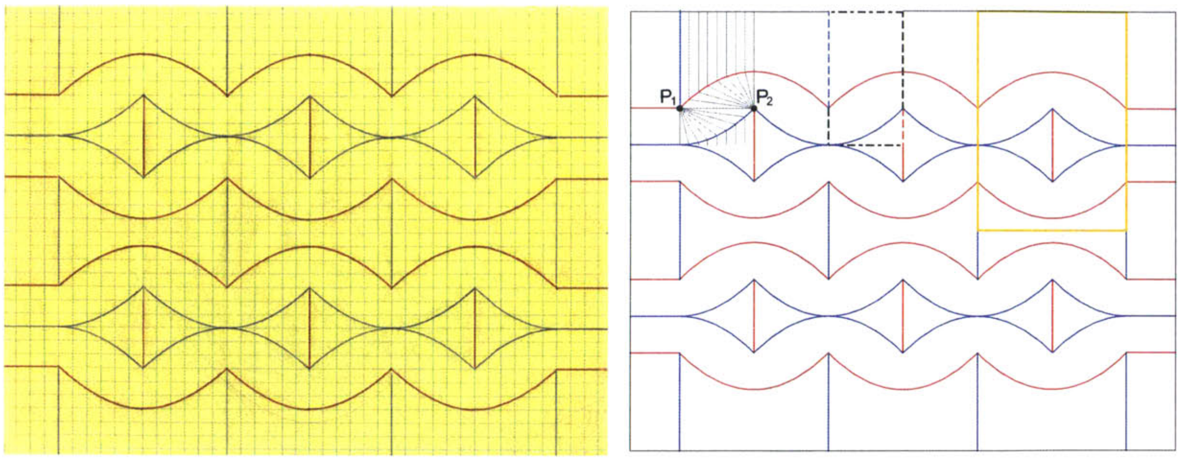


Fig 4.4.56 Paper model (undated, DAH [DK]), Crease pattern [DK]

sue of built-up pressure in the paper within the conical transition area.

Notes

The in-progess model shows the opposite side with the central are as convex cylinders (Fig 4.4.55). Huffman photographs the model not only during its making, but appears to take pleasure in creating artistic images with high contrasting shadows (Fig 4.4.55 left).

The above iteration adds a second row and reduces the inverted cylindrical area such that the parabolic curves touch each other (Fig 4.4.56). All main characteristics of the previous design are the same. The simulation folds well (Fig 4.4.57).

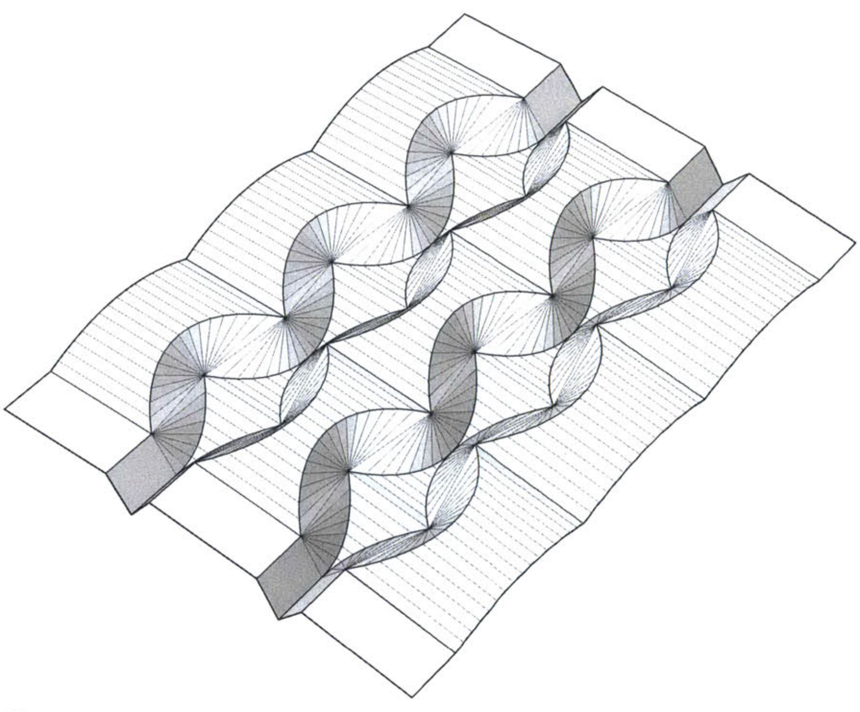


Fig 4.4.57 Simulated model [JH]

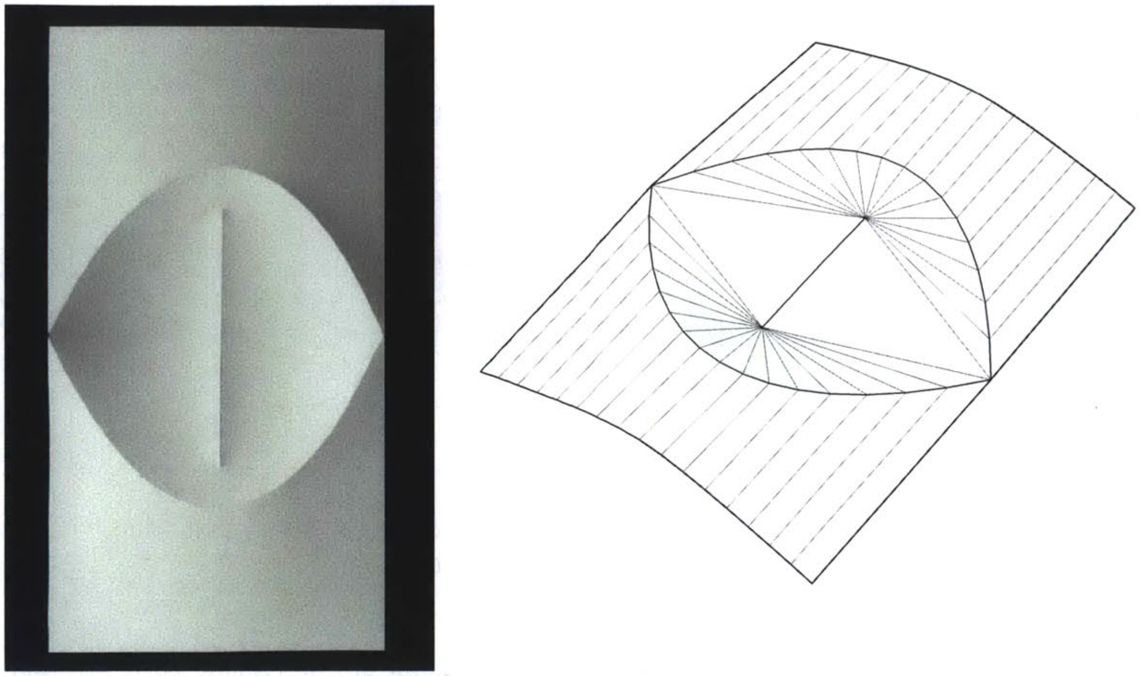


Fig 4.4.58 Vinyl model (undated, DAH [DK]), Simulated model [JH]

The 2 last examples in vinyl (Fig 4.4.58 left) and in paper (Fig 4.4.59 left), both appear to be study models. Huffman uses the single curve parabola gadget with a long parabola. He mirrors the tile centered on the main axis. The straight line segment in the center connects the 2 foci in the crease pattern. As a result the ruling configuration creates 2 flat triangles in the center (Fig 4.4.59 right).

The simulated model (Fig 4.4.58 right) uses the crease pattern of the second version as no sketch exists for the vinyl model. It folds well but the smooth connection is almost not achievable unless it is hardly folded, which is consistent with the appearance of Huffman's vinyl model.

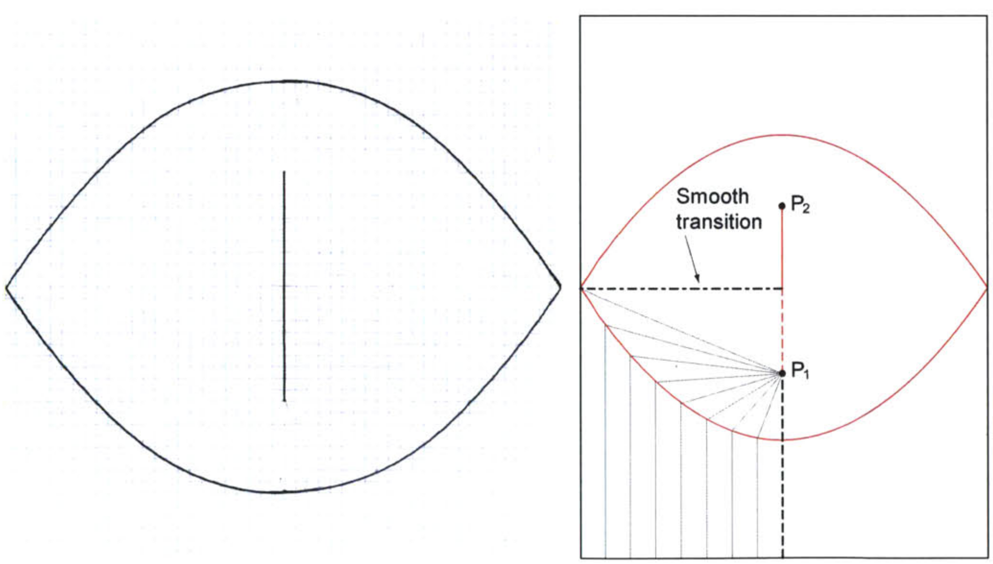


Fig 4.4.59 Paper model (undated, DAH [DK]), Crease pattern [DK]

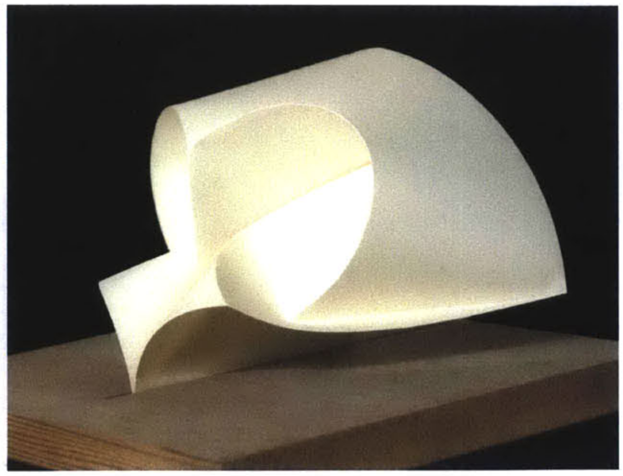


Fig 4.4.60 'Bird skull' (1977, DAH [DAH]), Identical model (1977, DAH [DK])

'Bird skull' of 1977 concludes this section on parabola gadgets with line segments. It remains unclear which curves Huffman draws for the design and reconstructions are inconclusive. The assumed crease pattern consists of parabolas with vertical axes, but no correct rulings can be constructed.

The model is first folded in a similar way to most of Huffman's designs, but then needs to be twisted to create the large tuck in the bottom center of the crease pattern. The bottom right and bottom left corners of the crease pattern touch in Huffman's paper model.

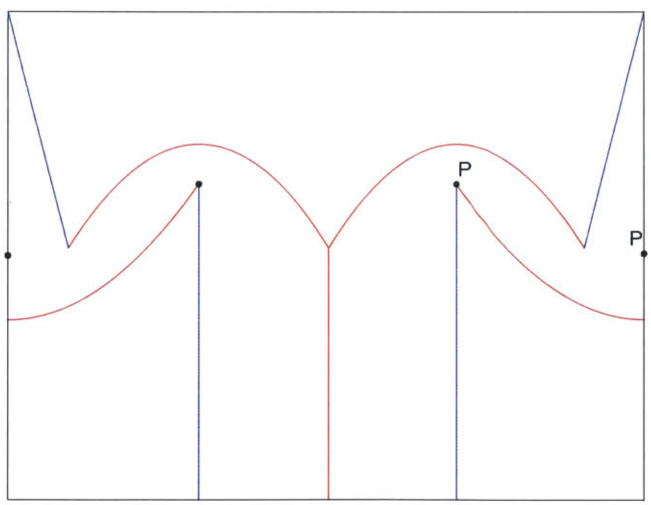


Fig 4.4.61 Crease pattern [DK])

Gadgets with parabolic splines

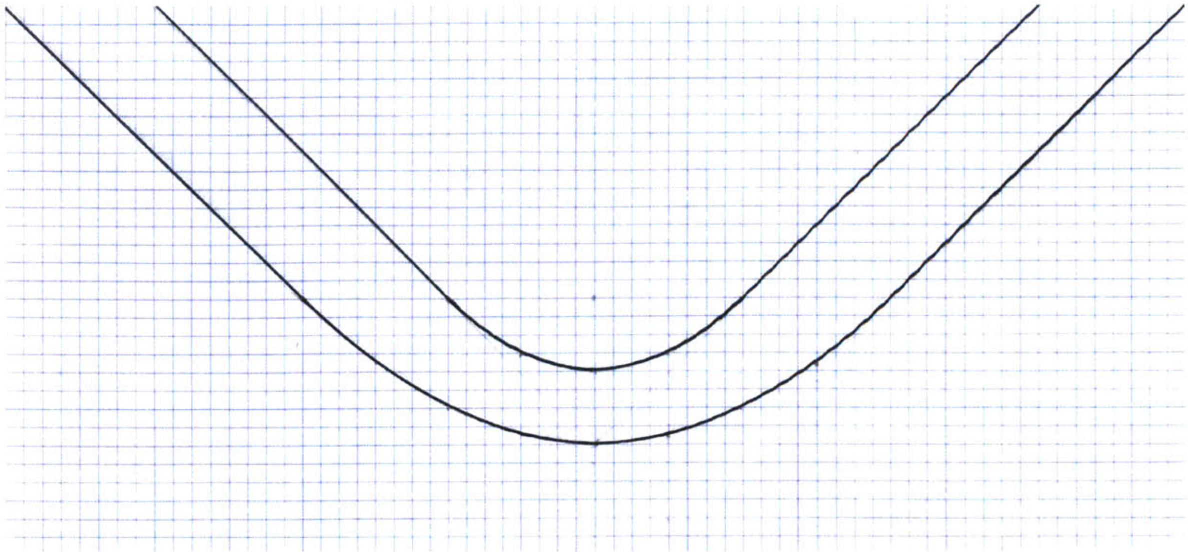


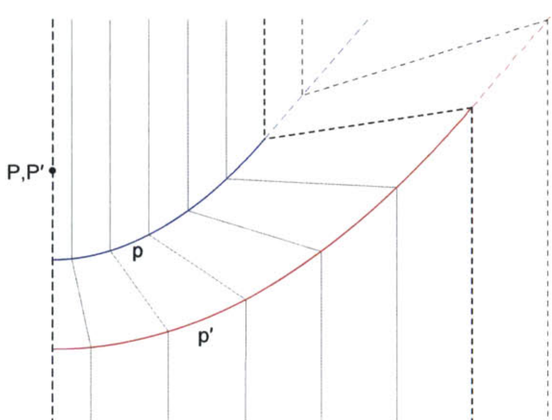
Fig 4.4.62 Paper model (undated, DAH [DK])

The term parabolic spline marks the beginning of this subsection that investigates quadratic splines with parabolas. It describes a piecewise smooth curve that consists of parabolas and/or line segments.

In the above design (Fig 4.4.62) Huffman draws linear extensions to the parabolas that also have be tangents at 45° (Fig 4.4.64). He uses parabolas as extensions in other cases (Fig 4.4.63 right). Another main characteristic of the designs in this section consists of general cylinders at the top an bottom.

The gadget

The gadget is based on the first parabola gadget in the taxonomy and introduces a second parabola *p'*. The parallel rulings above *p* form the previously mentioned cylinders and refract away from *P* once they pass through *p* (Fig 4.4.63). The second curve redirects the rulings such that



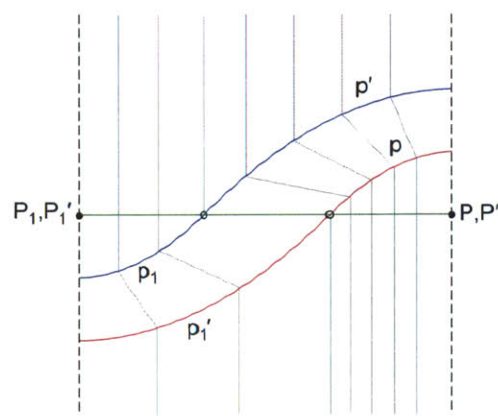


Fig 4.4.63 Gadgets [DK]

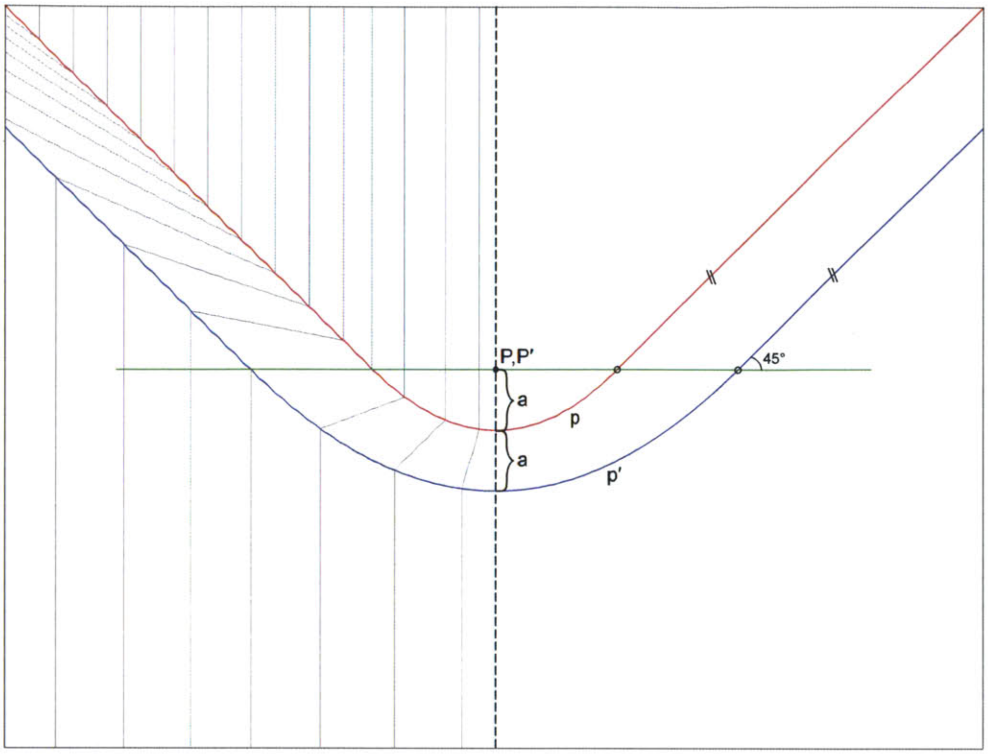


Fig 4.4.64 Crease pattern [DK]

they become parallel to the upper rulings. This results in the lower cylinders of the designs in this section. Connecting tiles have to follow the constraints of smooth connections.

Crease pattern and ruling analysis

The parabola reaches the latus rectum. The confocal pair of pleated curves is equidistantly distributed relative to *P*. Several of the following examples focus on the relationship between *P*, *P'* and *T* and *T'*, which here align on one horizontal line.

The simulated model folds well and clearly shows the 2 general cylinders at the top and bottom (Fig 4.4.65).

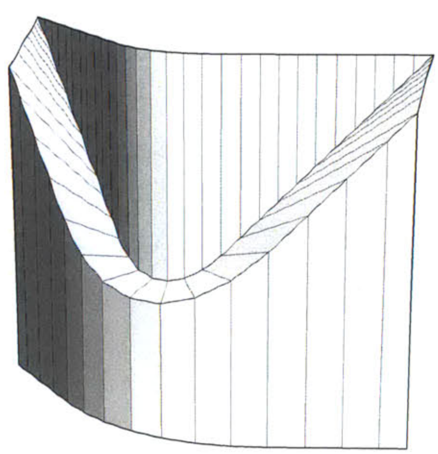


Fig 4.4.65 Simulated model [AH]

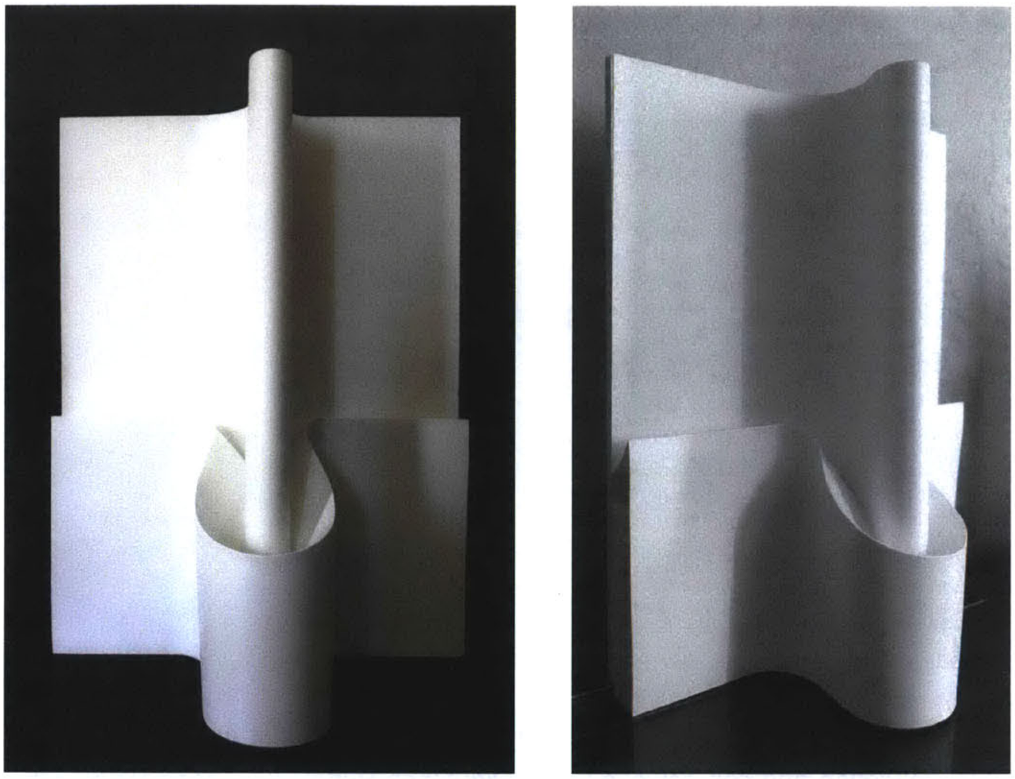


Fig 4.4.66 Vinyl model (undated, DAH [DK]), Identical model (undated, DAH [DK])

The above design can be thought of as a spline that starts and ends horizontally, Huffman refers to a similar model as 'columns' (Fig 4.4.66). Although no dates exist, it is likely from 1978.

Crease pattern and ruling analysis

The parabolas of the gadget connect smoothly to other rotated prototiles and create a parabolic spline (Fig 4.4.68). Huffman extends the splines to the edge of the paper with straight horizontal lines. The 6 foci are aligned on 1 horizontal axis and *a* and *b* are equal. The curved part of the

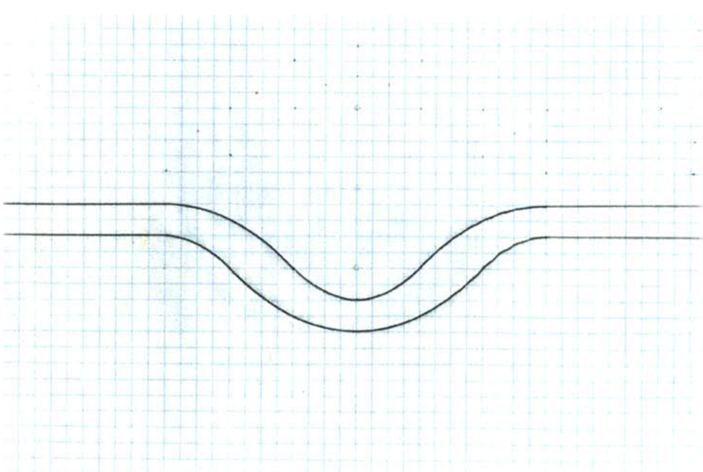


Fig 4.4.67 Paper model (undated, DAH [DK])

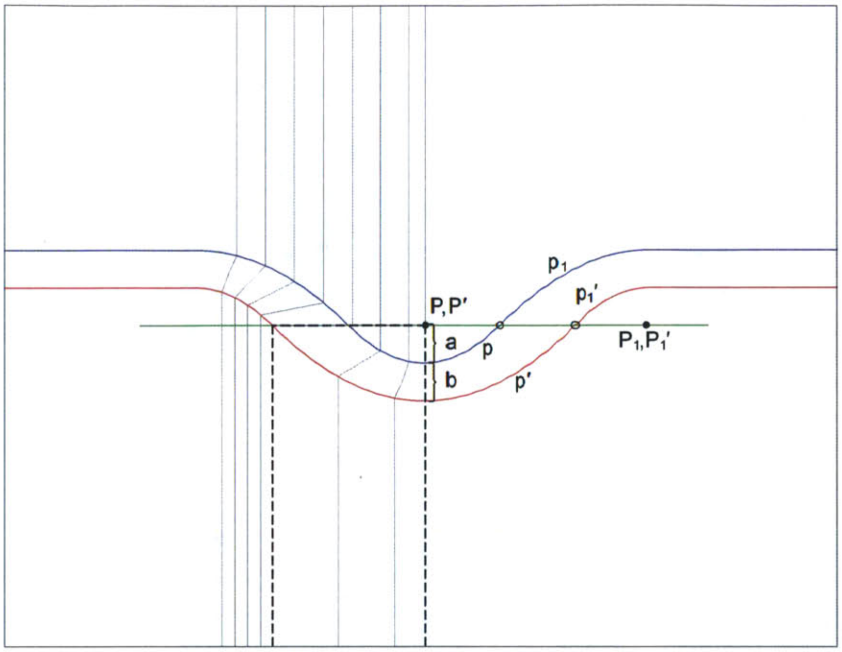


Fig 4.4.68 Crease pattern [DK]

design consists of a monohedral tiling with 4 prototiles that connects to flat quadrilaterals. The model folds well in simulation and can achieve the configuration Huffman chooses for his vinyl model (Fig 4.4.70 right).

Notes

This design appears to be a sketch model or trial version of a more elaborate design in the next subsection. Huffman's model is made of vinyl, but does not have a frame.

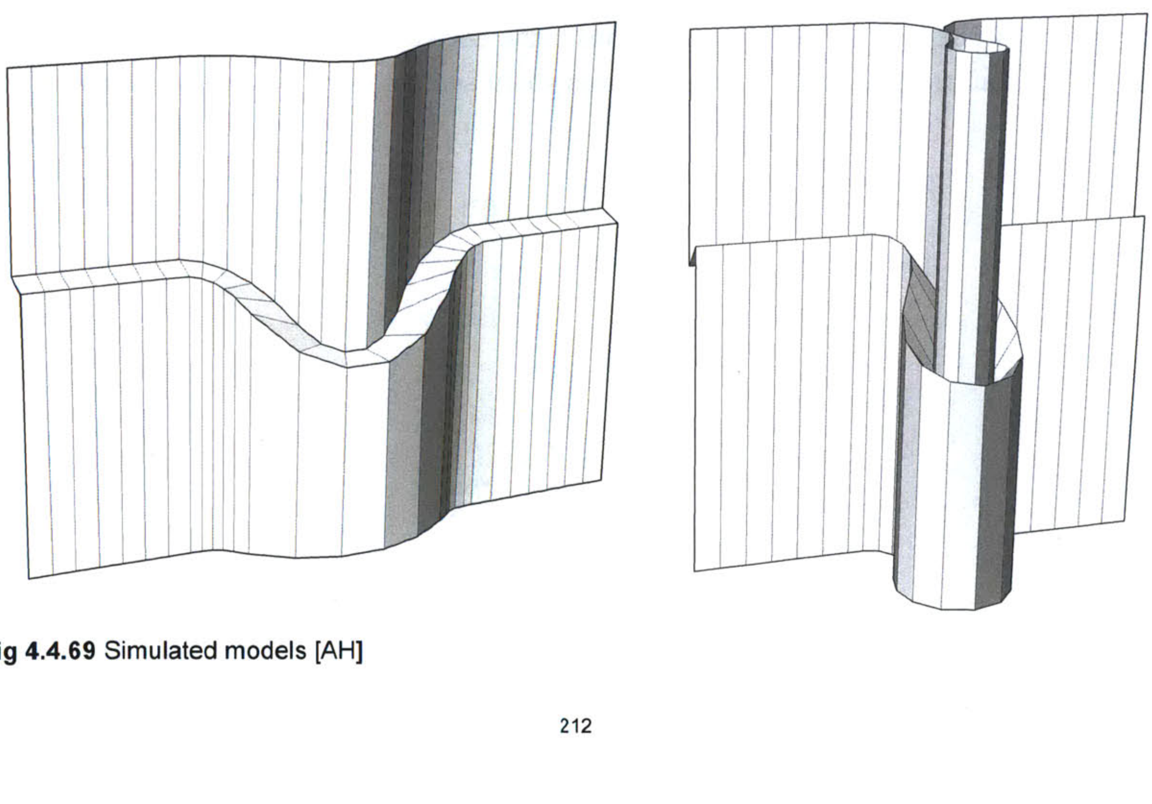


Fig 4.4.69 Simulated models [AH]

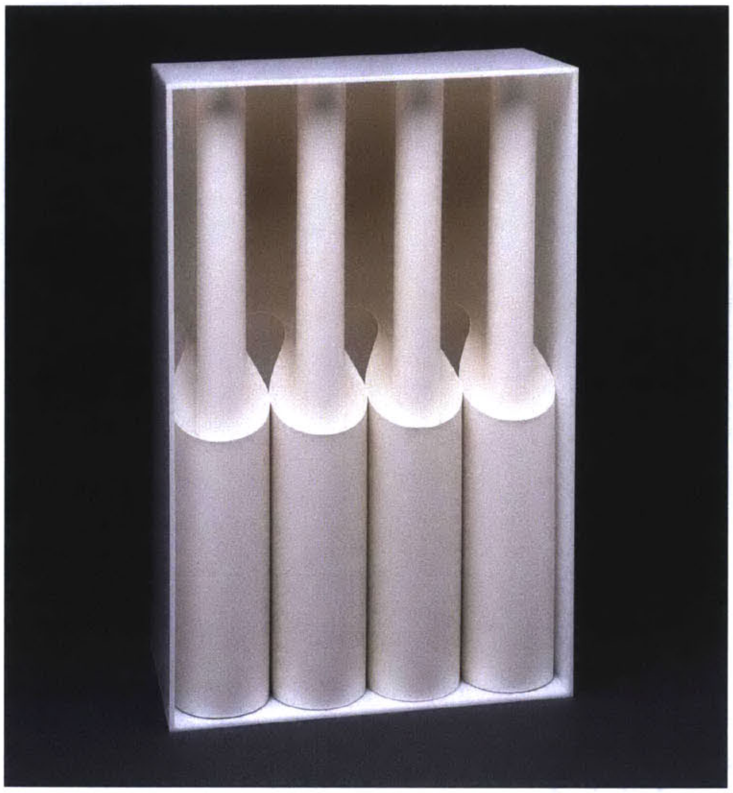


Fig 4.4.70 'Four columns' (1978, DAH [TG])

Huffman includes 'Four pipes' in his exhibition at UCSC in 1978 and completes the model with a white plexiglass frame (Fig 4.4.70). The paper model below appears to be a sketch for the design as it consists of a correctly scaled gadget, but does not undulate often enough to complete the tiling (Fig 4.4.71).

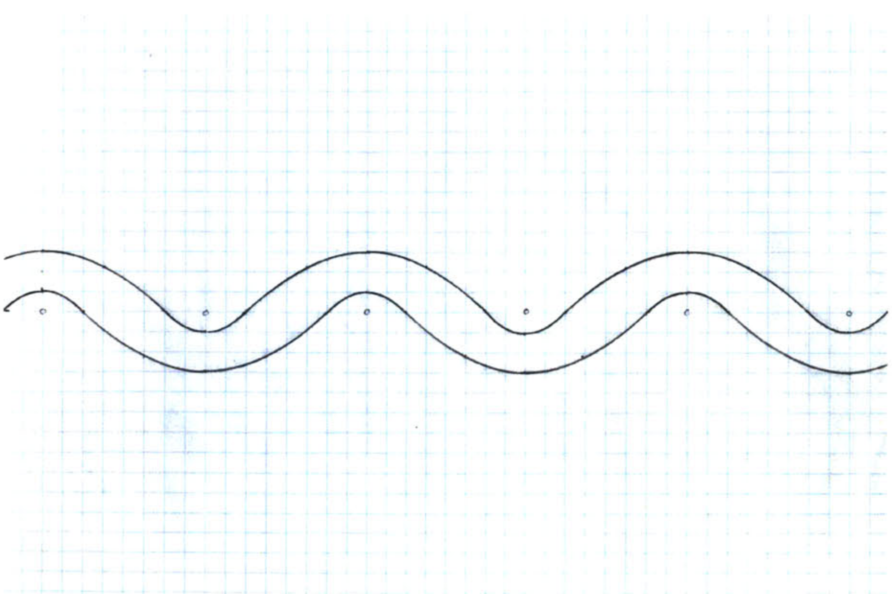


Fig 4.4.71 Paper model (undated, DAH [DK])

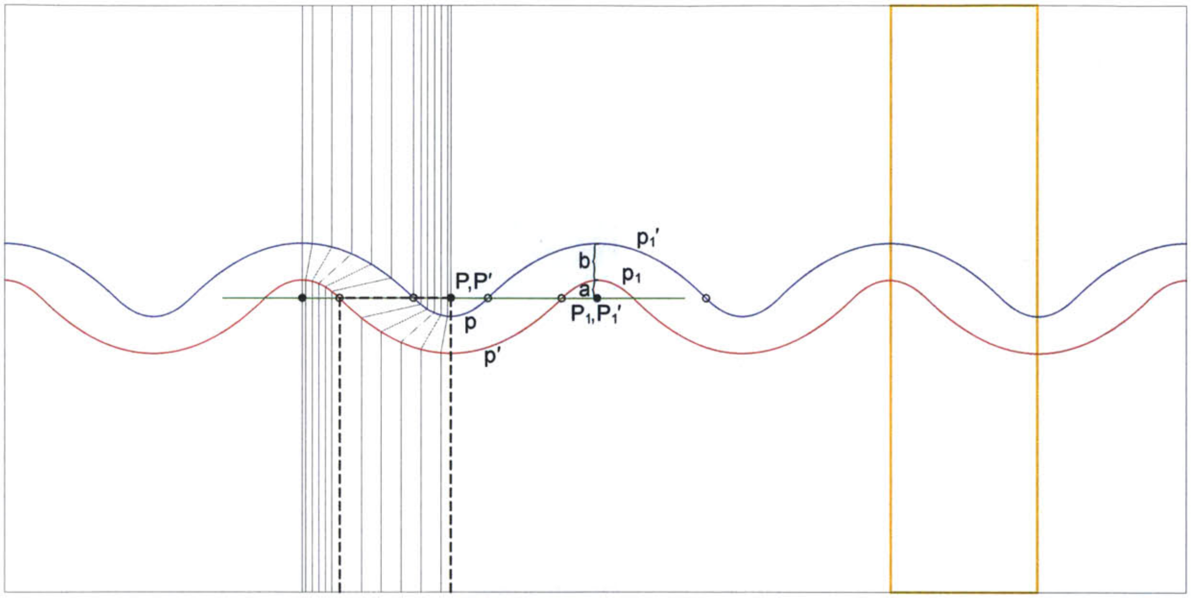


Fig 4.4.72 Crease pattern [DK]

Crease pattern and ruling analysis

All foci align on a horizontal axis and $b = 2a$ (Fig 4.4.72), which results in 2 rotationally symmetrical splines. The tiling consists of a total of 16 prototiles. It is debatable what the left and right edges of the design tile consist of, as the boundaries of a visual unit are ambiguous in this example.

The model folds well in the beginning (Fig 4.4.73 left). It can fold further to look like the configuration Huffman uses for his final model, but only with difficulty (Fig 4.4.73 right).

Notes

Huffman documents the making of the model and photographs it in an in-between state (Fig

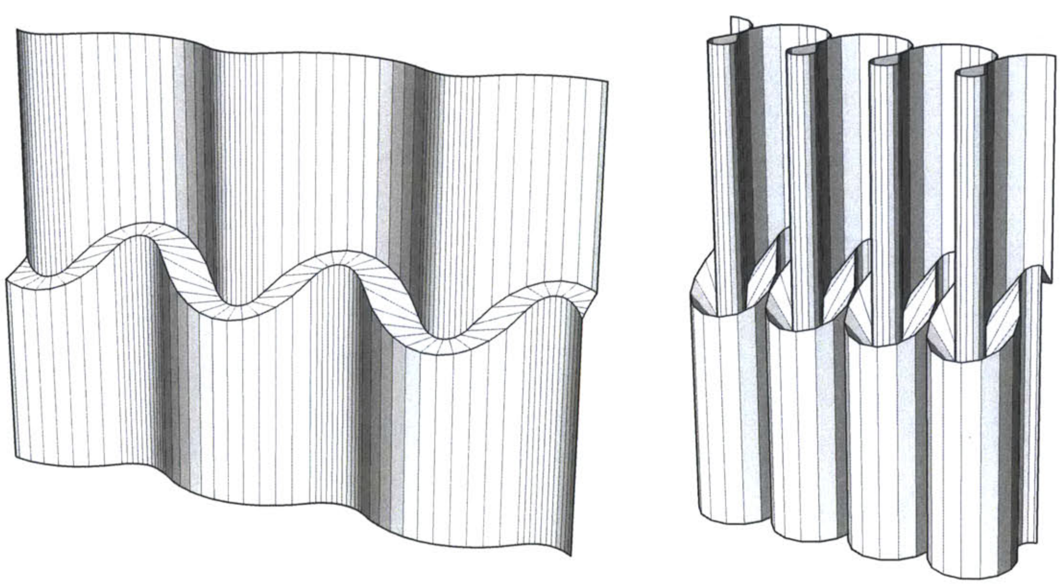


Fig 4.4.73 Simulated model [AH]

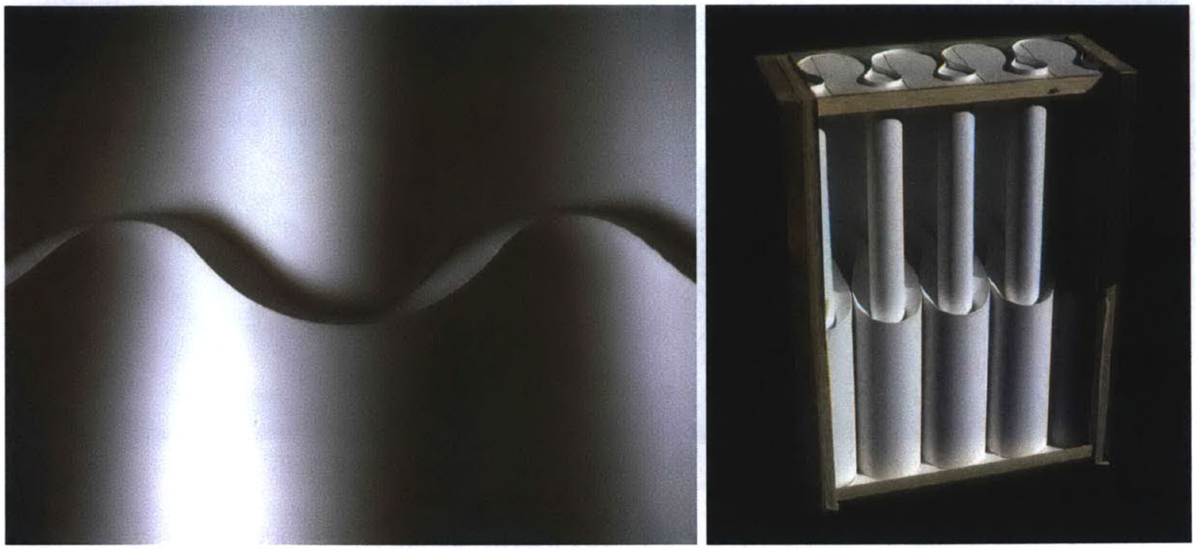


Fig 4.4.74 Vinyl model in progress (1978, DAH [DAH]), Identical model (1978, DAH [DAH])

4.4.74 left) similar to the simulation. He builds the model initially with cardboard templates at the top and bottom (Fig 4.4.74 right), which are held together by a wooden frame. After the vinyl has settled he constructs the plexiglass frame for the final result.

He makes several high-contrast photographs, but also takes a few images in which light is shining through the material (Fig 4.4.75 left). Both images are cropped here.

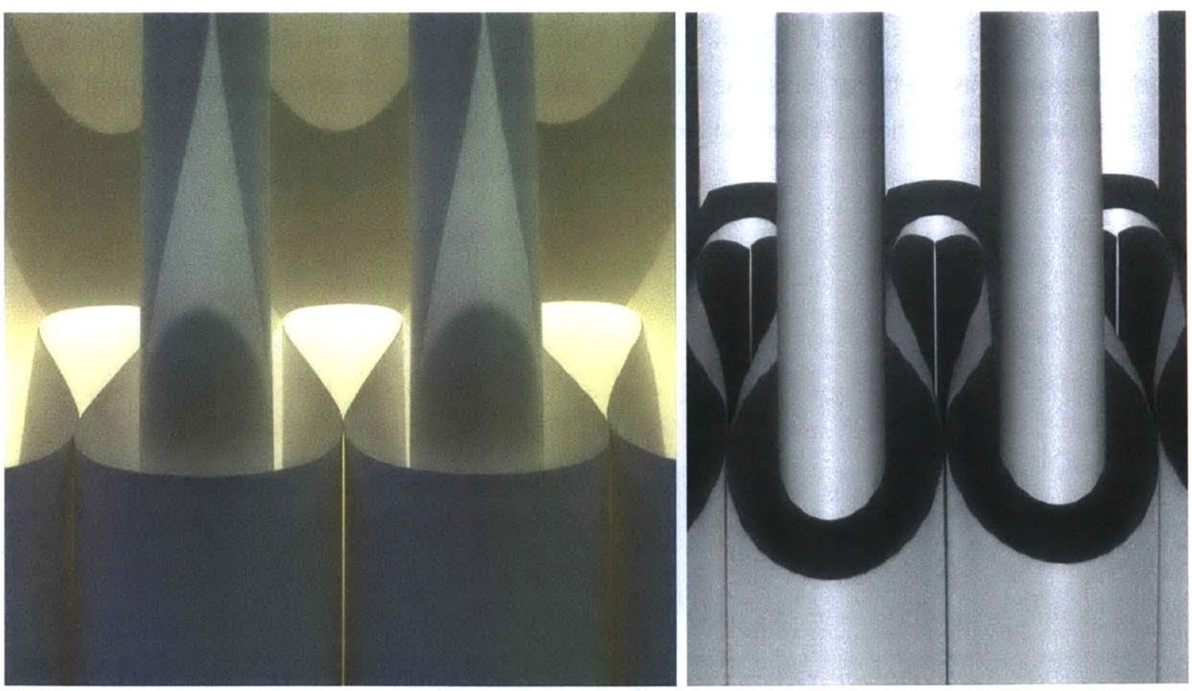


Fig 4.4.75 Vinyl model (1978, DAH [DAH]), Identical model (1978, DAH [DAH])

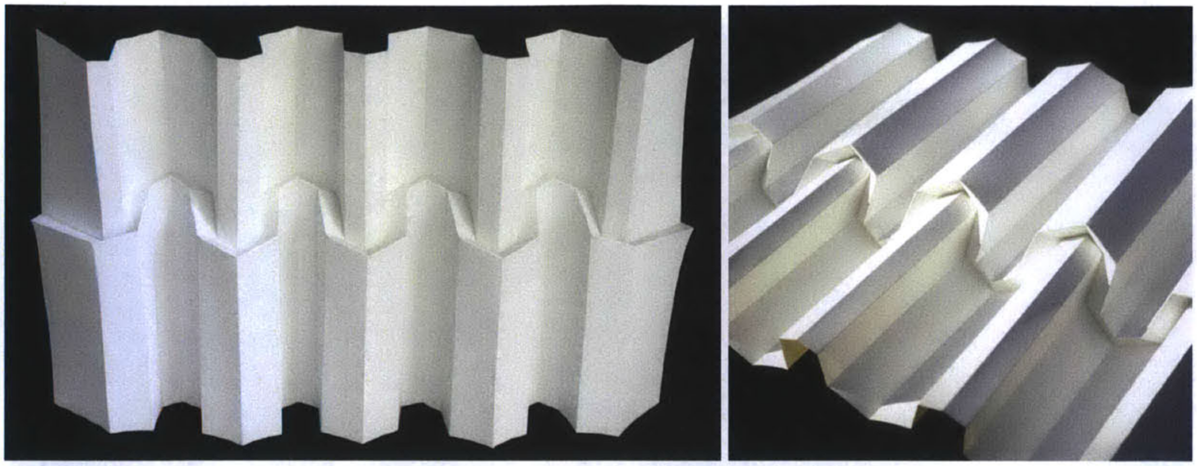


Fig 4.4.76 Vinyl model (undated, DAH [DK), Identical model (undated, DAH [EAH])

Huffman creates a discrete version of the previous column series and decides to make it in vinyl (Fig 4.4.76). The investigation consists of 2 very similar drawings and a model.

Crease pattern

The crease pattern is based on drawing the lower mountain crease first by using graph paper points. One vertex, the high point of the undulating poly-line, uses 1/2 a graph paper unit in the *y* direction (Fig 4.4.77). The valley crease is constructed by a parallel offset. As a result no vertices align with the graph paper grid in the *y* direction. The parallel vertical creases create approximations of the upper and lower general cylinders.

Huffman draws another version that consists of a very similar crease pattern except for a shorter rectangle in the tiling. I assume Huffman decides to create more depth for the undulation in the version depicted here. The simulation folds easily (Fig 4.4.77 right).

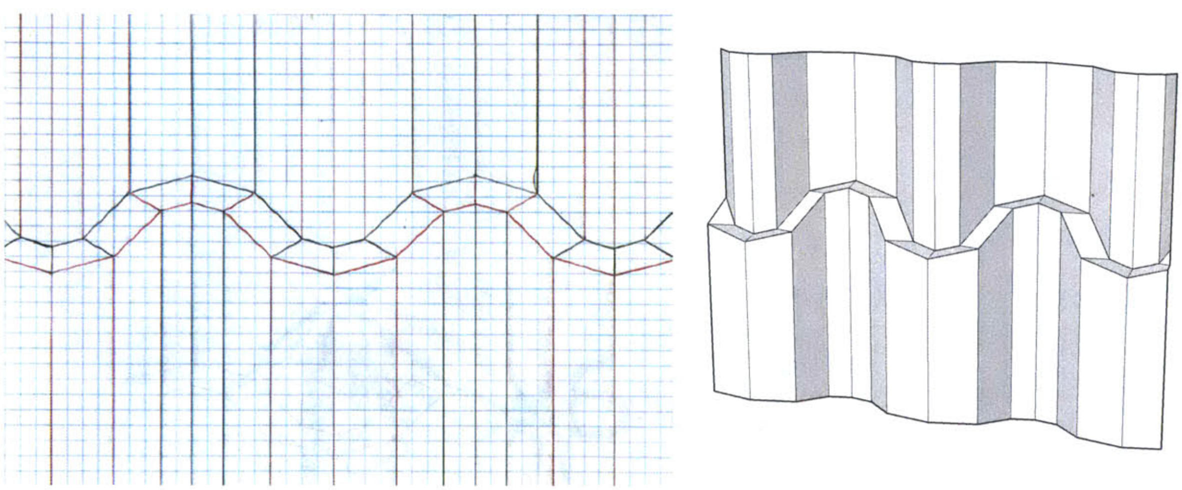


Fig 4.4.77 Paper model (undated, DAH [DK]), Simulated model [AH]

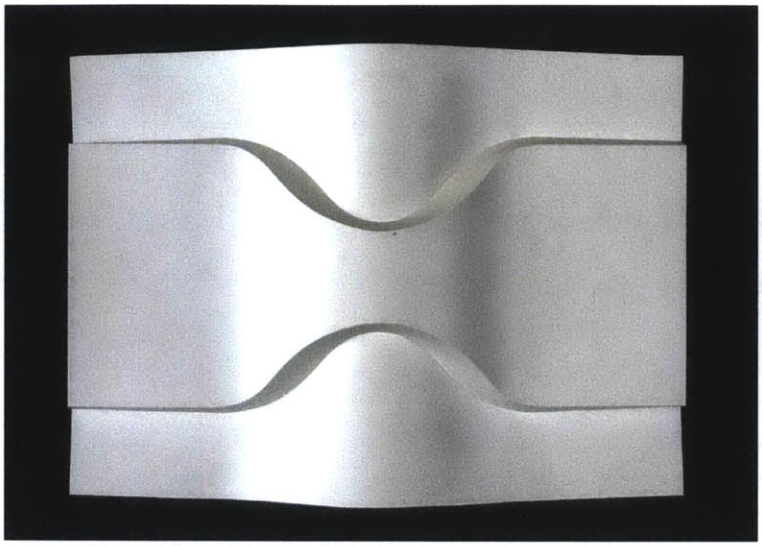


Fig 4.4.78 Vinyl model (undated, DAH [TG])

Huffman continues to uses a pair of quadratic splines with parallel line segments as extensions in the above design (Fig 4.4.78). However, in this case he mirrors the set of curves across a horizontal axis such that it creates large areas between the line segments. This subsection focuses on such mirrored tilings that use more gadgets than previous models.

Crease pattern and ruling analysis

The gadget again consists of a pleated pair of parabolas that reach the latus rectum. The tiling uses 4 gadgets made of 8 prototiles combined with 4 pairs or linear extensions. The general effect of redirecting parallel rulings back to parallel rulings is used throughout.

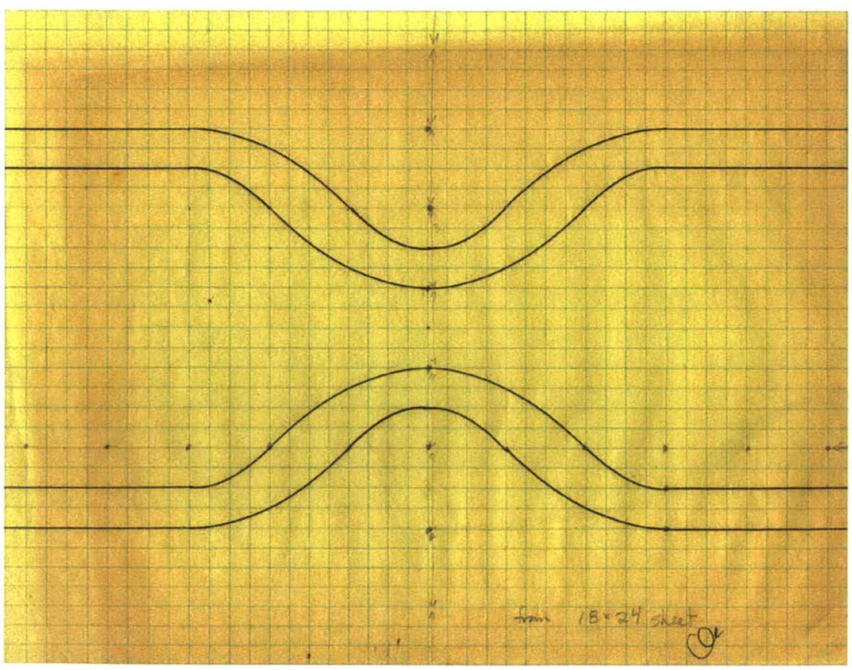


Fig 4.4.79 Paper model (undated, DAH [DK])

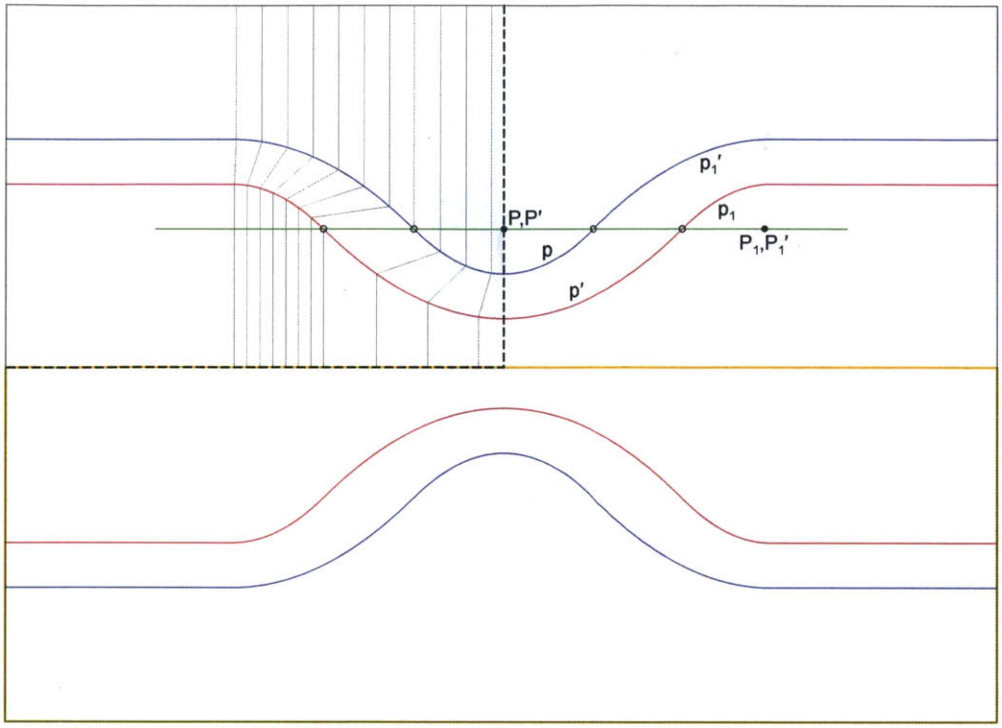


Fig 4.4.80 Crease pattern [DK]

Notes

The drawing is a scaled template and Huffman indicates dimensions for the 18″ by 24″ vinyl model (Fig 4.4.79). He decides to fold the design with inverted mountain and valley folds when compared to other designs in this subsection.

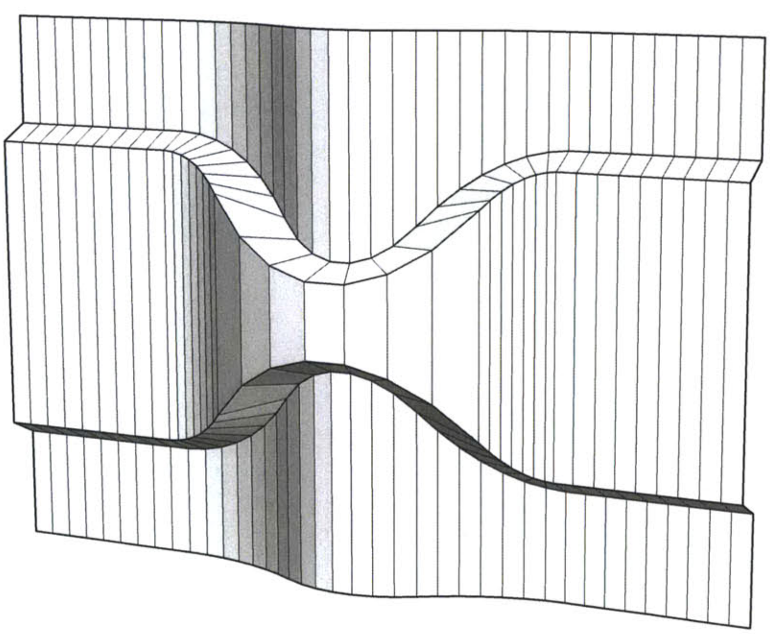


Fig 4.4.81 Simulated model [AH]

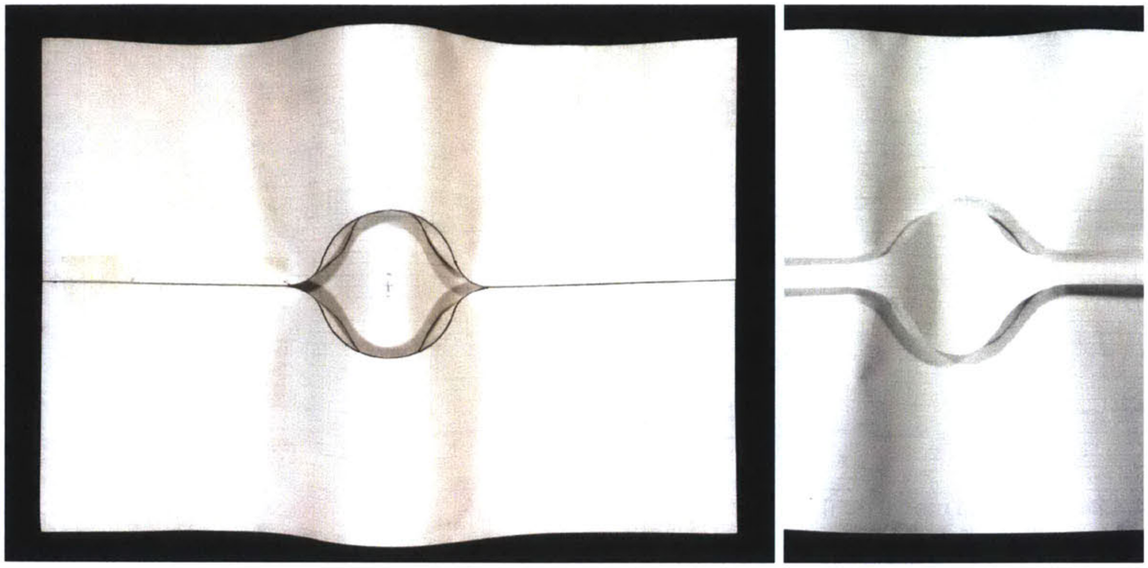


Fig 4.4.82 Paper model (undated, DAH [DK]), Identical model (undated, DAH [DK)

The above paper model uses similar parabolic splines as the previous model (Fig 4.4.82). However, Huffman mirrors the set across a different horizontal axis which results in a different center area. He designs a small series with this tiling.

Crease pattern and ruling analysis

We can describe the proportions of the gadget by counting graph paper units of the upper pair of splines (Fig 4.4.83). The distance from *P* to the y-intercept is *a* and the next distance to *p'* is *b*. The gap that consists of twice the distance of the lower spline to the mirror axis is *d*. Huffman plays with the parametric definitions of this design, but keeps only 1 of the versions in its folded

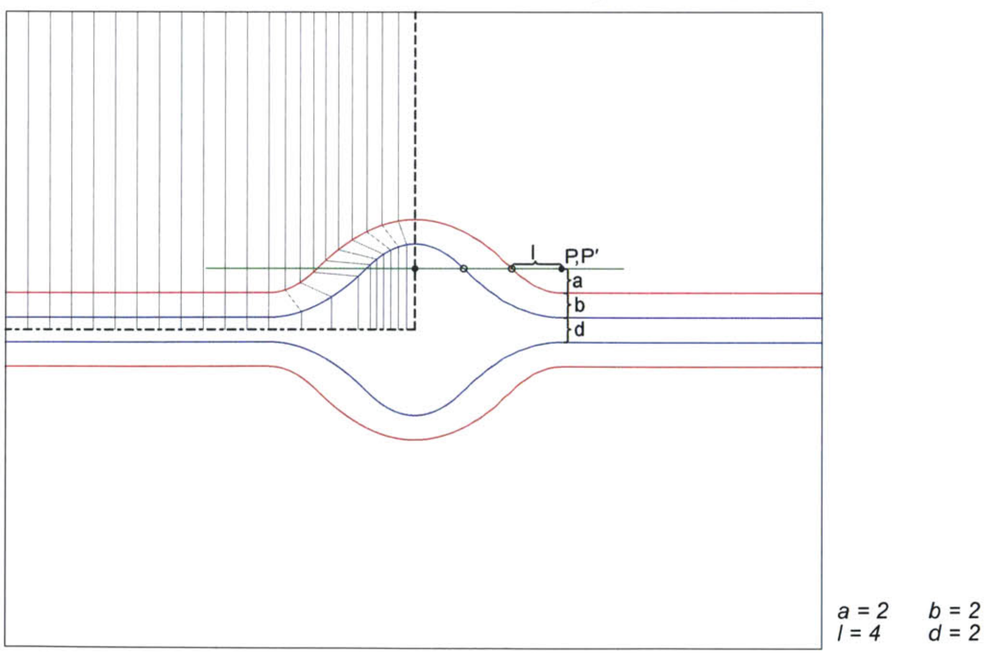


Fig 4.4.83 Crease pattern [DK]

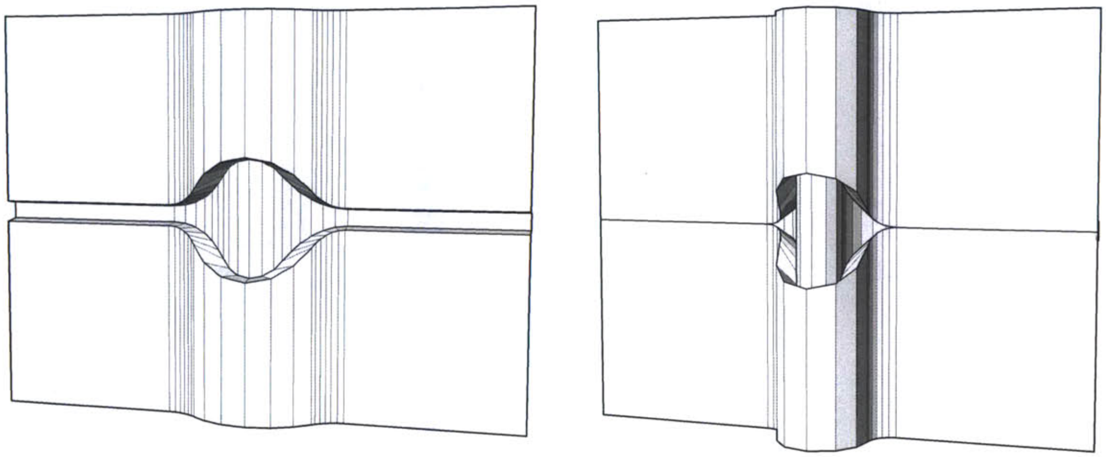


Fig 4.4.84 Simulated model [AH]

configuration (Fig 4.4.82).

The model folds well during simulation. Huffman's exact configuration that relies on adhesive tape on the front side of the paper model appears to be between the 2 simulated states (Fig 4.4.84). If the 2 line segments that Huffman tapes together are supposed to touch, the cylindrical parts folded much further than in his paper model.

The series he makes consist of 4 examples and we can extract the above mentioned parametric values by counting graph paper units (Fig 4.4.85). The values in square brackets are normalized such that *a = 2* and *l = 4* for ease of comparison. The example on the left with *d = 0* appears to also belong to a series of drawings in the next subsection.

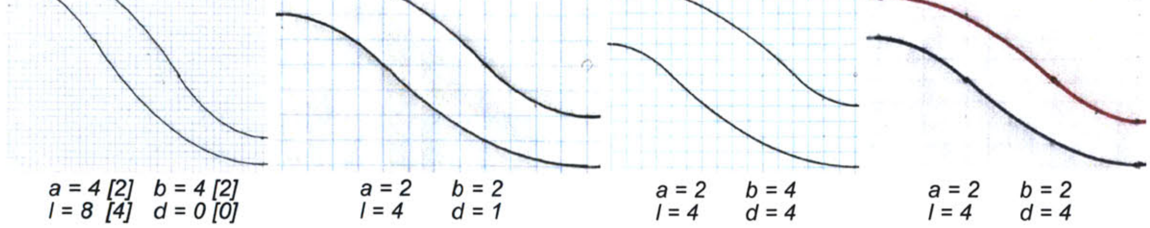


Fig 4.4.85 Paper models (undated, DAH [DK])

Gadgets with parabolic splines with inclined axis

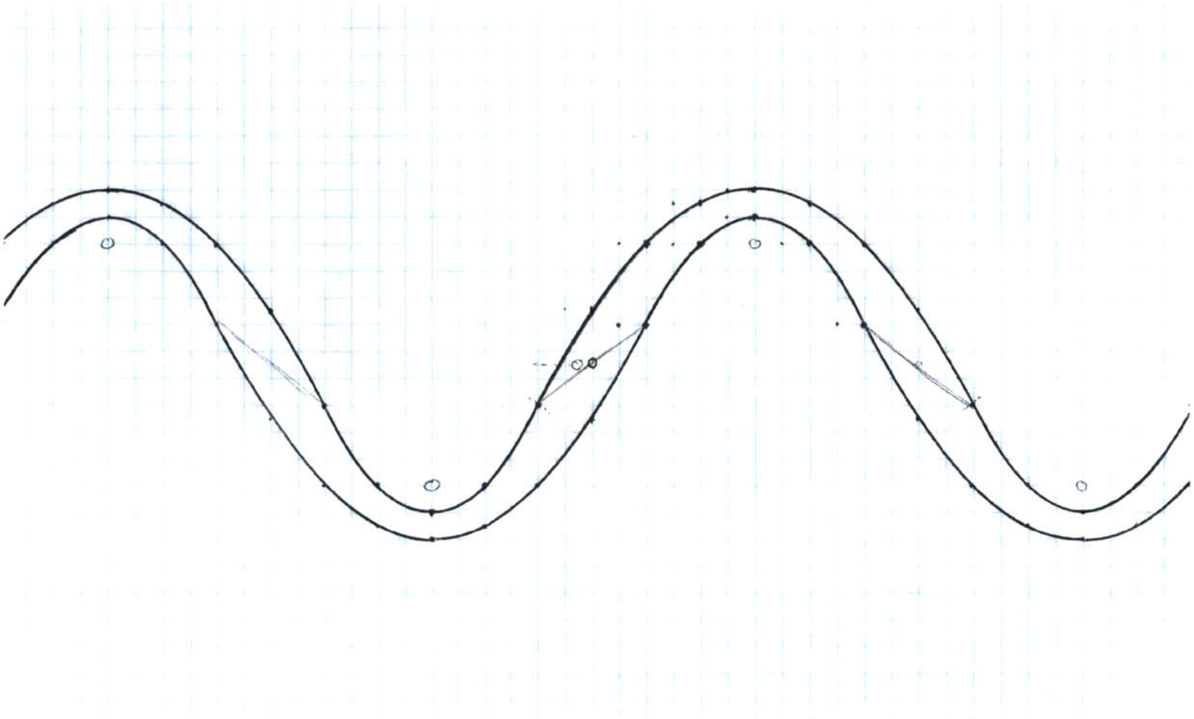


Fig 4.4.86 Paper model (undated, DAH [DK])

Huffman provides a more general case with parabolic splines that connect with angled tile edges in the following series. The above design provides a good case study to introduce the gadget (Fig 4.4.86).

The gadget

The previously horizontal axis between *P*, *P'* and *P1*, *P1'* now has a slope, but still locates the smooth connection between the partial parabolas.

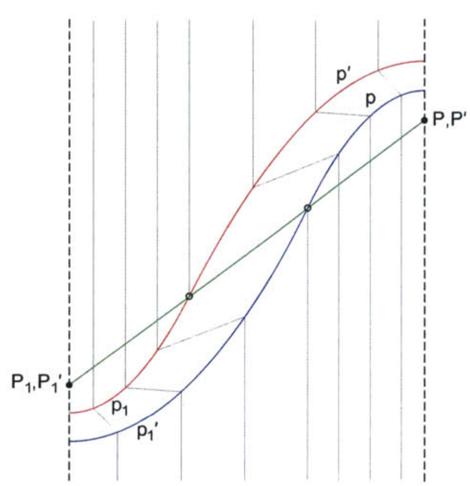


Fig 4.4.87 Gadget [DK]

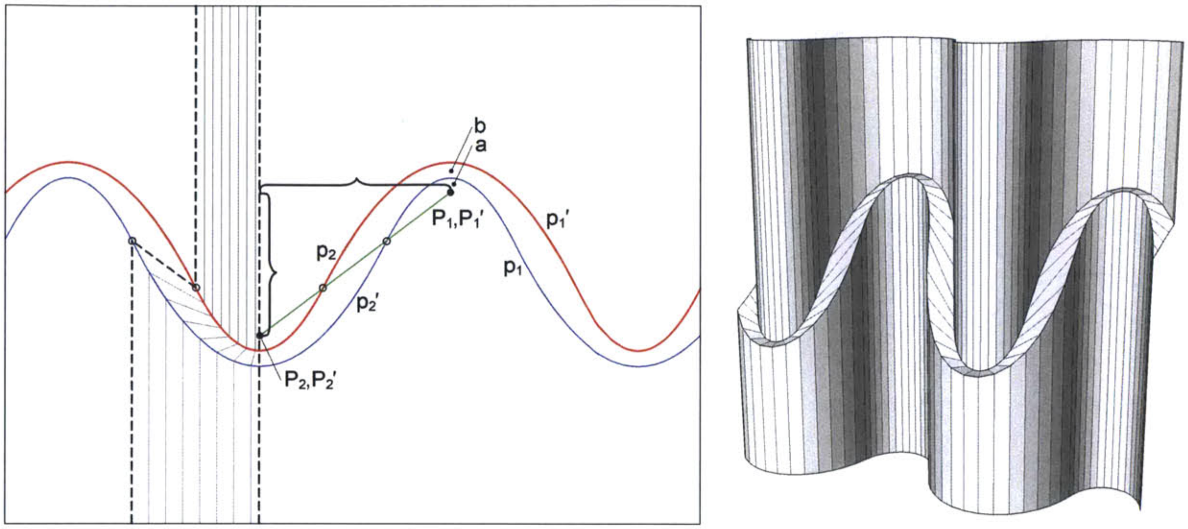


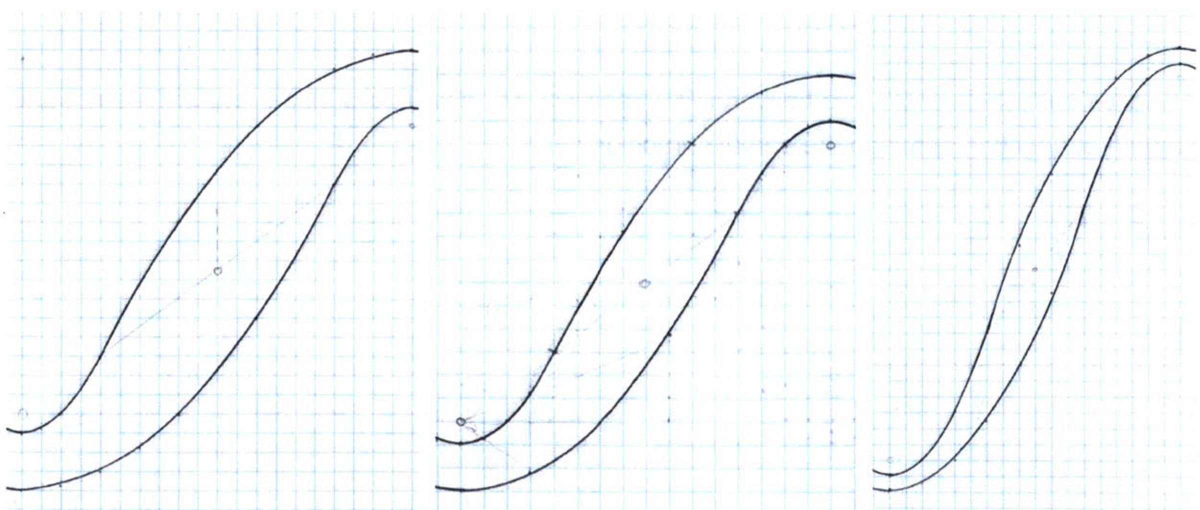
Fig 4.4.88 Crease pattern [DK], Simulated model [AH]

Crease pattern and ruling analysis

The crease pattern consists of 3 gadgets and 2 prototiles on either side. A total 8 prototiles create the tiling, but the 2 at the edges of the paper are cropped.

The model folds well during simulation (Fig 4.4.88 right).

Huffman creates several versions with the gadget by scaling the curves and changing the distance between them. He thereby also changes the slope of the gadget axis indicated with brackets in the crease pattern (Fig 4.4.88 left). The values for *a*, *b* and the *slope* help to identify the changing slope (Fig 4.4.89).



a = *1* *slope* = *20/15* *b* = *3*	*a* = *1* *slope* = *12/16* *b* = *2*	*a* = *1* *slope* = *18/24* *b* = *1*

Fig 4.4.89 Paper models (undated, DAH [DK])

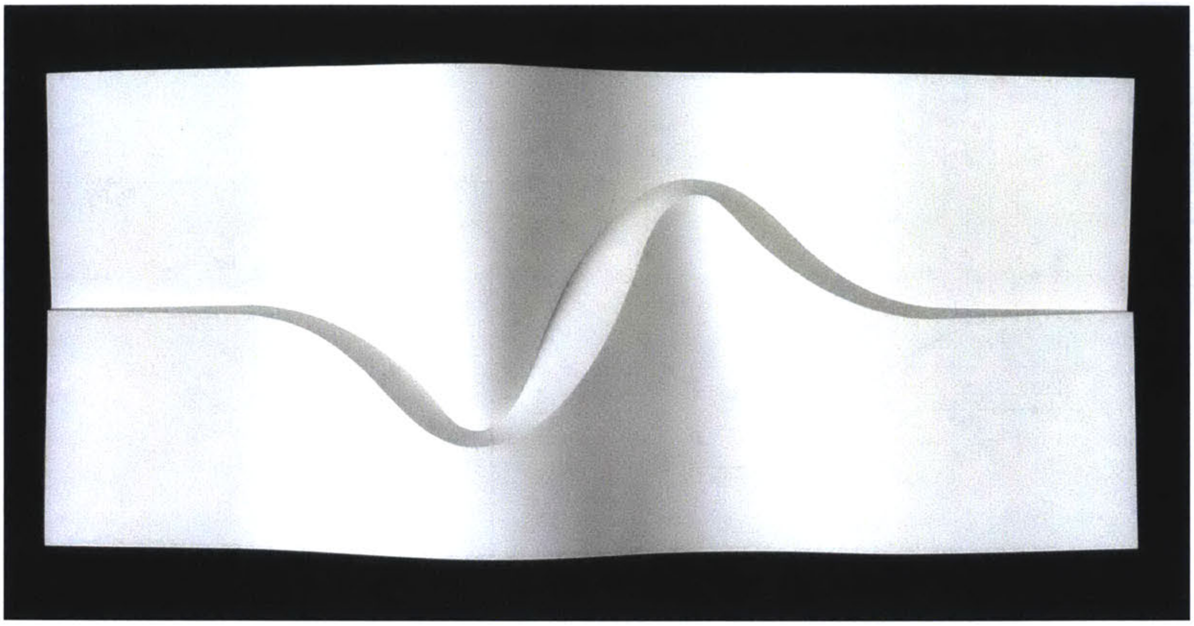


Fig 4.4.90 Vinyl model (1998, DAH [TG])

The more general parabolic spline gadget is placed in the center of the above design (Fig 4.4.90), which Huffman makes in vinyl in 1998. He produces a vast amount of sketches for this and the next model and appears to be investigating more general cases. The drawings are usually 8 1/2" by 11", but he splices two sheets together and cuts off a part of the second sheet in order to obtain the length he needs for the above design (Fig 4.4.91).

Crease pattern and ruling analysis

The crease pattern consists of only 2 prototiles (Fig 4.4.92). The gadget in the middle connects to scaled versions of the same gadget with a horizontal axis. The splines end with line segments that reach the edges of the paper on either side. The bottom crease is a mountain and the second

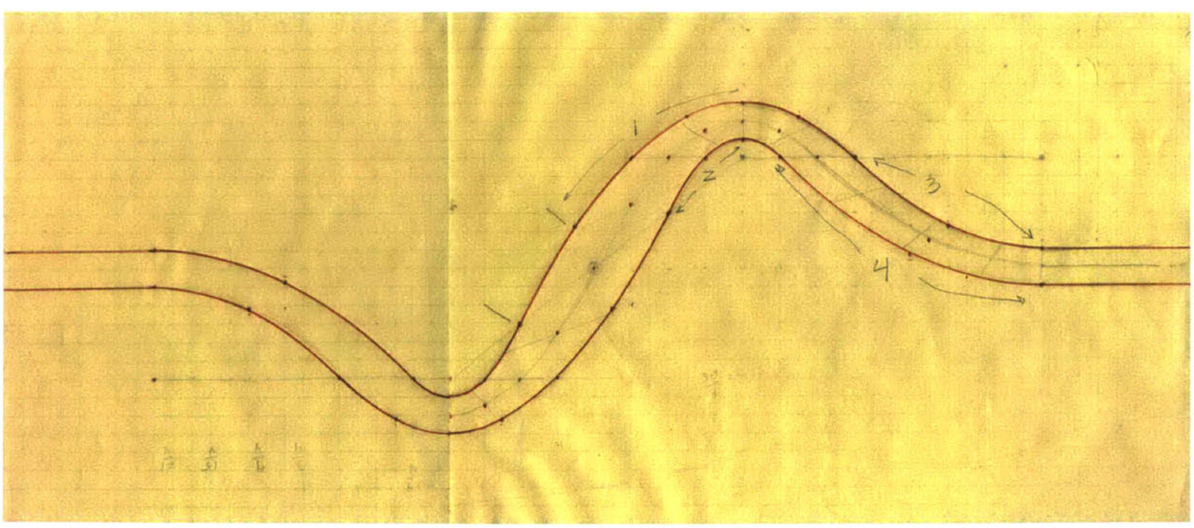


Fig 4.4.91 Drawing (undated, DAH [DK])

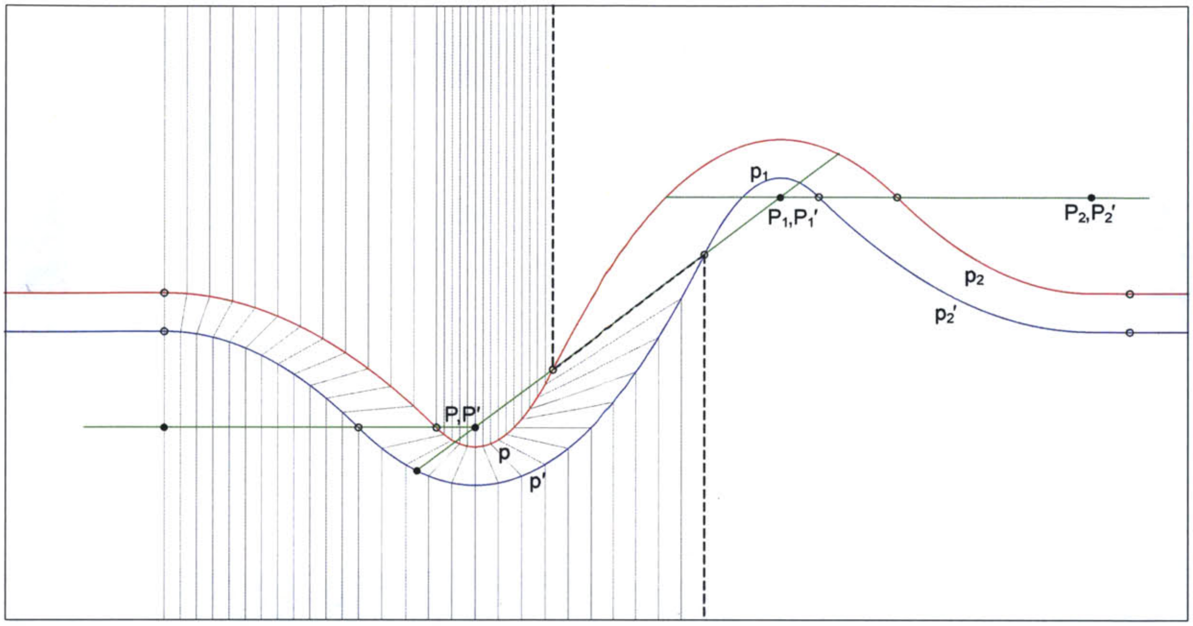


Fig 4.4.92 Crease pattern [DK]

spline is a valley crease.

The model folds without issues during simulation as most parabola models do (Fig 4.4.93).

Notes

The small sketch (Fig 4.4.93) indicates the proportions Huffman wants to use for the design. He draws the sketch with the undulation backwards relative to the vinyl model.

Huffman explores several combinations of the gadget and uses longer parabolas and line segments (Fig 4.4.94 and 4.4.96). All models fold as expected and are visually closely related even with different kinds of undulations. All designs consist of 2 prototiles with edges along the rulings.

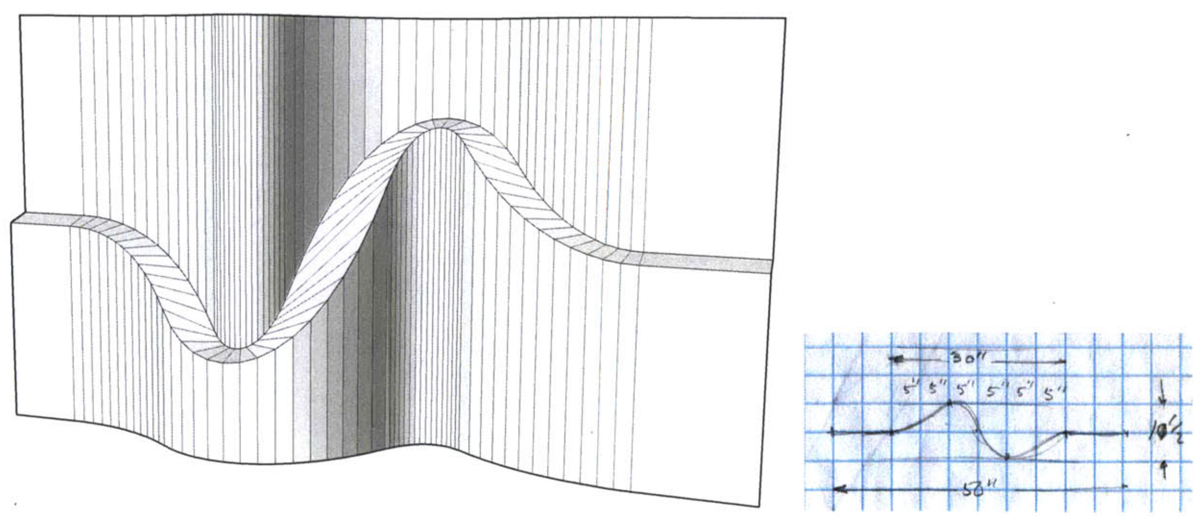
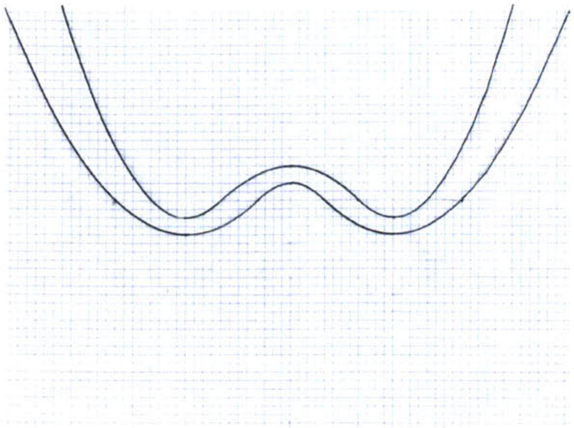
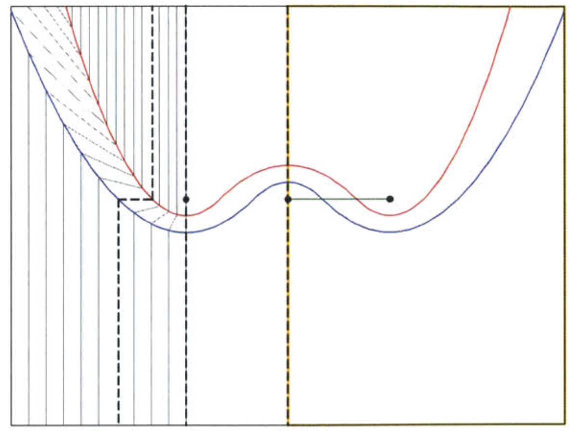
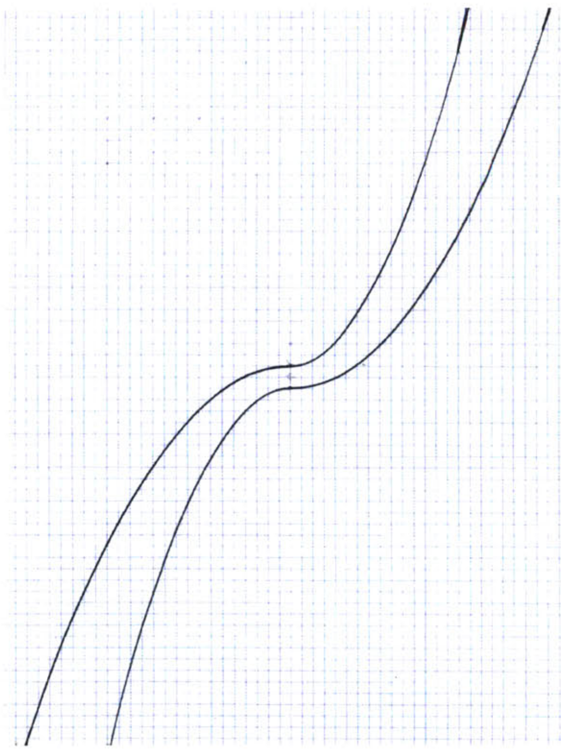
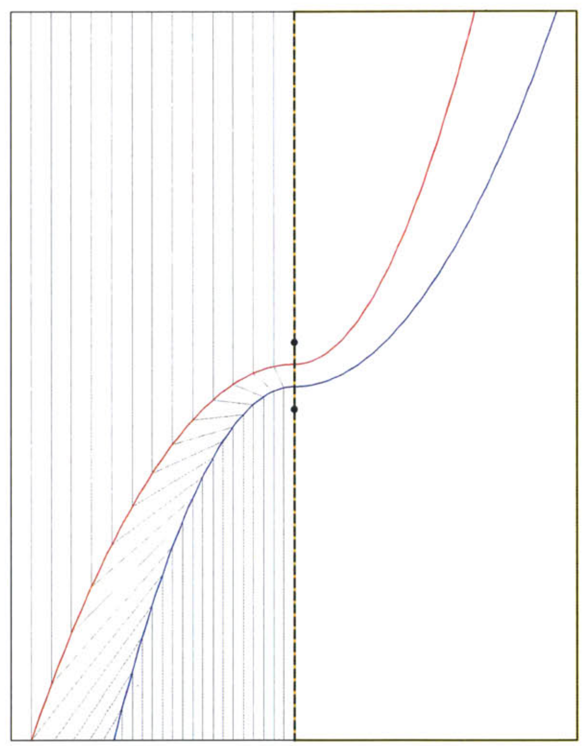


Fig 4.4.93 Simulated model [AH], Sketch (undated, DAH [DK])

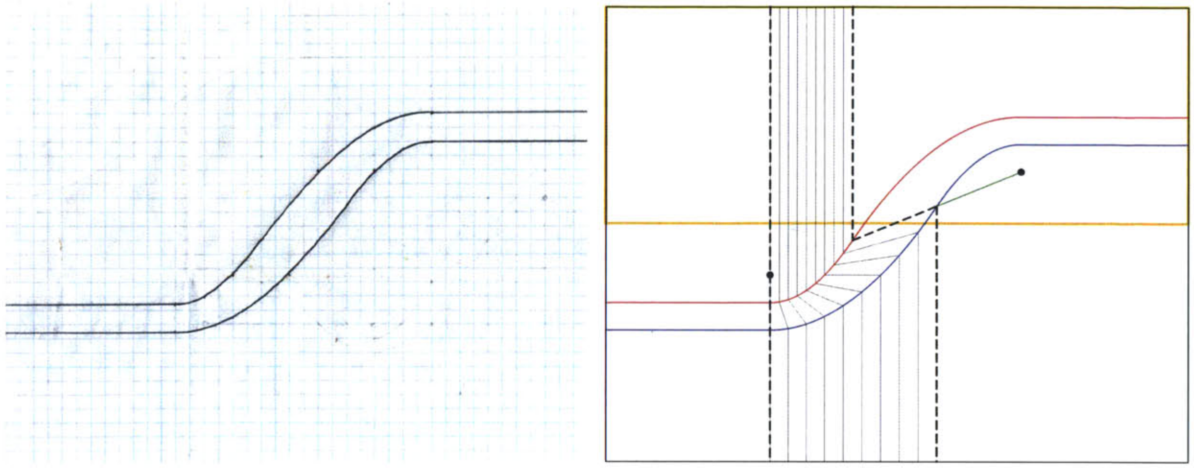


a = 2 *slope = 0*
b = 2

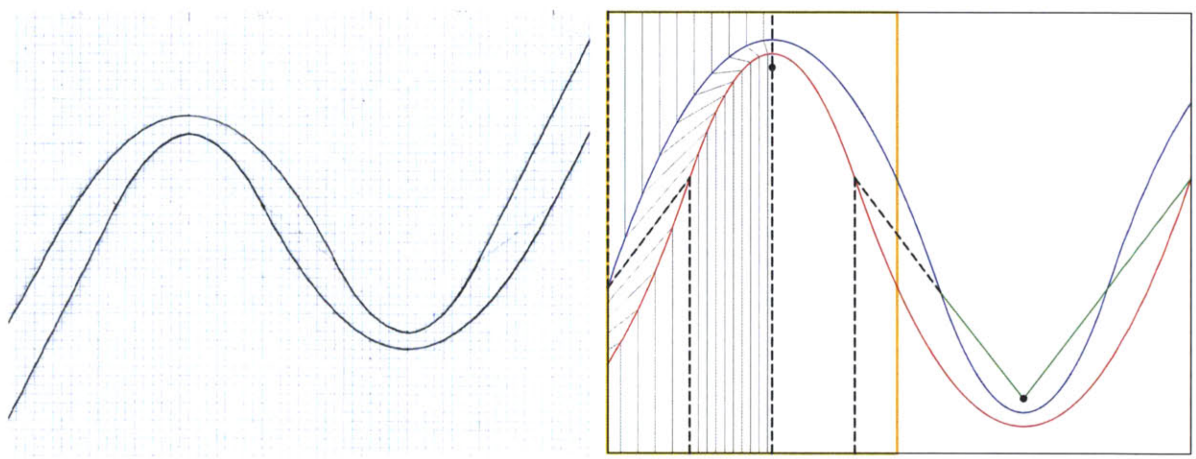


a = 2 *slope = 0*
b = 2

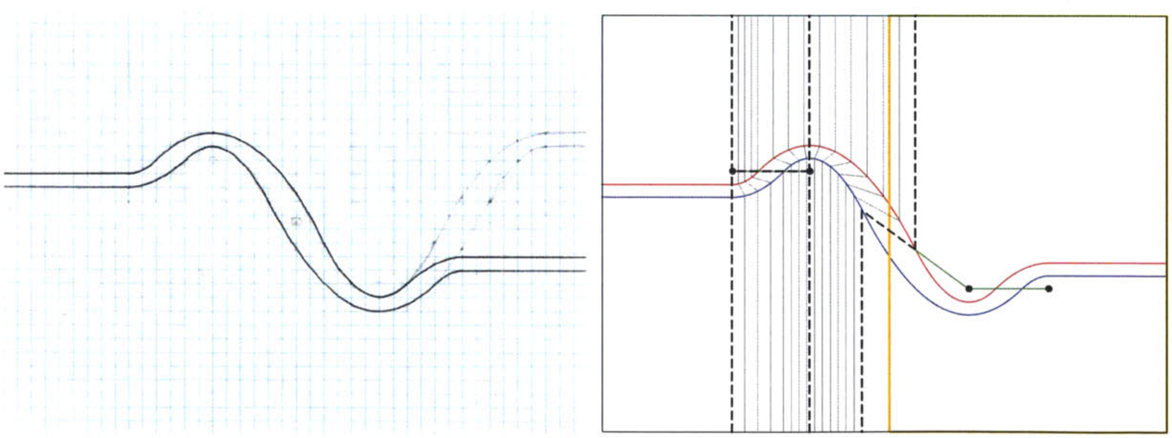
Fig 4.4.94 Paper models (undated, DAH [DK])



a = 2 *slope* = 7.5/18
b = 2



a = 2 *slope* = 6/8
b = 2



a = 2 *slope* = 2/4
b = 2

Fig 4.4.95 Paper models (undated, DAH [DK])

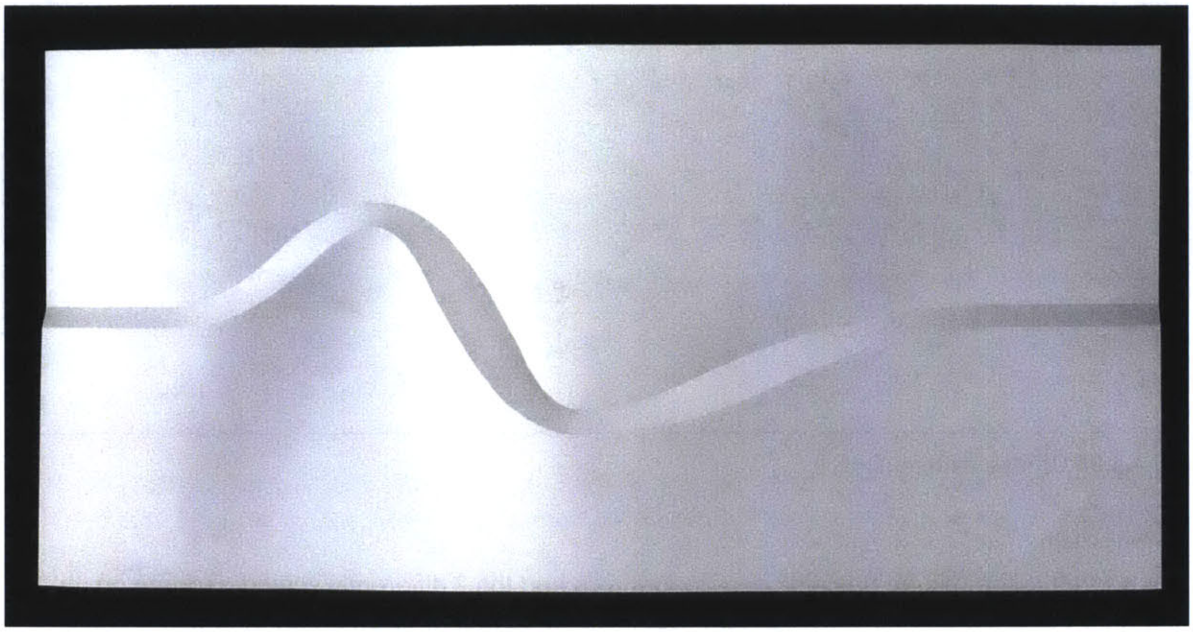


Fig 4.4.96 Vinyl model (1998, DAH [TG])

Another general case with the use of parabolic splines can be studied in the above example (Fig 4.4.96). This model is a very rare case in which Huffman does not design a symmetrical crease pattern.

No single sheet with a clearly identifiable crease pattern exists, but among the many sketches I was able to find a set of matching curves across 2 sheets (Fig 4.4.97). It is unlikely that Huffman used these exact sheets for the design, but many duplicates of the gadget exist and this design was most likely designed piece-by-piece, before he transfers the pattern onto vinyl.

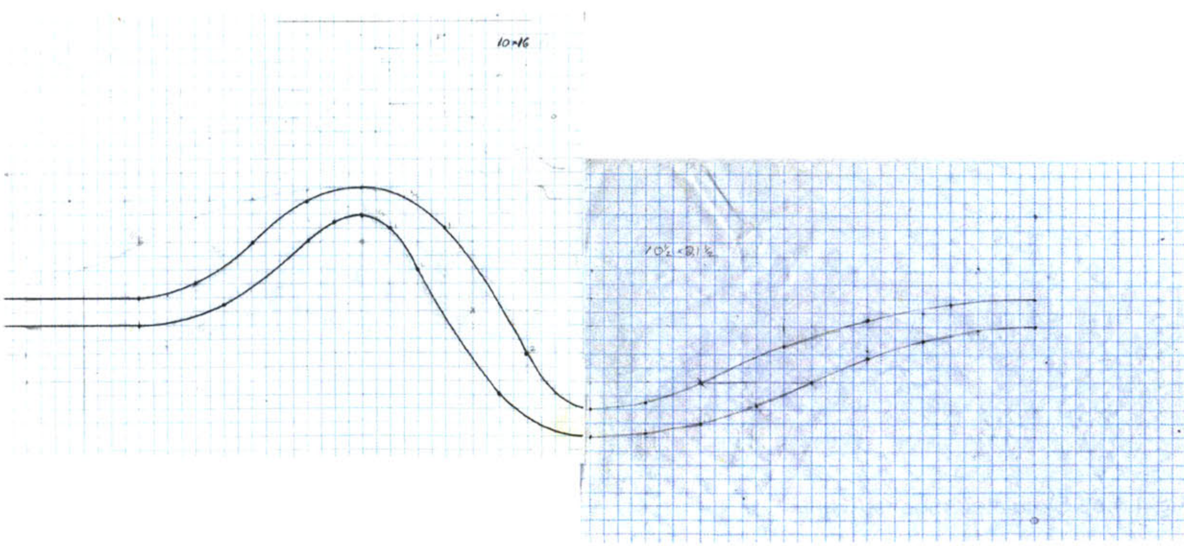


Fig 4.4.97 Drawings (undated, DAH [DK])

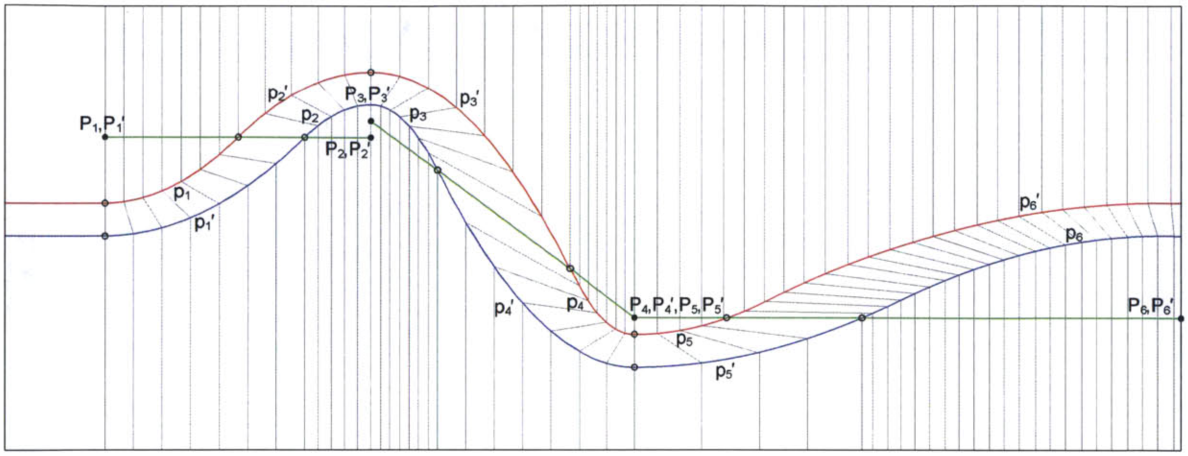


Fig 4.4.98 Crease pattern [DK]

Crease pattern

The gadget in the middle has a sloped gadget axis and the 2 differently scaled gadgets on the left and right have a horizontal axis (Fig 4.4.98). The left side is a familiar shape in this chapter, but the gadget on the right consists of a set of parabolas that appear to be scaled by a factor of *0.25*. It is unclear which curves exactly Huffman uses for that part of the design. He marks several points on the right sheet that could have been alternate foci. Line segments on either side complete the crease pattern.

Ruling analysis

The model does not fold well in the region of the sloped gadget and the scaled parabolas. We can see ripples in the simulation (Fig 4.4.99) that uses the above crease pattern. Other reconstructions with different foci simulate just as poorly.

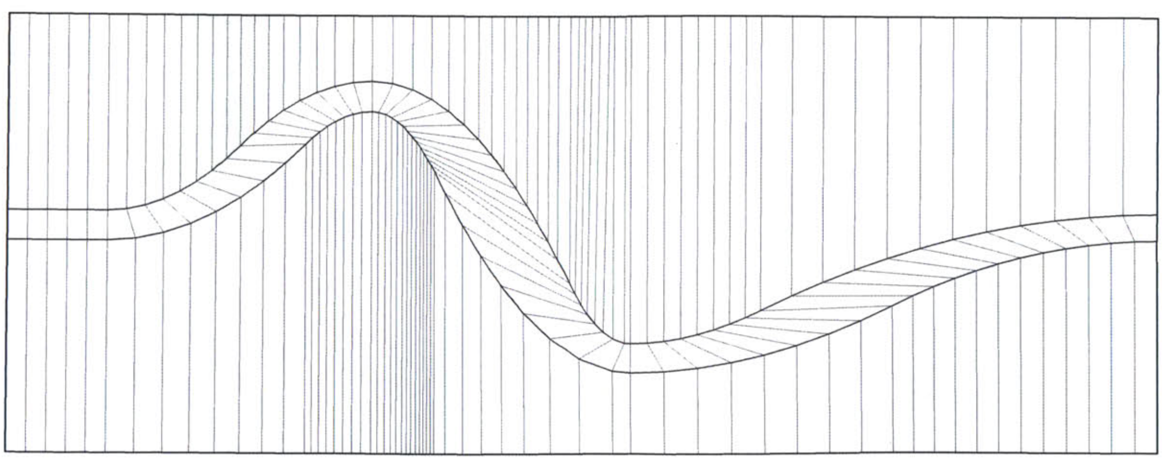


Fig 4.4.99 Simulated model [AH]

Gadgets with parabolic splines and circles

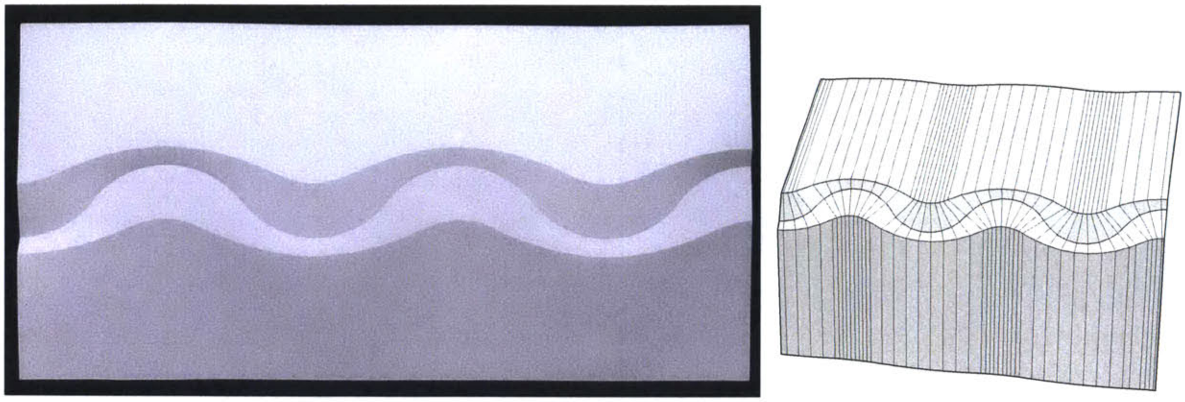


Fig 4.4.100 Vinyl model (undated, DAH [TG]), Simulated model [AH]

The gadget in this chapter consists of 2 parabolas and a new third non-dominant curve, namely a circle (Fig 4.4.102 left). Circles will occur more frequently in the following sections where Huffman combines several conics in a single design.

In the above example made in vinyl Huffman places the circle arc between the 2 parabolic splines (Fig 4.4.100).

Crease pattern and ruling analysis

Huffman stops the parabola at a 45° angle in the crease pattern in (Fig 4.4.102 right) and places the circle arc at 3 graph paper units away from p on the vertical toward p'. The tiling consists of 10 prototiles, mirrored vertically and horizontally, which results in a horizontal configuration.

The arc has no consequence for the direction of the rulings as they simply pass through, but the crease pattern needs to adjust in terms of its mountain and valley assignments. The added

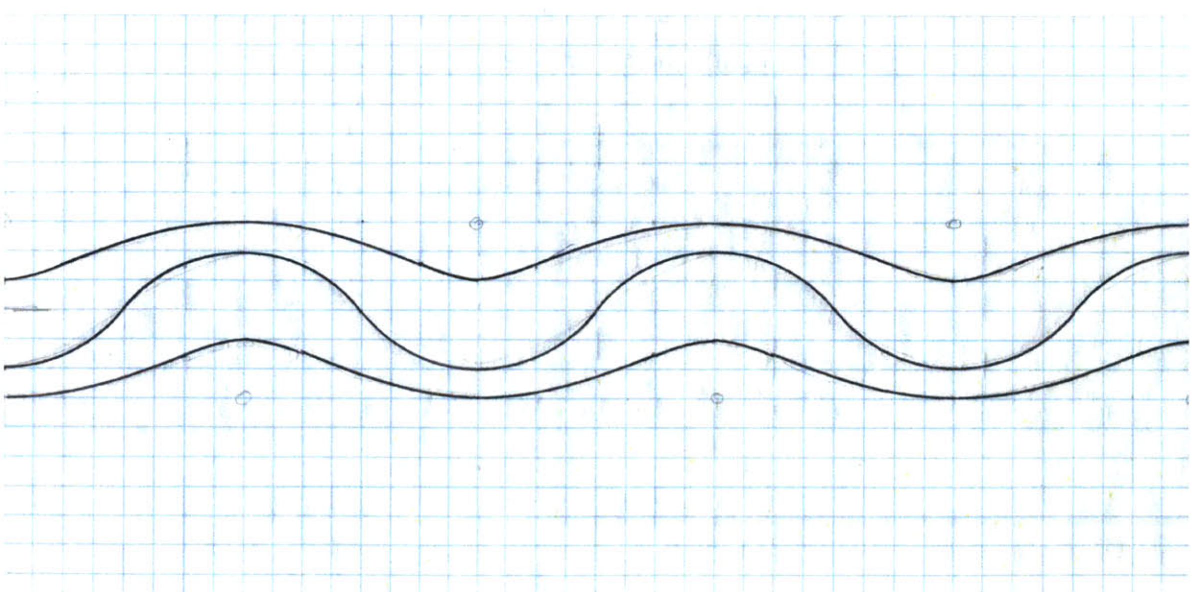


Fig 4.4.101 Paper model (undated, DAH [DK])

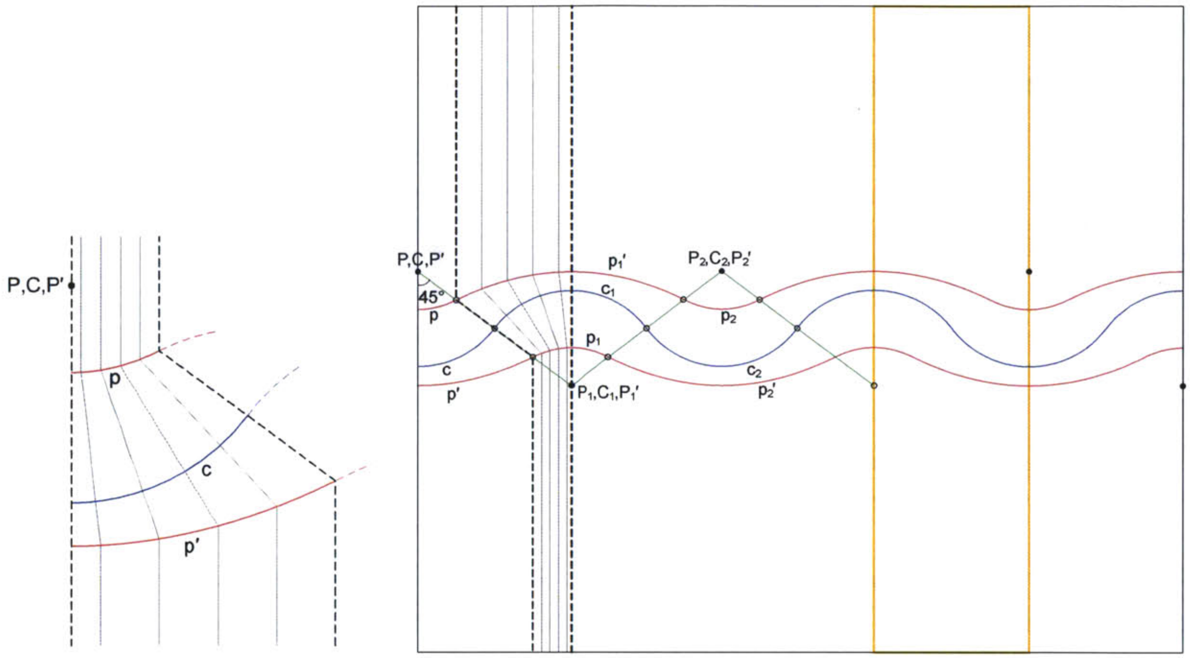


Fig 4.4.102 Gadget [DK], Crease pattern [DK]

crease turns the upper spline into a mountain which makes theses designs look very different to previous designs.

The 3d configuration in the simulation consists of general cylinders at the top and bottom, but their rulings do not remain parallel (Fig 4.4.100 right). The simulated model is folded more than Huffman's model and shows the effect in a more pronounced way.

Huffman creates a more symmetrical iteration, also made in vinyl, which looks very flat. The design is most likely from the 1990s (Fig 4.4.103 left).

He places the circle such that it is closer to the closer to the 2 splines and draws only a partial crease pattern (Fig 4.4.103 right). The conical area is more narrow in this design.

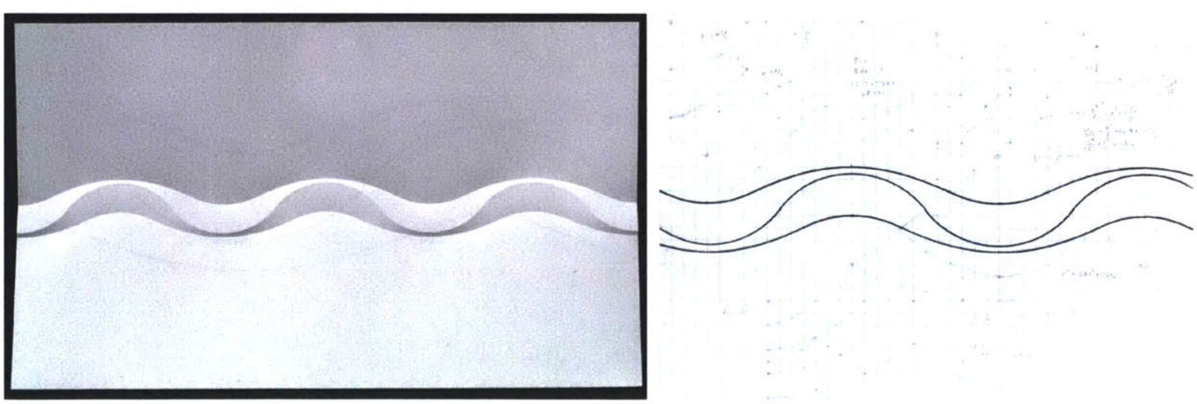


Fig 4.4.103 Vinyl model (undated DAH [DK]), Paper model (undated DAH [DK])

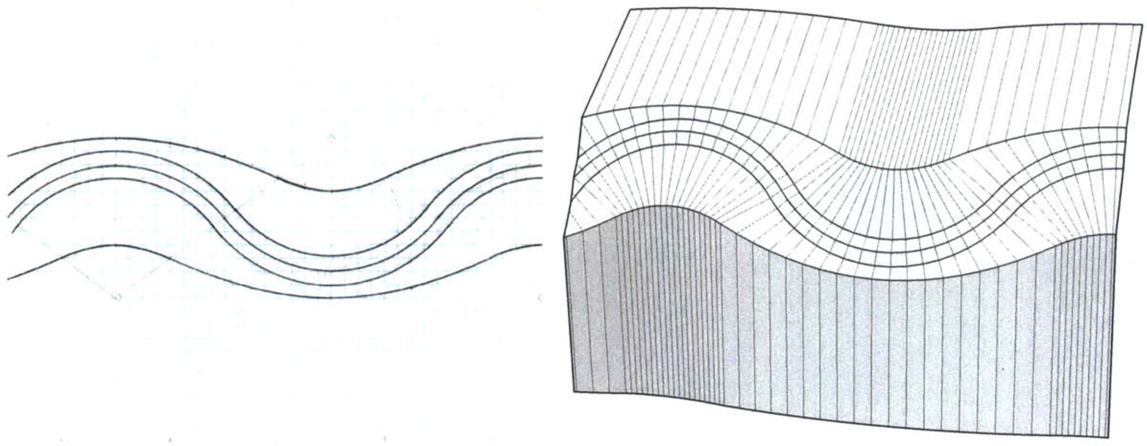


Fig 4.4.104 Paper model (undated, DAH [TG]), Simulated model [AH]

The last example in this subsection uses 3 circles in the conic transition (Fig 4.4.104 left).

Crease pattern and ruling analysis

The non-dominant curves have no effect on the ruling direction and as the number of pleats is odd we also don't incur a change in mountain or valley assignments for the 2 splines. Huffman draws the edges of the axes of the gadget in pencil in (Fig 4.4.105).

The model folds reasonably well in simulation and we can see that the upper and lower general cylinder do not have parallel rulings (Fig 4.4.104 right).

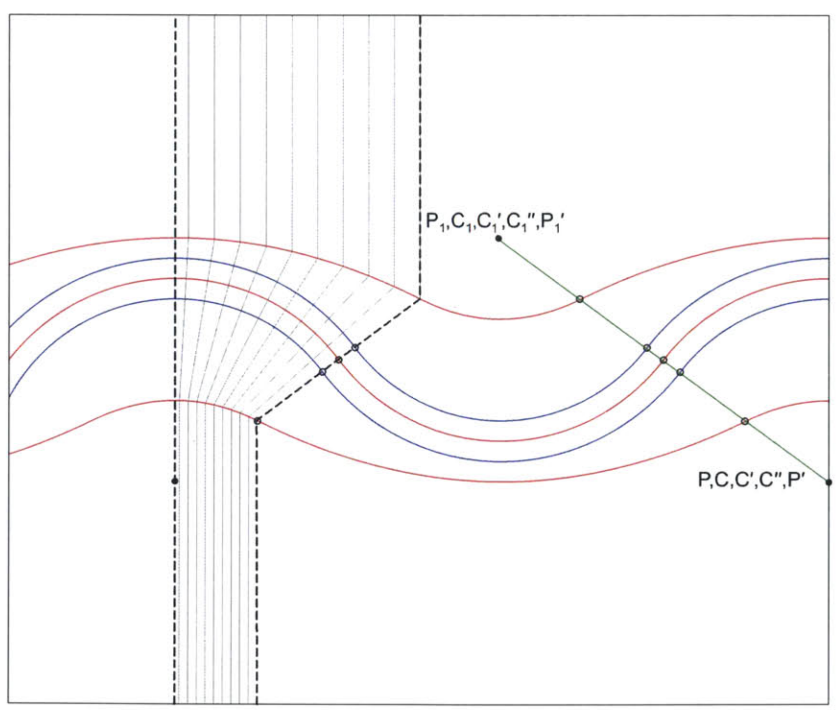


Fig 4.4.105 Crease pattern [DK]

Gadgets with pleated parabolic splines

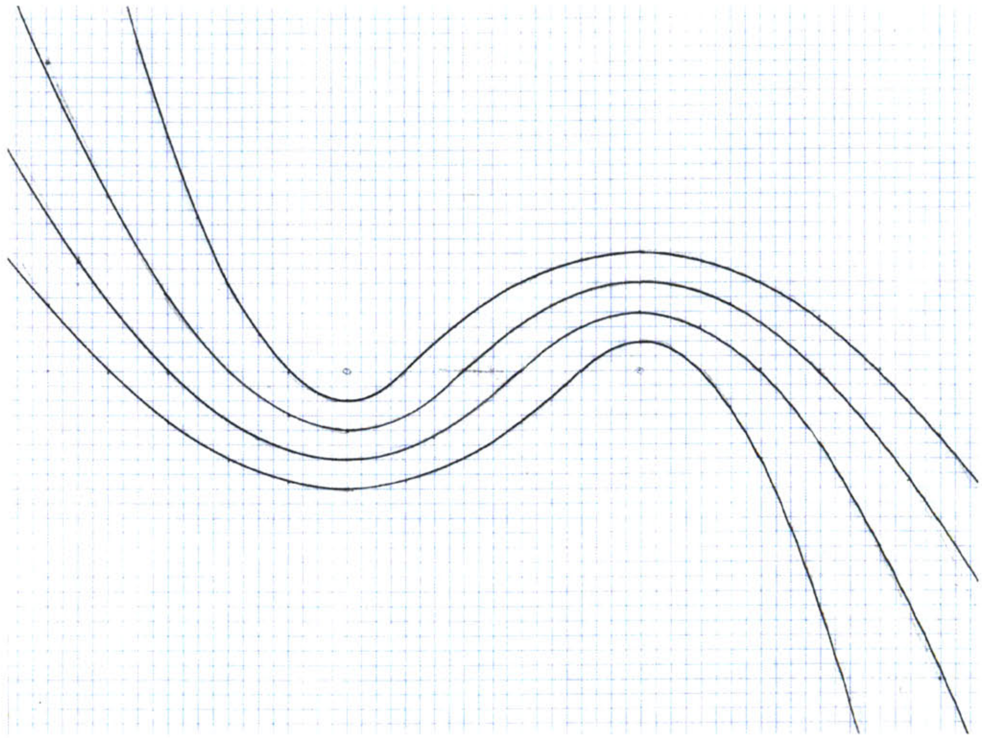


Fig 4.4.106 Paper model (undated, DAH [DK])

Huffman is pleating parabolas with the previous gadget that has a horizontal gadget axis and uses a set of 4 splines in the above design (Fig 4.4.106).

Crease pattern and ruling analysis

The parabolas on the left and right side of the gadget that is in the center extend to the edge of

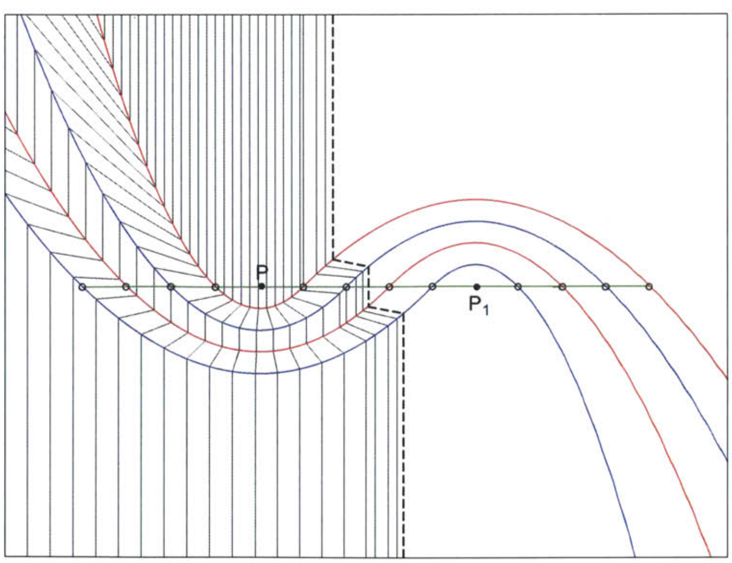


Fig 4.4.107 Crease pattern [DK]

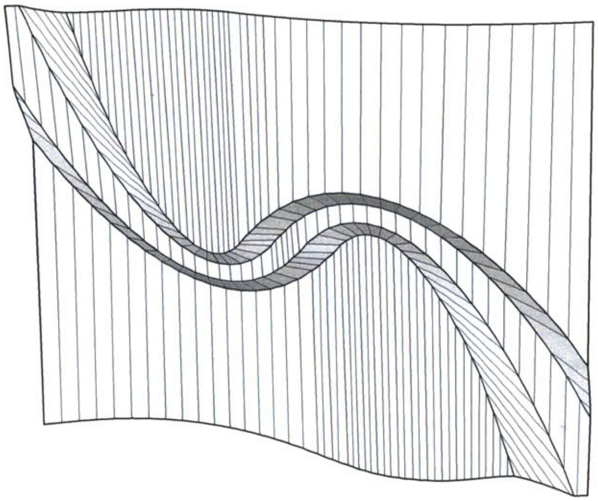


Fig 4.4.108 Simulated model [AH]

the page. The right edges of the left prototile have to follow the rulings and result in a stepped configuration.

Ruling analysis

The gadget is used with an even number of creases and thus creates a transition from a general undulating cylinder at the top to another general undulating cylinder at the bottom. The model folds reasonably well in simulation, but only remains stable in a hardly folded state (Fig 4.4.108).

Huffman draws a similar design that consists of a set of pleated parabolas that are rotated 180° around the center of the crease pattern (Fig 4.4.109). The sketch, even though incomplete and not folded, is presented here as it provides a segue to the following subsection of pleated designs,

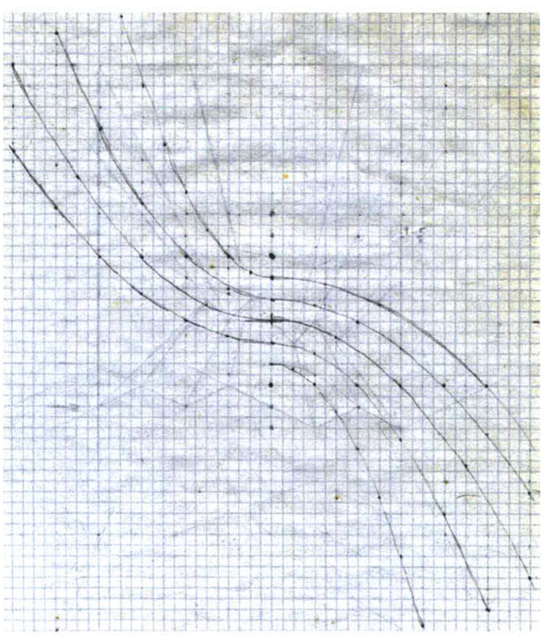


Fig 4.4.109 Sketch (undated, DAH [DK])

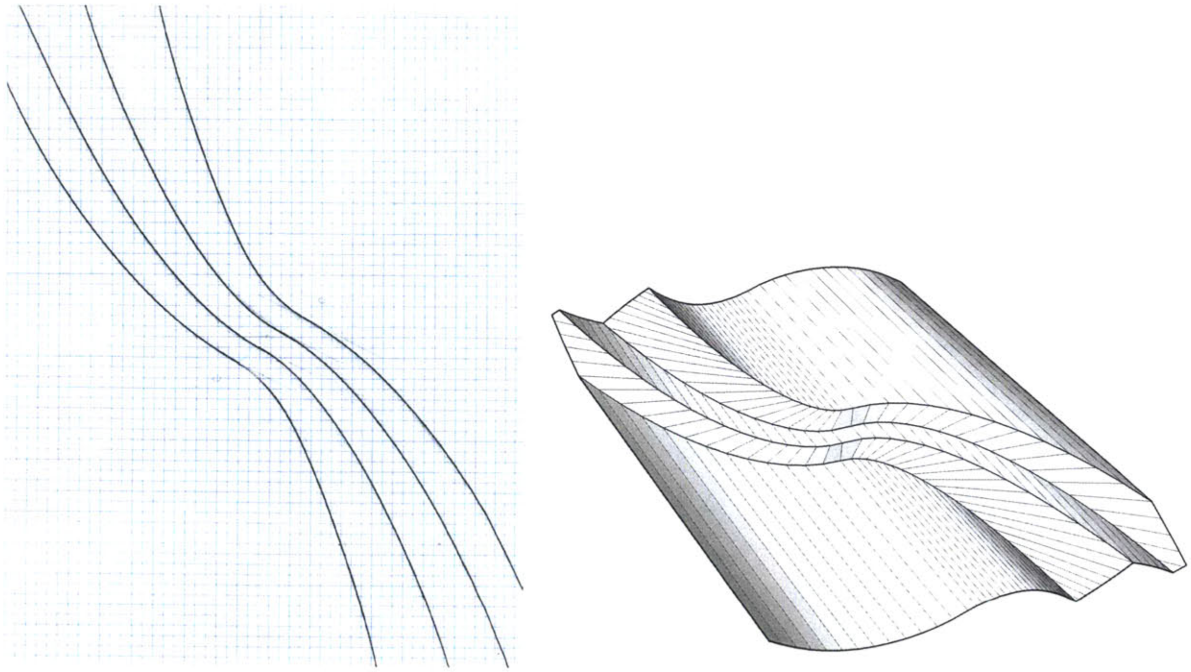


Fig 4.4.110 Paper model (undated, DAH [DK]), Simulated model [AH]

in which the gadget axis rotates further (Fig 4.4.110). We can think of the slope of the gadget axis being negative or having a slope in the other direction in (Fig 4.4.111).

The 3d configuration only works with alternating general cylinders, meaning the upper and lower general cylinders have to undulate similar to a sine curve (Fig 4.4.110 right).

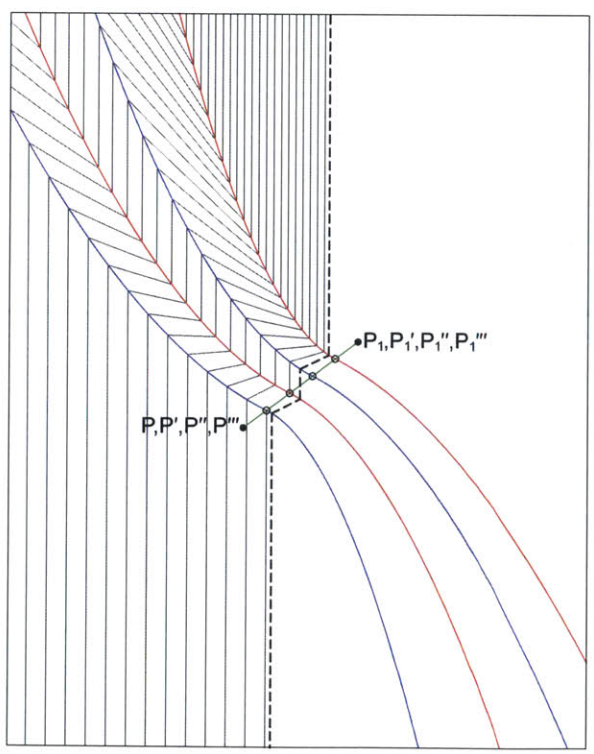


Fig 4.4.111 Crease pattern [DK]

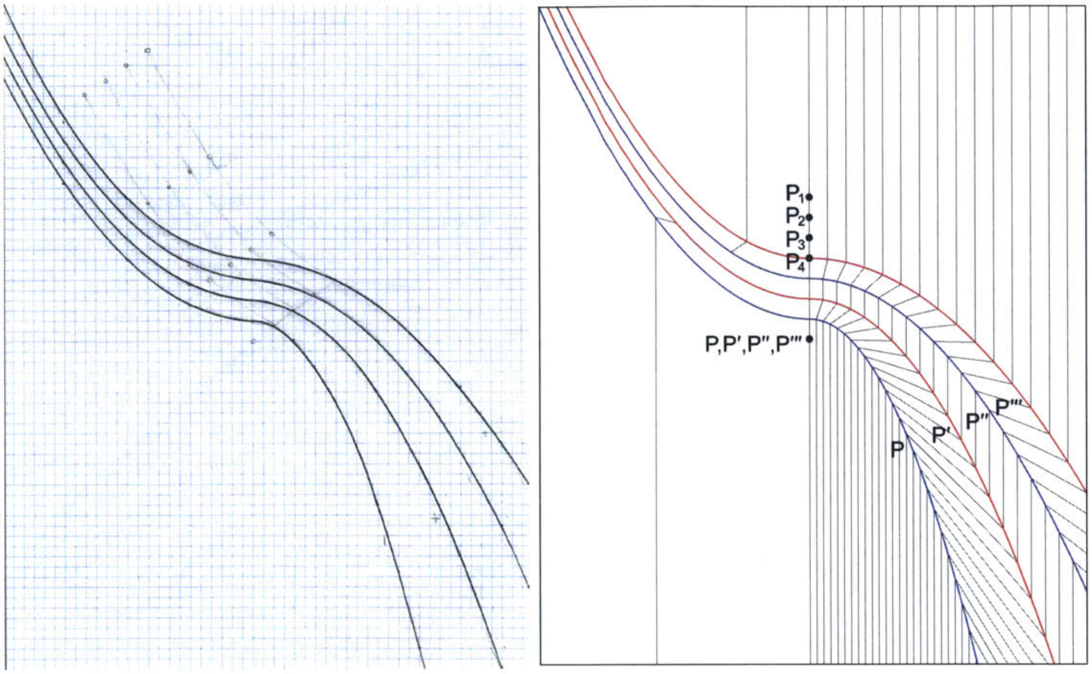


Fig 4.4.112 Sketch (undated, DAH [DK]), Crease pattern [DK]

Huffman investigates further examples with parabolas and pleating, which don't appear to have resolved solutions for rulings. The 2 examples are included in this subsection as the construction of their crease patterns is interesting.

Huffman combines the previous gadget on the right with parabolas that have vertically shifted foci on the left (Fig 4.4.112). All foci in the crease pattern below are shifted, but they remain pair-wise related and have parallel gadget axes (Fig 4.4.113).

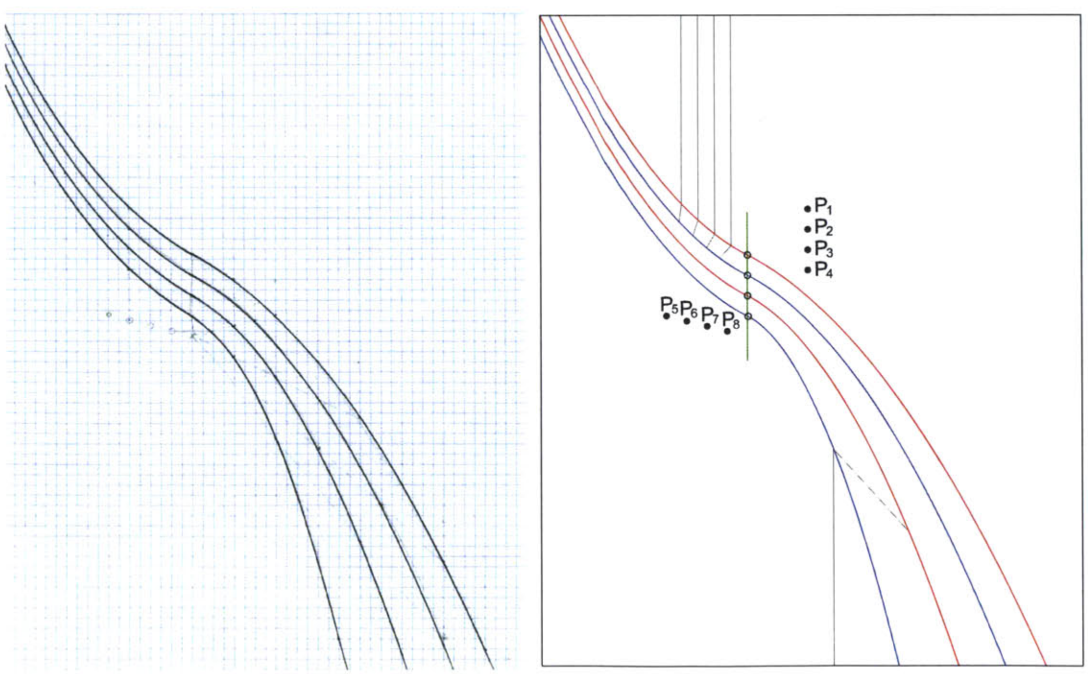


Fig 4.4.113 Sketch (undated, DAH [DK]), Crease pattern [DK]

Gadgets with parabolic splines and line segments

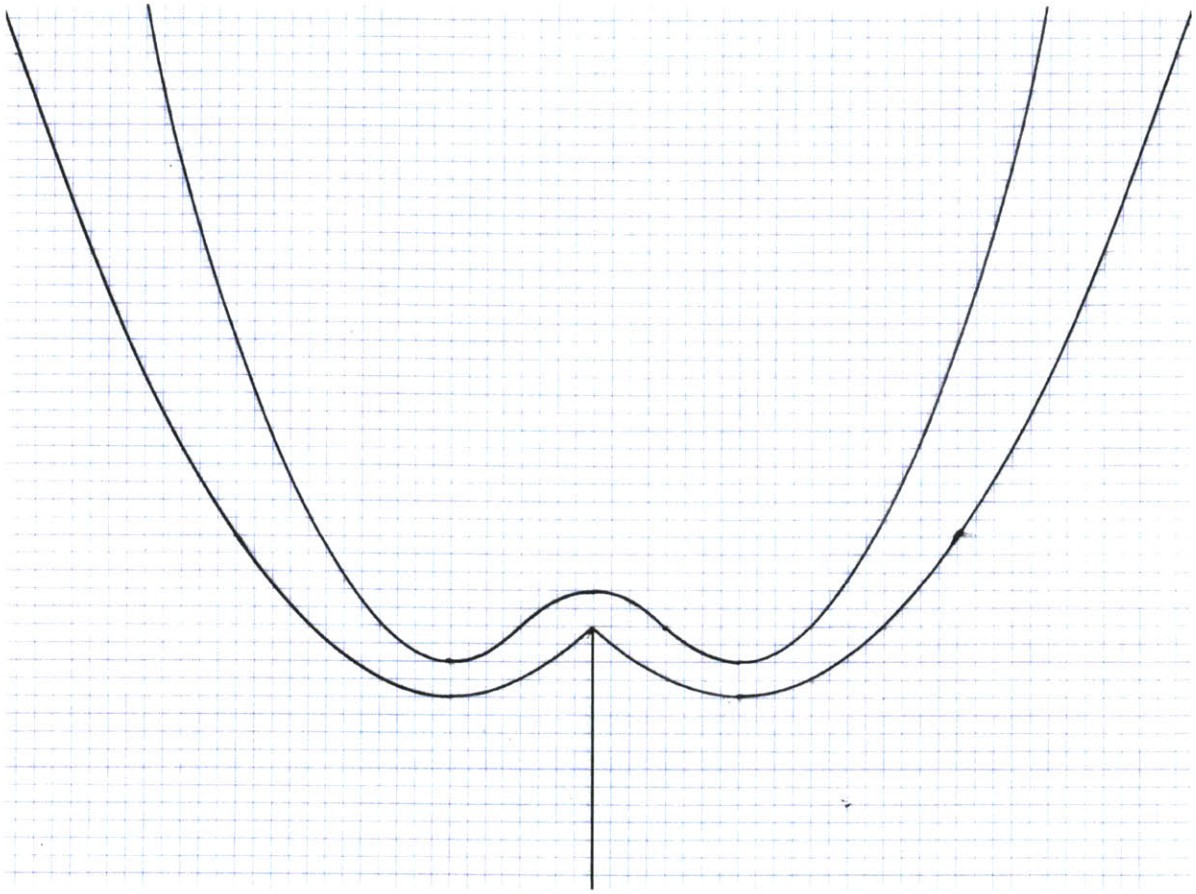


Fig 4.4.114 Paper model (undated, DAH [DK])

This subsection discusses designs that combine parabolic splines with parabolas and line segments. The 2 intersecting parabolas define the location of a vertex, where Huffman often places a linear crease. This special characteristic gives the designs their distinct visual appearance. All models in this section only exist as sketches and paper models. Huffman uses this technique to design a commissioned model for Donald Knuth, acclaimed computer scientist and colleague, but never completes the work.

The above design has one straight crease and is only symmetrical along the vertical axis (Fig 4.4.114). In the folded state a surface consisting of general cones operates as transition between an undulating cylinder above and 2 cylinders below (Fig 4.4.116).

Crease pattern and ruling analysis

The gadget consists of 2 parabolas that turn parallel rulings at the top of the crease pattern into parallel rulings after they passed through the second parabola. Huffman extends the parabolas to the edge of the paper on the right and left side of the crease pattern (Fig 4.4.115). The result consists of 2 prototiles.

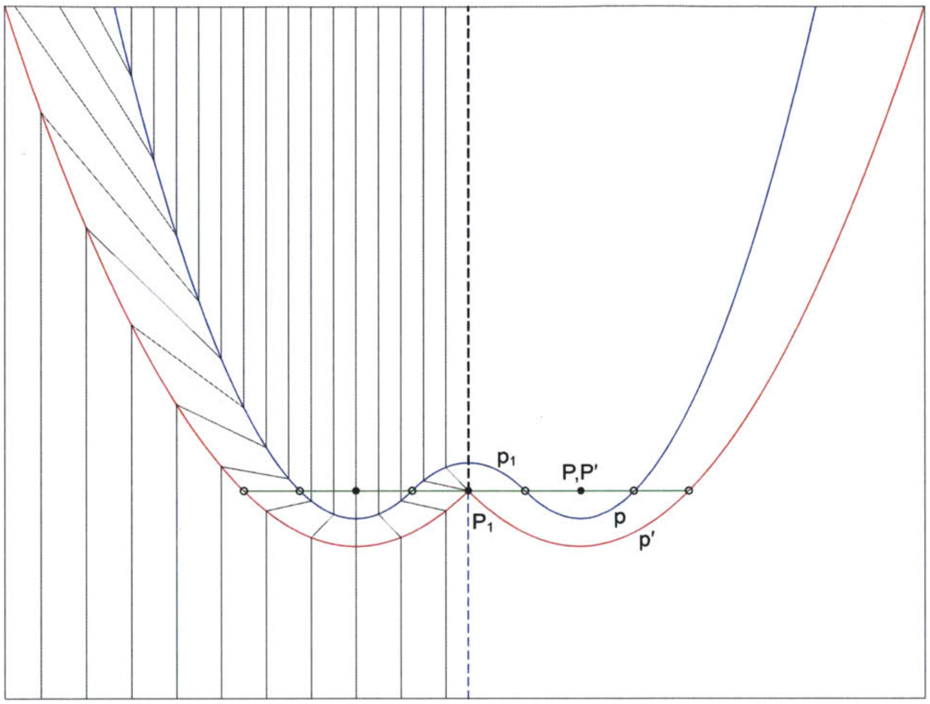


Fig 4.4.115 Crease pattern [DK]

The model folds very well during simulation and can be folded further than depicted (Fig 4.4.116). The upper undulating general cylinder stands in contrast to the 2 separate cylinders below.

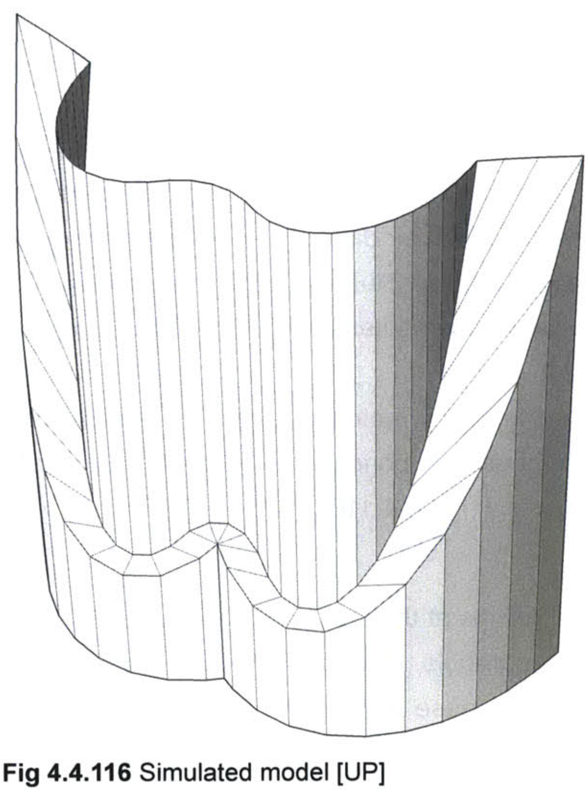


Fig 4.4.116 Simulated model [UP]

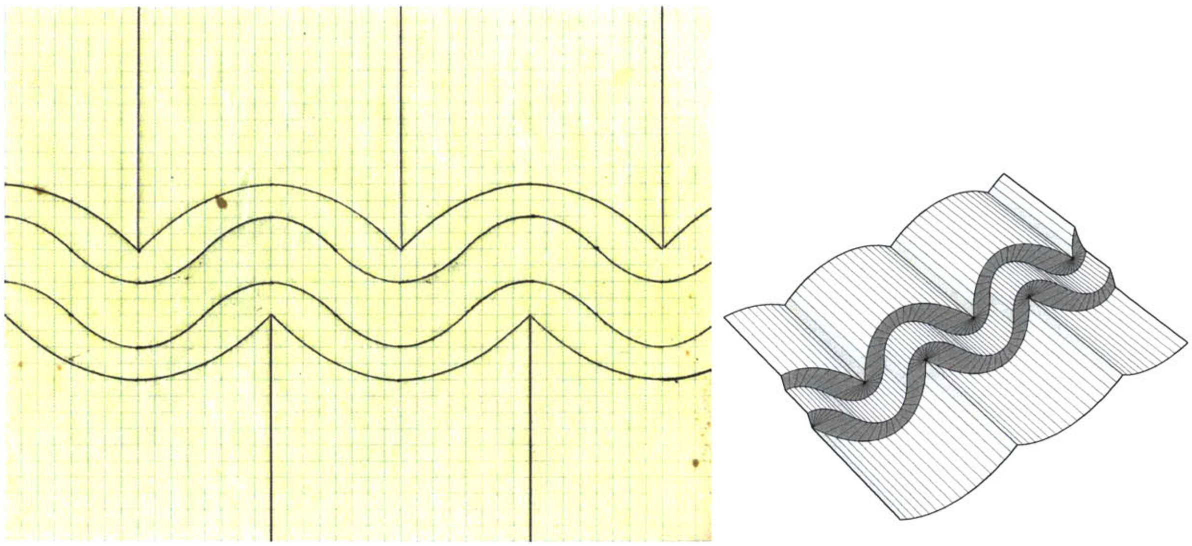


Fig 4.4.117 Paper model (undated, DAH [DK]), Simulated model [AH]

The next example consists of 4 partial cylinders at the top and 3 partial cylinders at the bottom. The 2 splines are located in the central area, which creates the undulating cylindrical surface typical for this series.

Crease pattern and ruling analysis

10 complete and 2 cropped prototiles generate the crease pattern (Fig 4.4.117). The design is remarkably different to many others in this section as the mountain and valley assignments force the upper cylinders to be convex and the lower ones to be concave.

The model folds very well in simulation Fig 4.4.117 right).

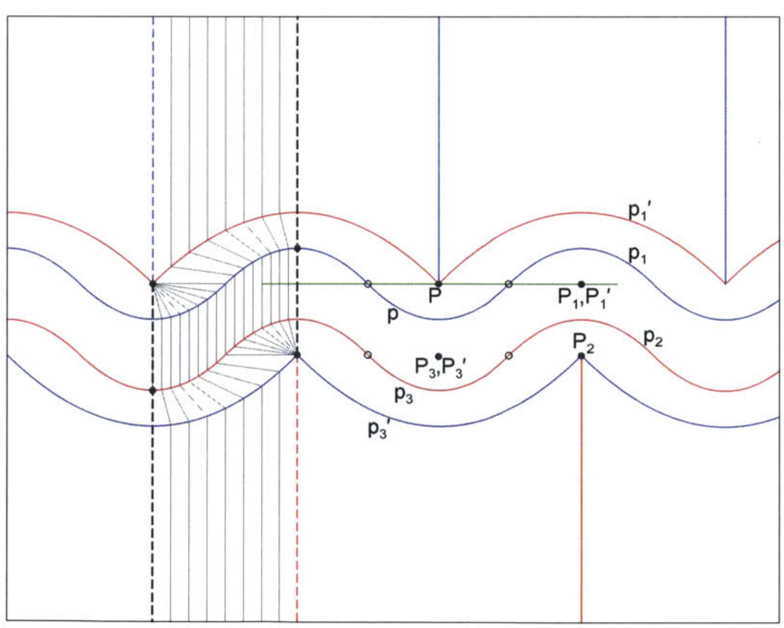


Fig 4.4.118 Crease pattern [DK]

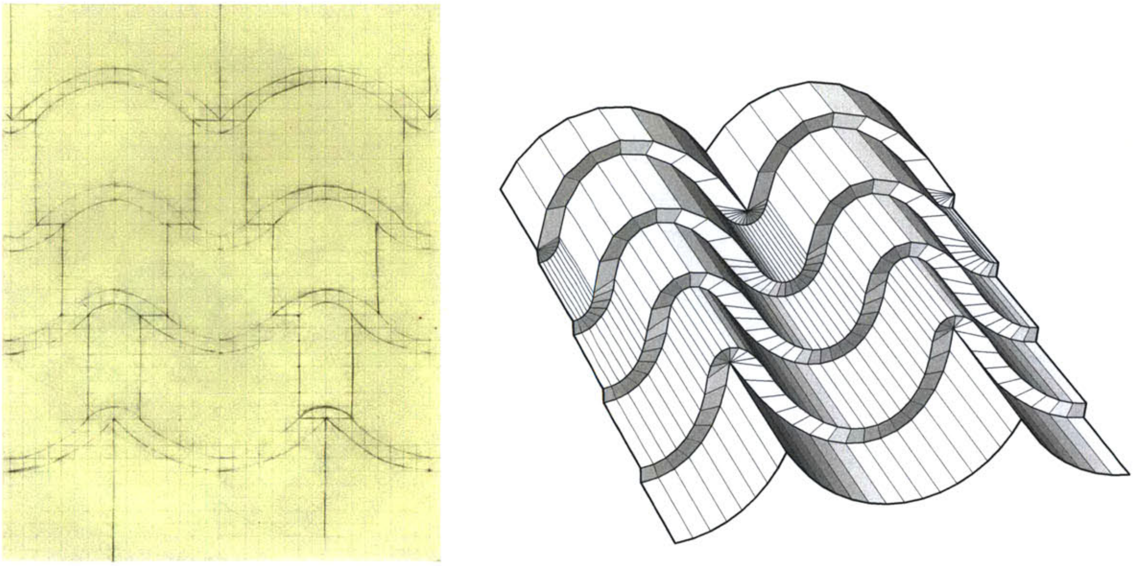


Fig 4.4.119 Drawing (undated, DAH [DK]), Simulated model [AH]

The above design uses fairly large prototiles (Fig 4.4.119). It is unclear what Huffman's intention is regarding mountain and valley assignments as the drawing is drawn with very faint pencil lines. If we use another sketch as a guide, lines separate the splines into parts with different curvature, but their connections might be smooth (Fig 4.4.120 right). Only a single line at the top and 2 lines at the bottom would be actual creases.

Crease pattern and ruling analysis

8 prototiles generate the crease pattern with the above mentioned assumptions regarding mountains and valleys (Fig 4.4.120). The model folds reasonably well in simulation (Fig 4.4.119 right).

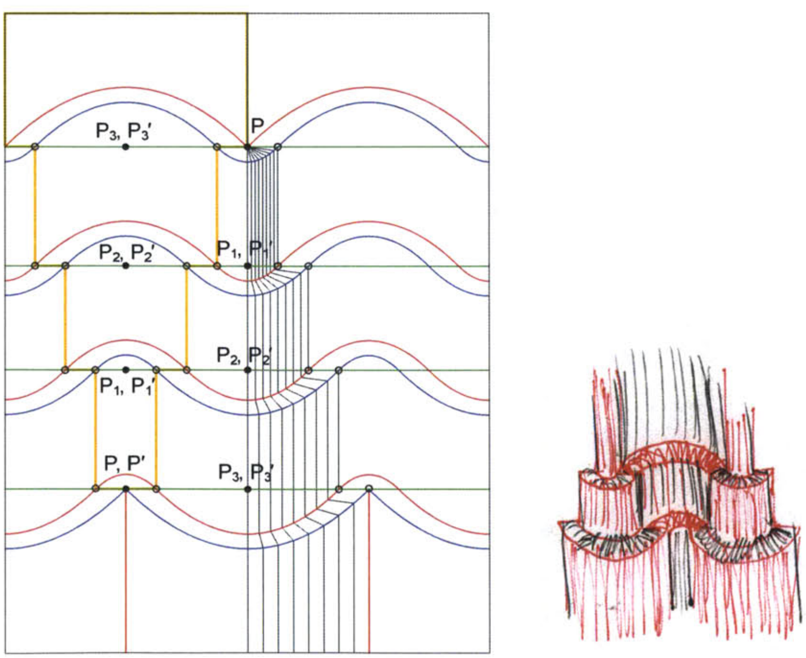


Fig 4.4.120 Sketch (undated, DAH [DK]), Crease pattern [DK]

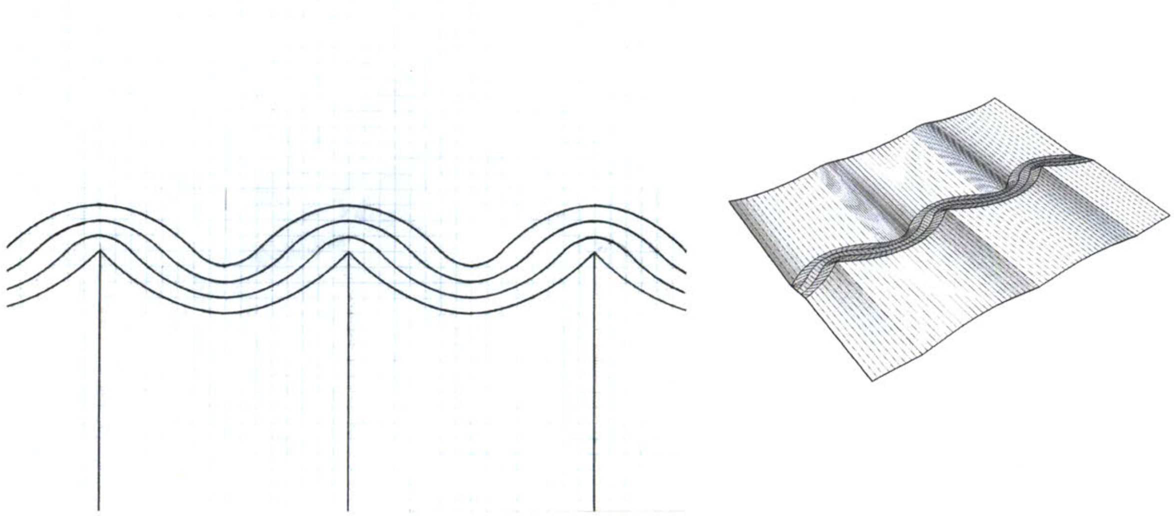


Fig 4.4.121 Paper model (undated, DAH [DK]), Simulated model [PC]

Another design in the series has an unusual characteristic at first, which is that the upper half of the page is blank (Fig 4.4.121). The gadget pleats the parabolas in 4 steps, which means we should expect 1 undulating cylinder at the top and 4 partial cylinders at the bottom.

Crease pattern and ruling analysis

Huffman needs 6 prototiles to complete the crease pattern and all foci and smooth connections align on a horizontal line in the center of the page (Fig 4.4.122).

The model folds reasonably well during simulation in (Fig 4.4.121) on the right.

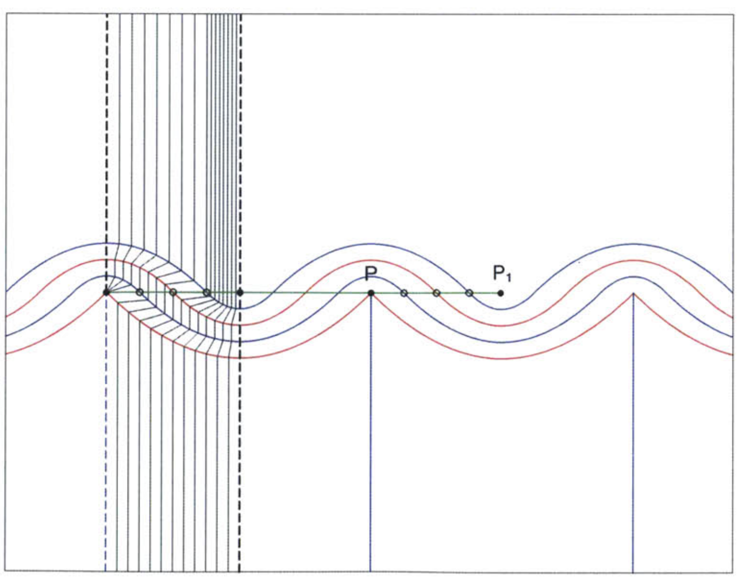


Fig 4.4.122 Crease pattern [DK]

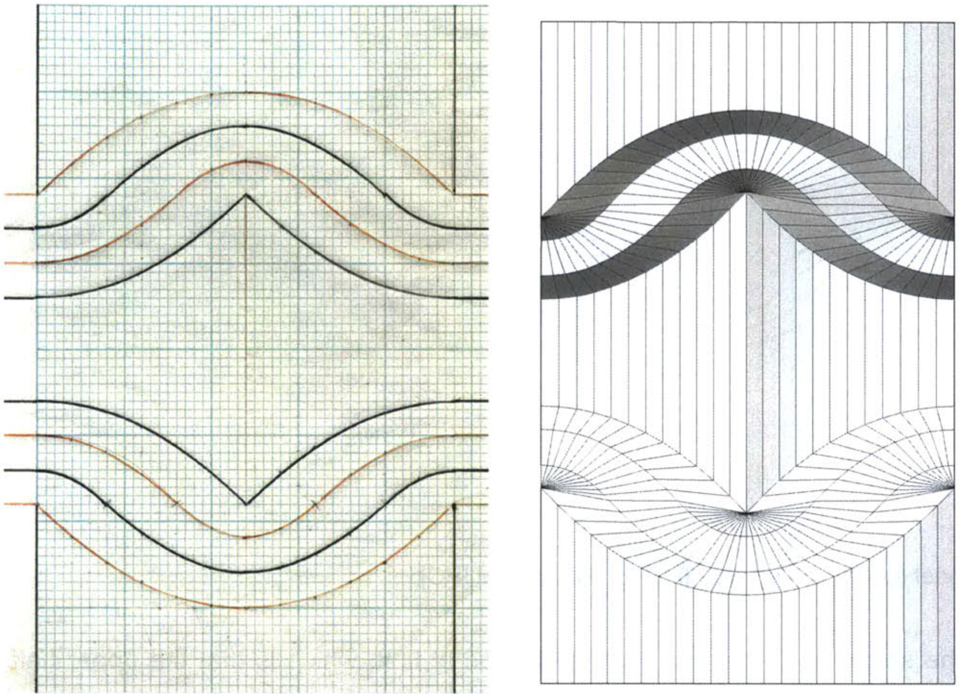


Fig 4.4.123 Paper model (undated, DAH [DK]), Simulated model [UP]

Huffman draws the above crease pattern on graph paper with two colors (Fig 4.4.123). The design pleats 4 parabolas such that the prototile creates individual cylinders at the top and bottom (Fig 4.4.124). The model folds reasonably well during simulation (Fig 4.4.123 right).

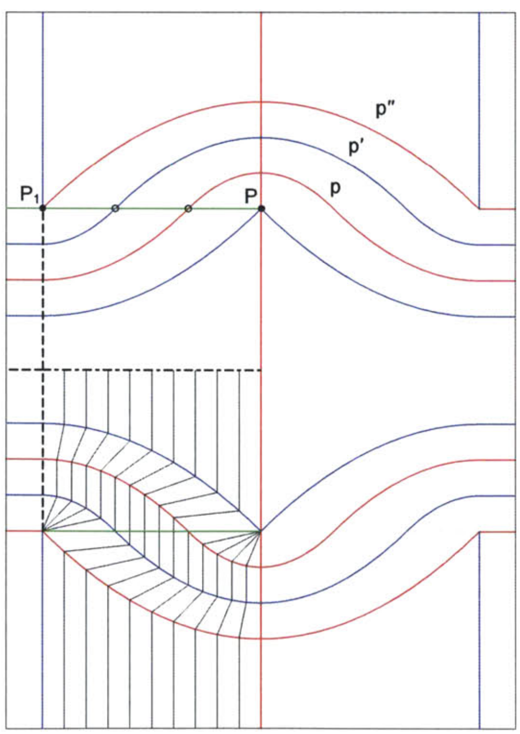


Fig 4.4.124 Crease pattern [DK]

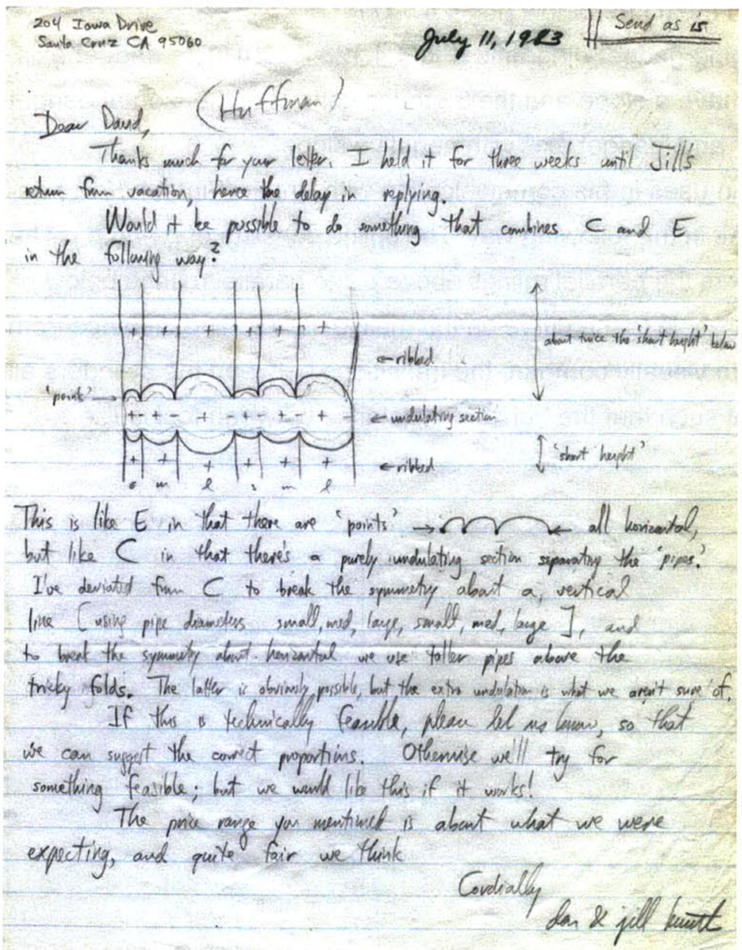
Gadgets with parabolic splines and line segments for Donald Knuth

The examples in this subsection relate to a commissioned sculpture that Huffman designs for Donald Knuth in the early 1980s. Knuth asks Huffman to design and make a model for him and his wife Jill, which is probably the only 'art commission' Huffman has.

The contemporaries exchange letters with drawings, diagrams and photocopied folded sketch models (Fig 4.4.125, 4.4.126), in which they discuss design options. Huffman privileges symmetrical designs in his letters to Knuth, who prefers to break the overall symmetry of the piece. Knuth seems to be looking for a random distribution of elements, which might have resulted in one of the few irregular works by Huffman.

The conversation lasts for several letters, but Knuth eventually retracts the commission as he and his wife feel that they are burdened by the things they own. They prefer to pay him for his services, but not receive the commissioned piece. Huffman never completes the work, but produces a significant amount of sketches and drawings, which I think are all related to this effort.

This subsection presents all crease patterns Huffman creates and displays several tiling


204 Iowa Drive
Santa Cruz CA 95060

July 11, 1983 || Send as is

Dear David, (Huffman)

Thanks much for your letter. I held it for three weeks until Jill's return from vacation, hence the delay in replying.

Would it be possible to do something that combines C and E in the following way?

This is like E in that there are 'points' all horizontal, but like C in that there's a purely undulating section separating the 'pipes.' I've deviated from C to break the symmetry about a vertical line [using pipe diameters small, med, large, small, med, large], and to break the symmetry about horizontal we use taller pipes above the tricky folds. The latter is obviously possible, but the extra undulation is what we aren't sure of.

If this is technically feasible, please let us know, so that we can suggest the correct proportions. Otherwise we'll try for something feasible; but we would like this if it works!

The price range you mentioned is about what we were expecting, and quite fair we think.

Cordially
don & jill knuth

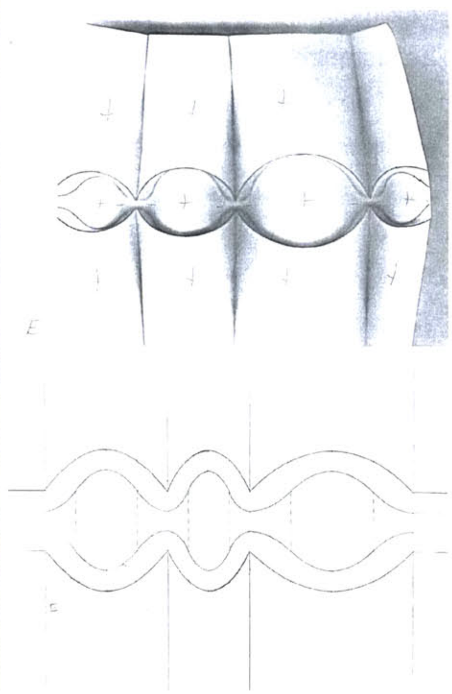


Fig 4.4.125 Correspondence between Huffman and Knuth (1983 [DK])

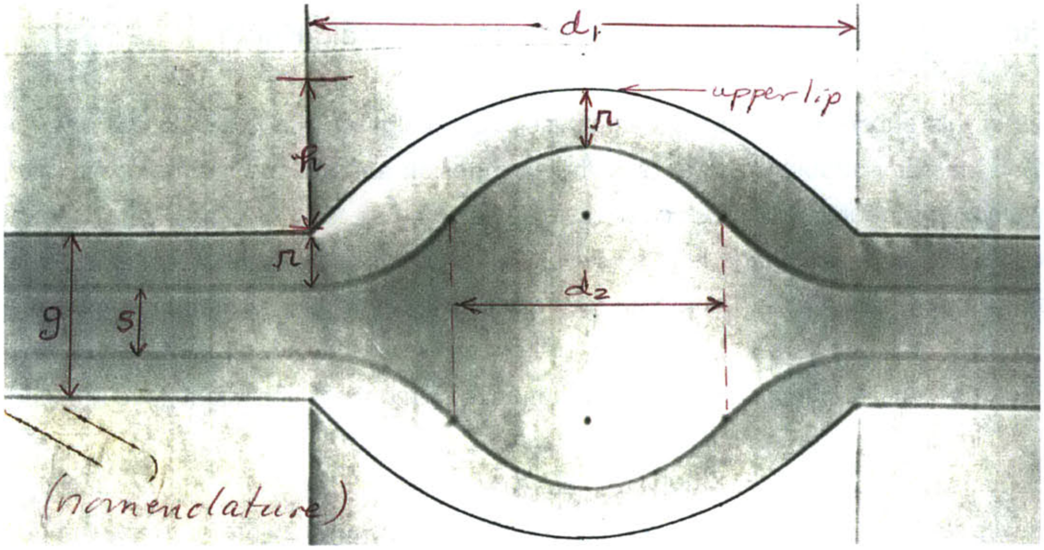


Fig 4.4.126 Paper model (undated, DAH [DK])

options, but does not show every example he draws as his drawings and sketches are very repetitive. I categorize them in 3 groups that use the gadget axis as ordering principle. Representative examples of the 3 cases are shown in the figure below (Fig 4.4.127).

They correspond to the following gadget diagrams (Fig 4.4.128 - 130). The gadget axis, drawn in green, can be horizontal or have a slope and the 3 groups consist of 'horizontal gadget axis', 'gadget axis with positive slope' and 'gadget axis with negative slope'.

Similar to the notation Huffman uses in his communication with Knuth (Fig 4.4.126) I define parameters and relevant distances in the following way. The spline consists of p and p_1. The lower mountain crease is p' as it refracts the parallel rulings above p into parallel rulings below p'. The distance from the focus of the parabola to the curve on the main axis is a. The distance from p to p' on the main axis is b. In order to visually compare the transitions between the cylinders all of Huffman's drawings are normalized such that the horizontal distance between foci is $l' = 1$.

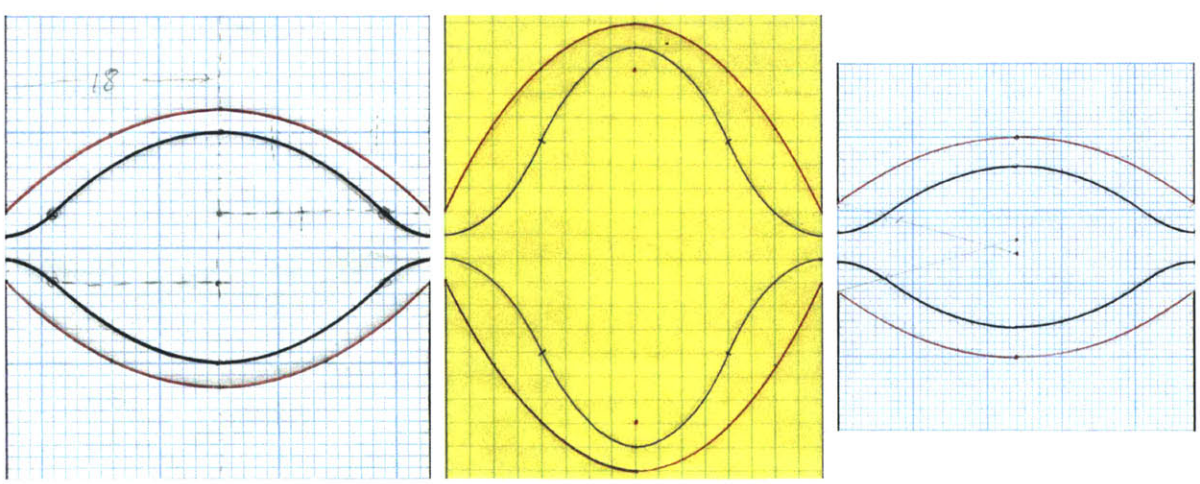
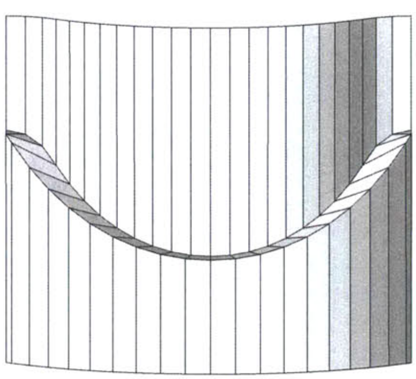


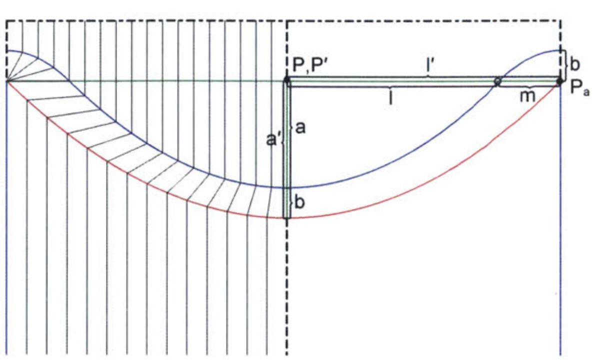
Fig 4.4.127 Paper models (undated, DAH [DK])

Horizontal gadget axis

Definitions and parameters:

$a' = a + b$
$l' = l + m$
$l' = 1$
$slope = d/l'$





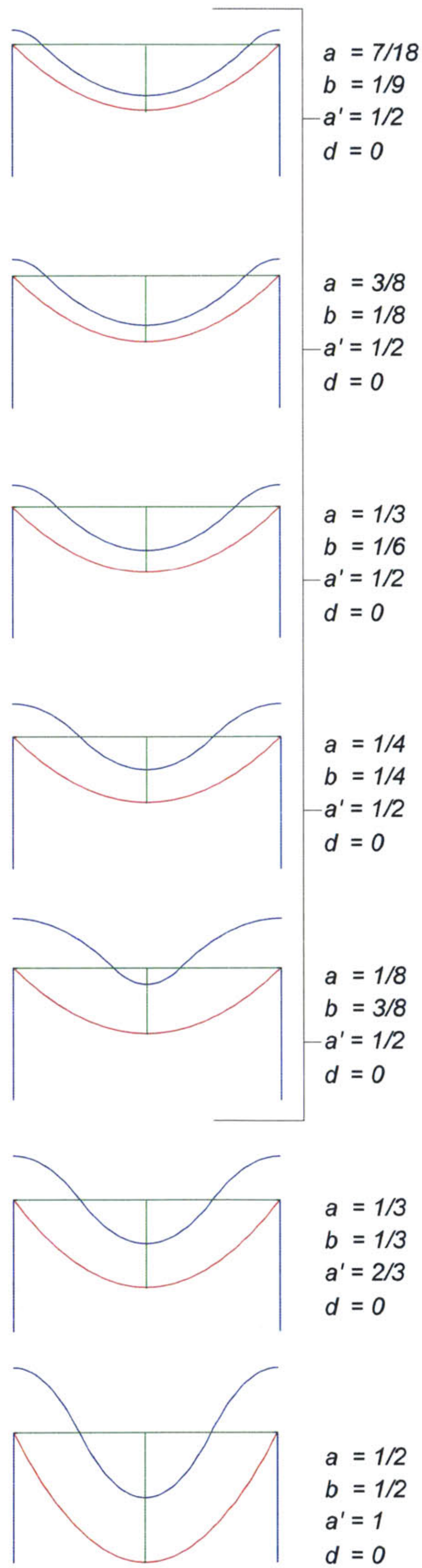
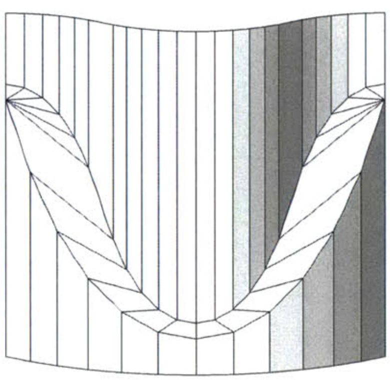


Fig 4.4.128 Gadget [DK], Simulated model [UP], Variations [DK]

Gadget axis with positive slope

Definitions and parameters:

a' = a + b
l' = l + m
l' = 1
slope = d/l'



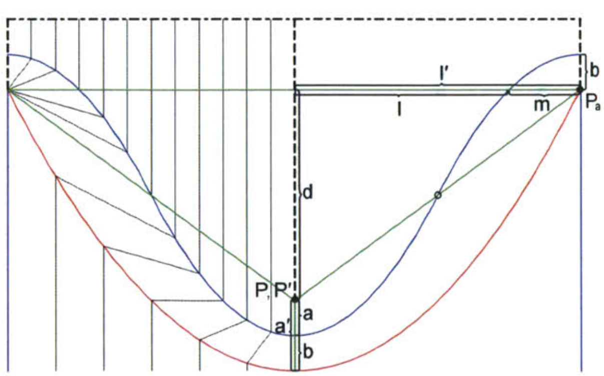
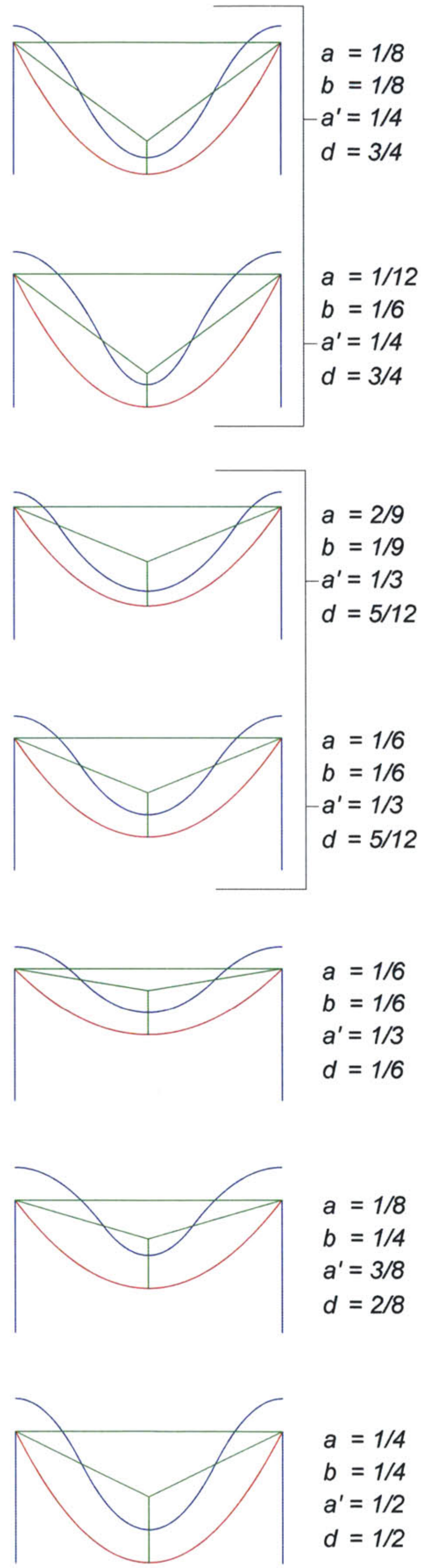


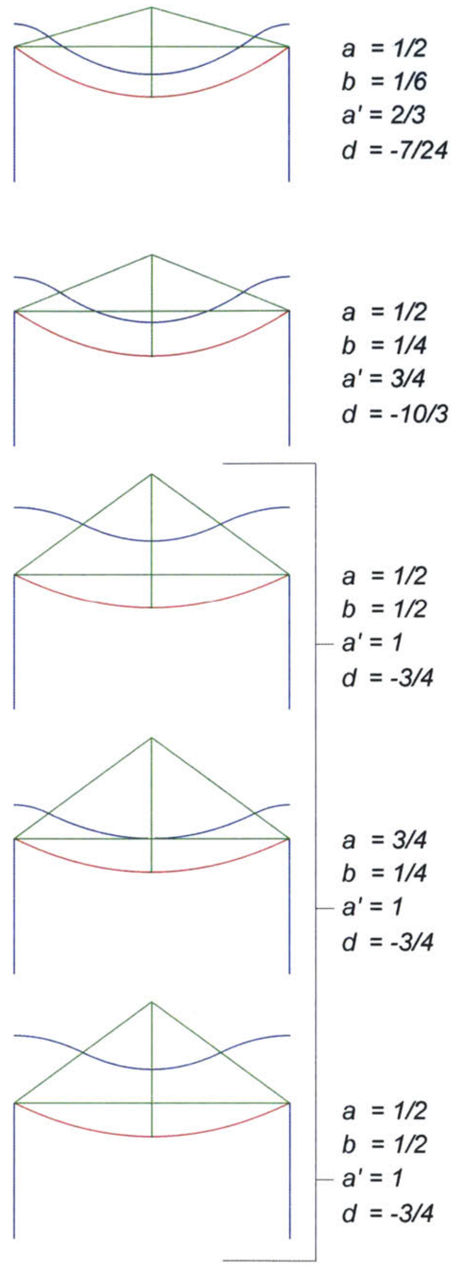
Fig 4.4.129 Gadget [DK], Simulated model [UP], Variations [DK]

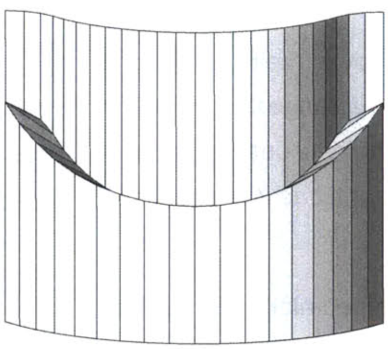


Gadget axis with negative slope

Definitions and parameters:

a = a + b
l' = l + m
l' = 1
slope = -d/l'





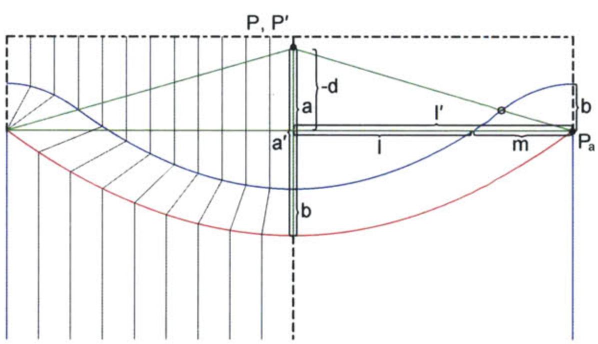


Fig 4.4.130 Gadget [DK], Simulated model [UP], Variations [DK]

Some of Huffman's designs with combinations of the gadget are presented in the following examples. The 3 designs use less symmetrical tilings and it is unclear which design path the contemporaries would have taken, had the project been completed.

Once the gadget is mirrored horizontally we obtain a shape visually reminiscent of organ pipes and I refer to the shape as 'pipes'. The complete body of work of Huffman's endeavor to design a sculpture for Donald Knuth requires a more in depth study as more documentation might exist.

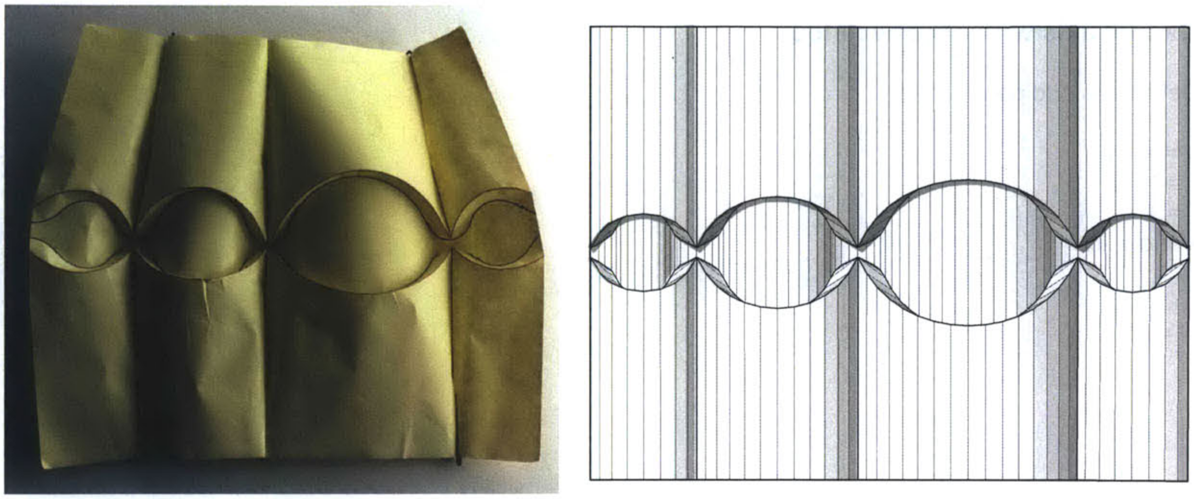


Fig 4.4.131 Paper model (undated, DAH [DK]), Simulated model [AH]

The above sketch model (Fig 4.4.131) corresponds to a photo copy in their correspondence (Fig 4.4.125). Huffman sizes the 4 'pipes' with different diameters and arranges them asymmetrically, which is probably in response to Knuth's request. This model is the only design Huffman keeps in a folded state in his archive and it appears to have traveled quite a bit as it is unusually wrinkled.

Crease pattern and ruling analysis

A single pipe consists of 4 prototiles and the pipes can only be joined along the vertical if *b* matches. The gadget axis is horizontal in all cases. The design causes no problems during simulation (Fig 4.4.131 right).

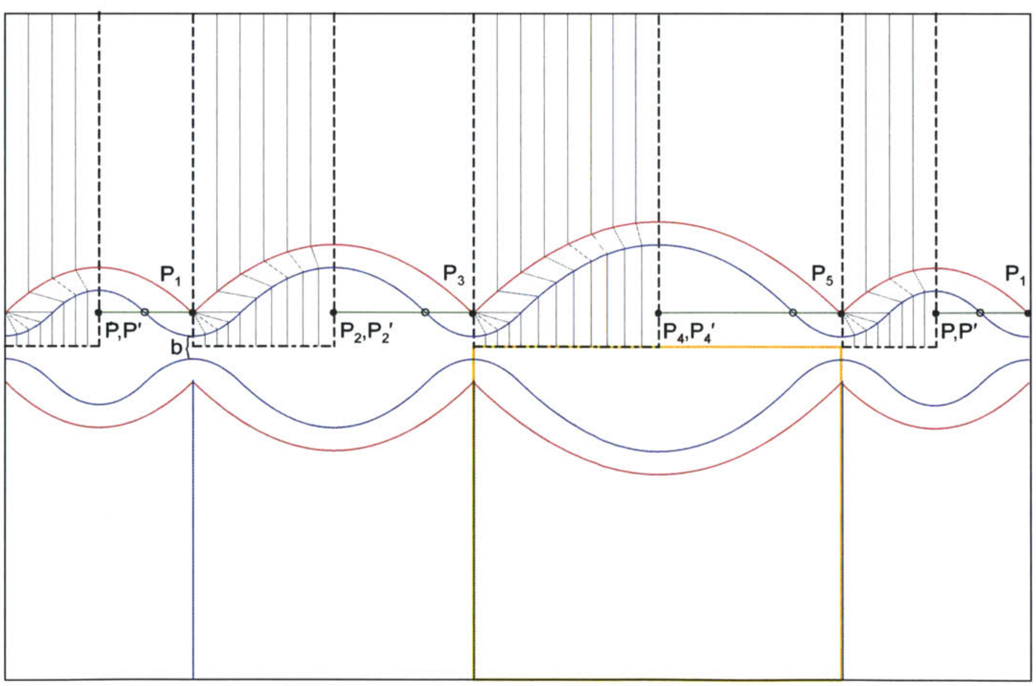


Fig 4.4.132 Crease pattern [DK]

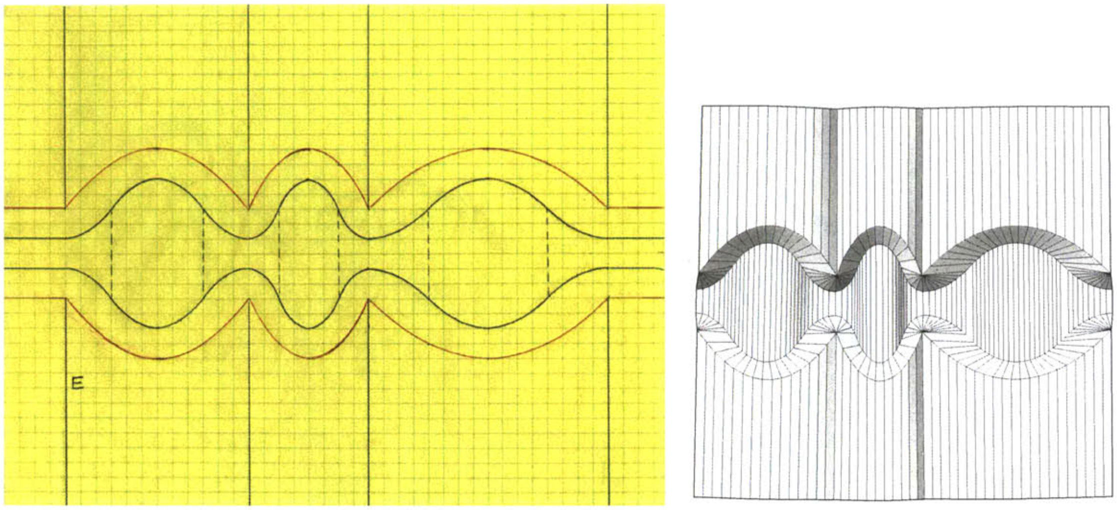


Fig 4.4.133 Paper model (undated, DAH [DK]), Simulated model [PC]

The next paper model in this series (Fig 4.4.133) also corresponds to one of the photo copies (Fig 4.4.125), but Huffman keeps this version in its flat state. The 3 horizontally aligned pipes have varying sizes. As the pipes are different we obtain an asymmetrical design. A small sketch (Fig 4.4.134) shows a symmetrical design with otherwise similar characteristics.

Crease pattern and ruling analysis

Huffman uses 4 prototiles per pipe and aligns all horizontal gadget axes along the same line. He adds flat areas on the left and right, most probably to stabilize the model. The model folds well during simulation (Fig 4.4.133 right).

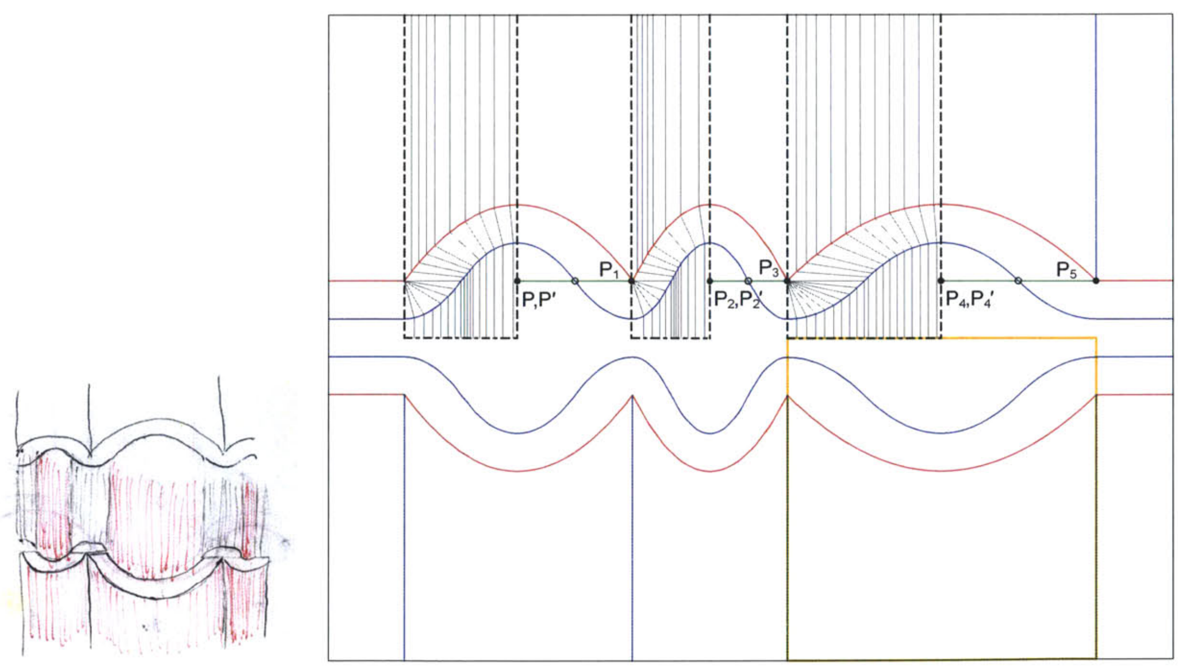


Fig 4.4.134 Sketch (undated, DAH [DK]), Crease pattern [DK]

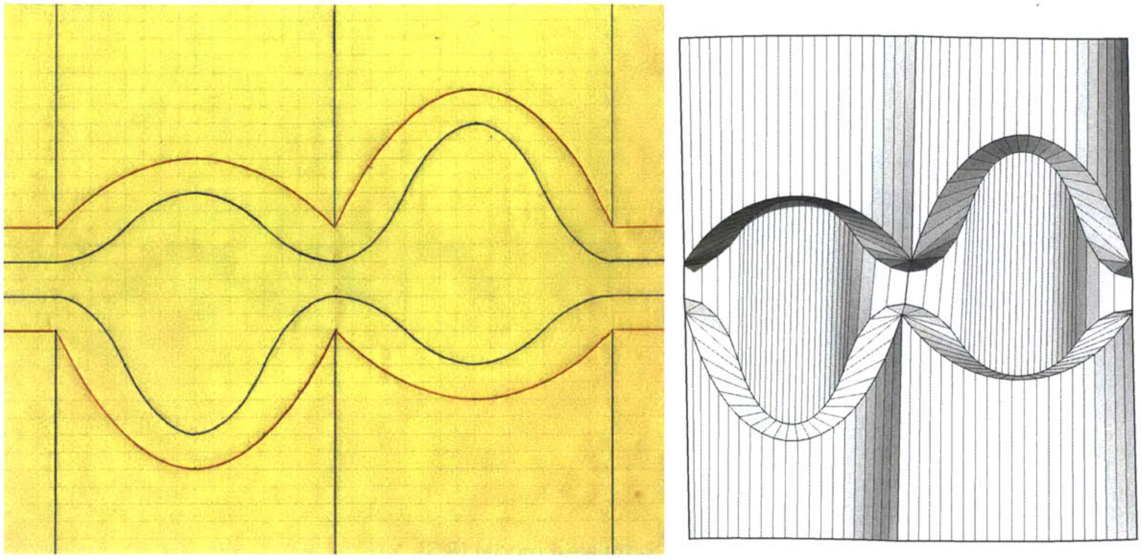


Fig 4.4.135 Paper model (undated, DAH [DK]), Simulated model [PC]

The third model in the series (Fig 4.4.135), also part of the correspondence between Knuth and Huffman, only consists of 2 pipes. It is not clear if it is meant as a complete arrangement or just as a detail of a design. The 2 pipes have different top and bottom splines, which renders them less symmetrical to previous examples as the pipe itself is only *d1*.

Crease pattern and ruling analysis

The 2 identical pipes, rotated around the center, consist of 2 prototiles each (Fig 4.4.136). The gadget axes have different angles and Huffman adds flat areas on the left and right side. The cylindrical surfaces do not remain parallel during simulation (Fig 4.4.135 right).

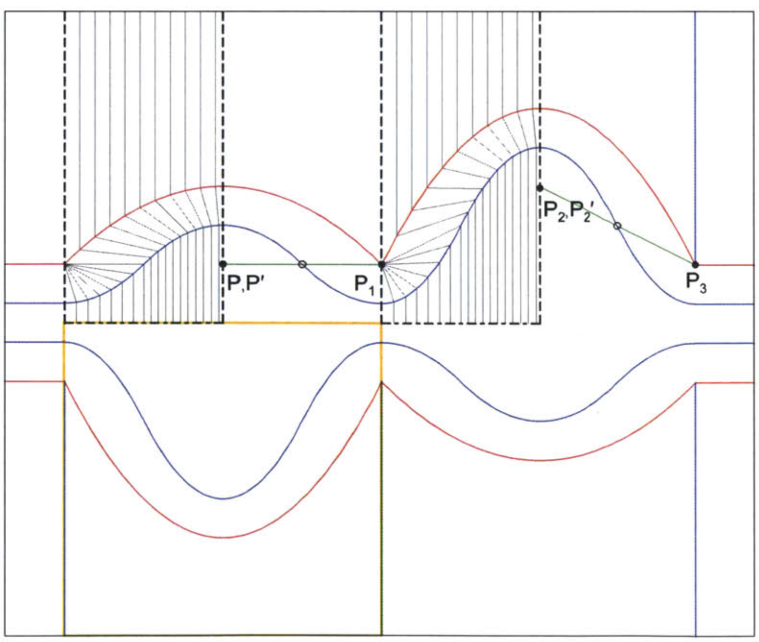
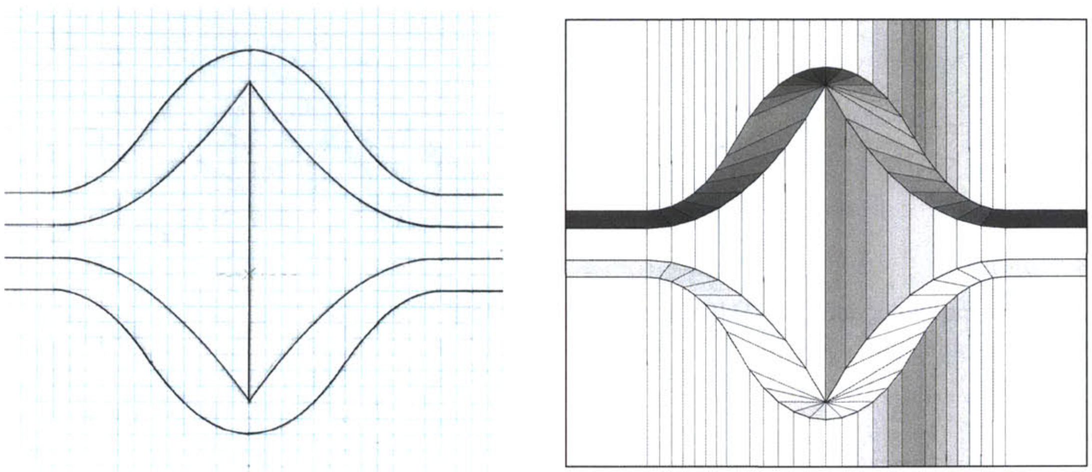


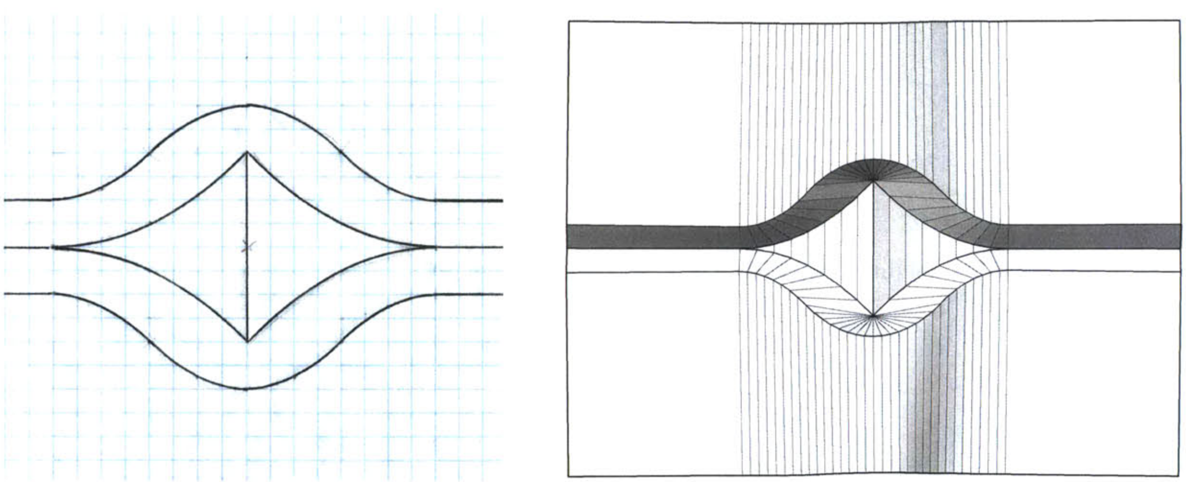
Fig 4.4.136 Crease pattern [DK]

Huffman continues to explore several options for Knuth's piece by introducing a mountain creases within a pipe, but they both appear to dislike that option. It is unclear if the following series is made for the sculpture, but the examples include the mountain crease at the center (Fig 4.4.137-141).



a = *1/6*
b = *1/6*
a' = *1/3* *d* = *5/12*

Fig 4.4.137 Paper model (undated, DAH [DK]), Simulated model [PC]



a = *1/4*
b = *1/4*
a' = *1/2*

Fig 4.4.138 Paper model (undated, DAH [DK]), Simulated model [PC]

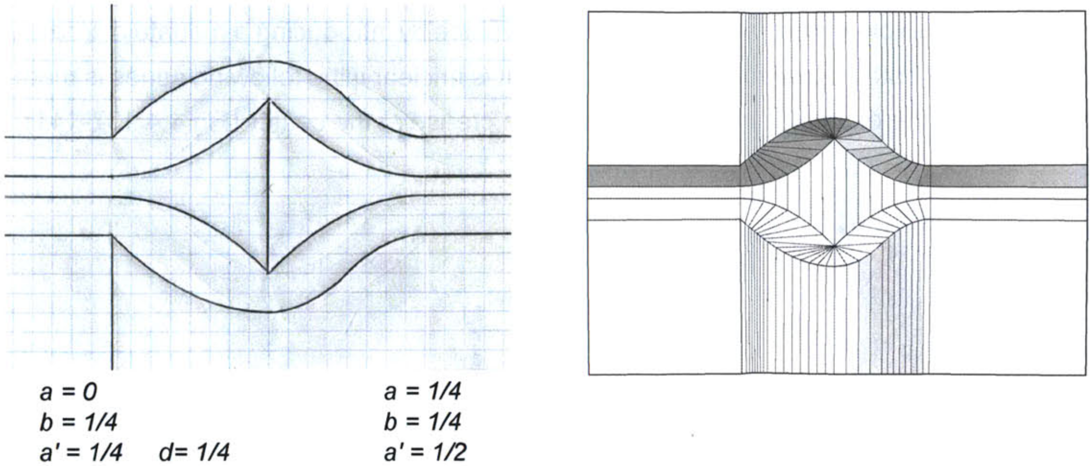


Fig 4.4.139 Paper model (undated, DAH [DK]), Simulated model [PC]

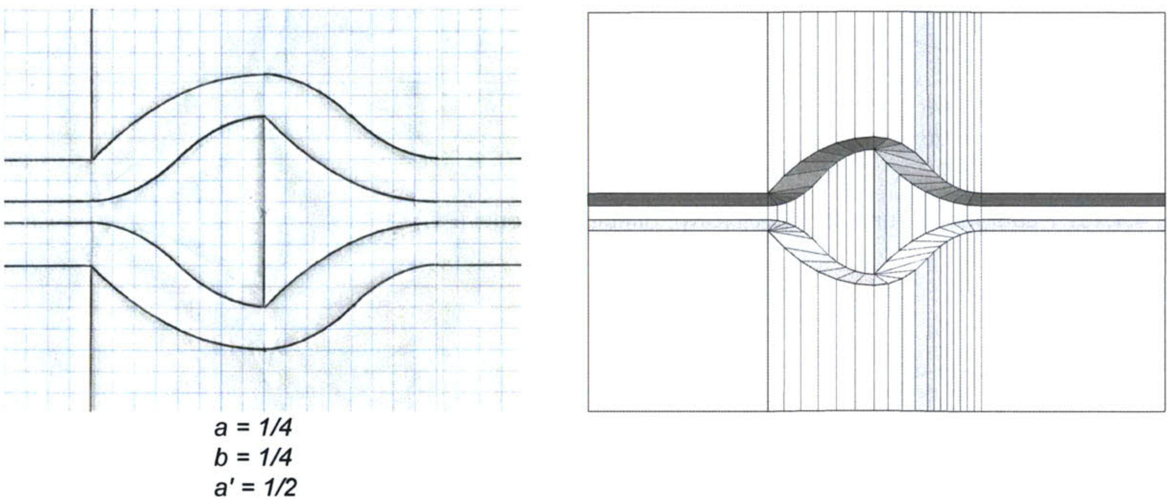


Fig 4.4.140 Paper model (undated, DAH [DK]), Simulated model [PC]

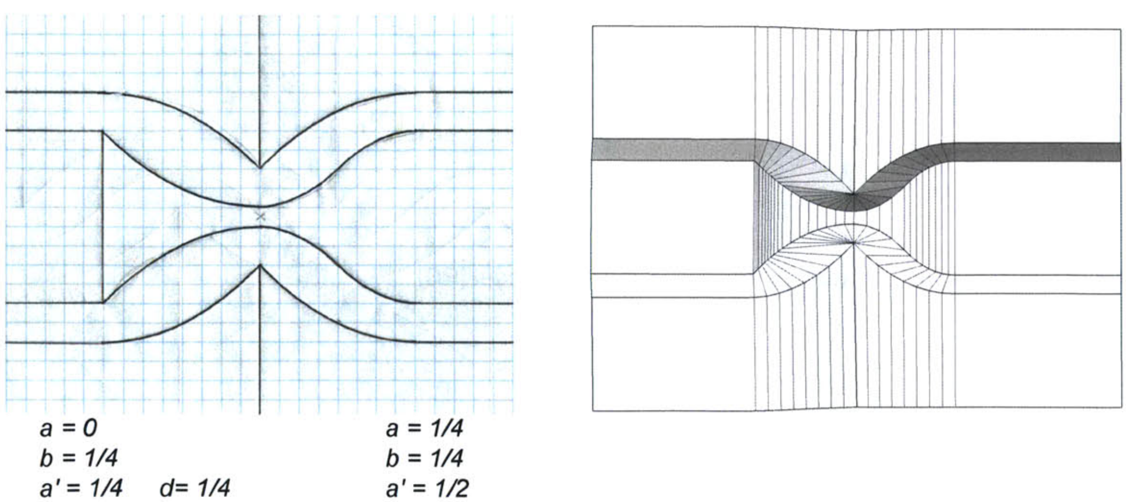


Fig 4.4.141 Paper model (undated, DAH [DK]), Simulated model [PC]

Gadgets with combined quadratic splines and line segments

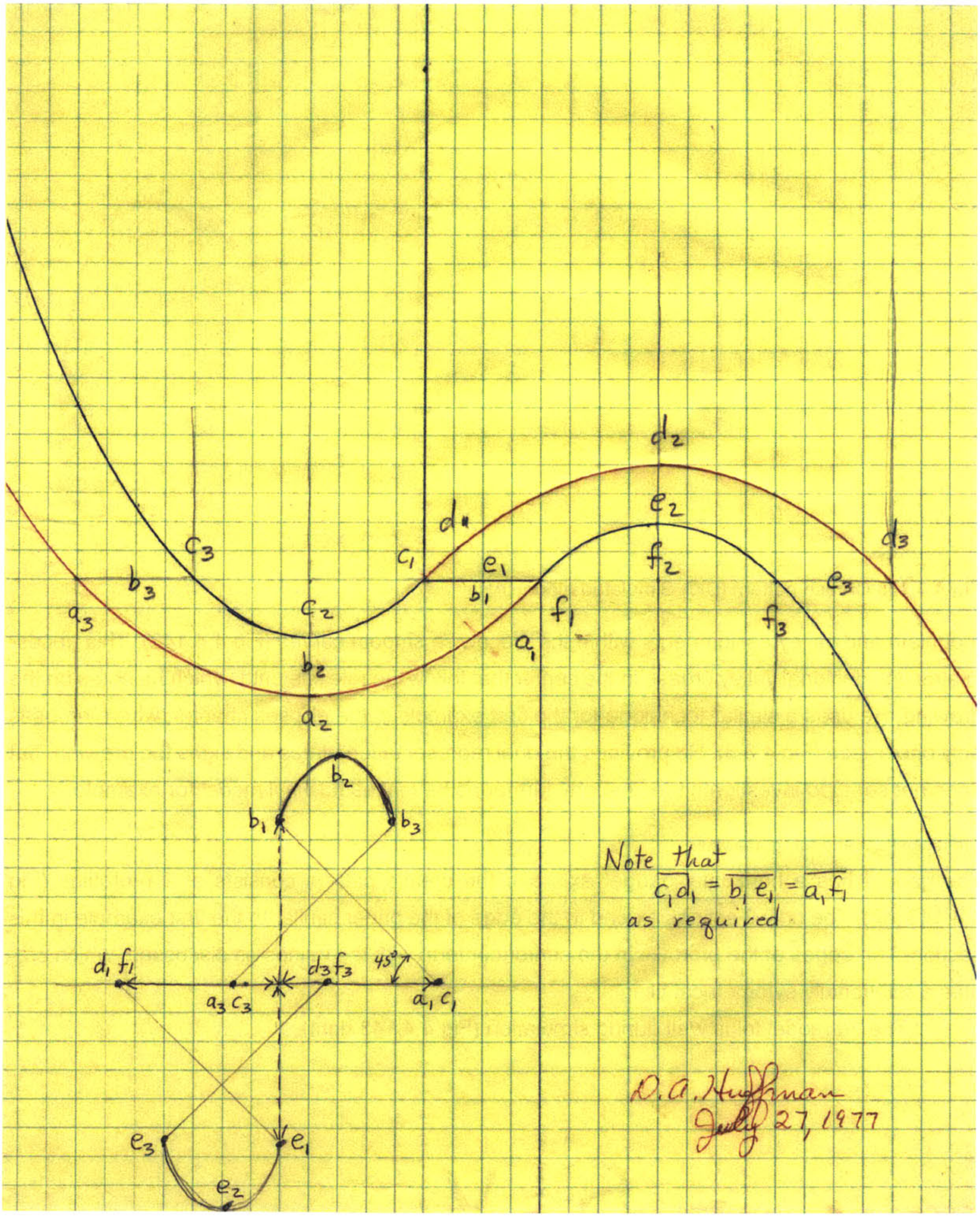


Fig 4.4.142 Paper model (1977, DAH [DK])

This short section presents splines with several curve types and different mountain and valley assignments. Huffman deploys 2 parabolic splines in the above design, but decides on alternating

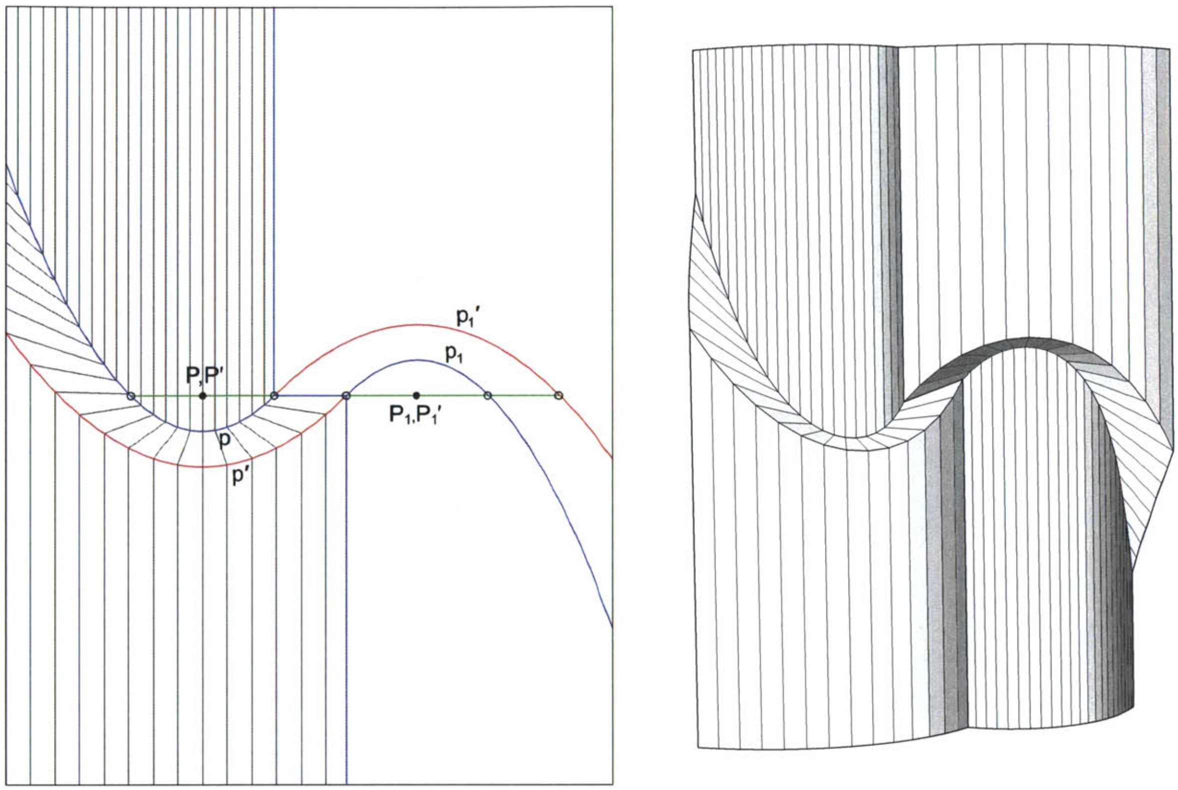


Fig 4.4.143 Crease pattern [DK], Simulated model [AH]

mountain and valley assignments within the piecewise smooth curves (Fig 4.4.142). This necessitates an additional valley crease in the center that follow the rule line path, which he notes on the drawing. He uses a similar technique for the first example in the ellipses chapter, where he uses only one ellipse (Fig 4.3.3). He provides the dual representation, dates and signs the drawing, but does not seem to investigate the idea any further as he makes no vinyl model for example.

Crease pattern and ruling analysis

The gadget has a horizontal gadget axis and the crease pattern consists of 2 prototiles (Fig 4.4.143 left). The only 2 splines extend to the edge of the paper similar to the first example in this section. The edges of the prototile in the center conform with the rulings as discussed, which creates a rotationally symmetrical tiling .

The 3d model folds well during simulation (Fig 4.4.143 right).

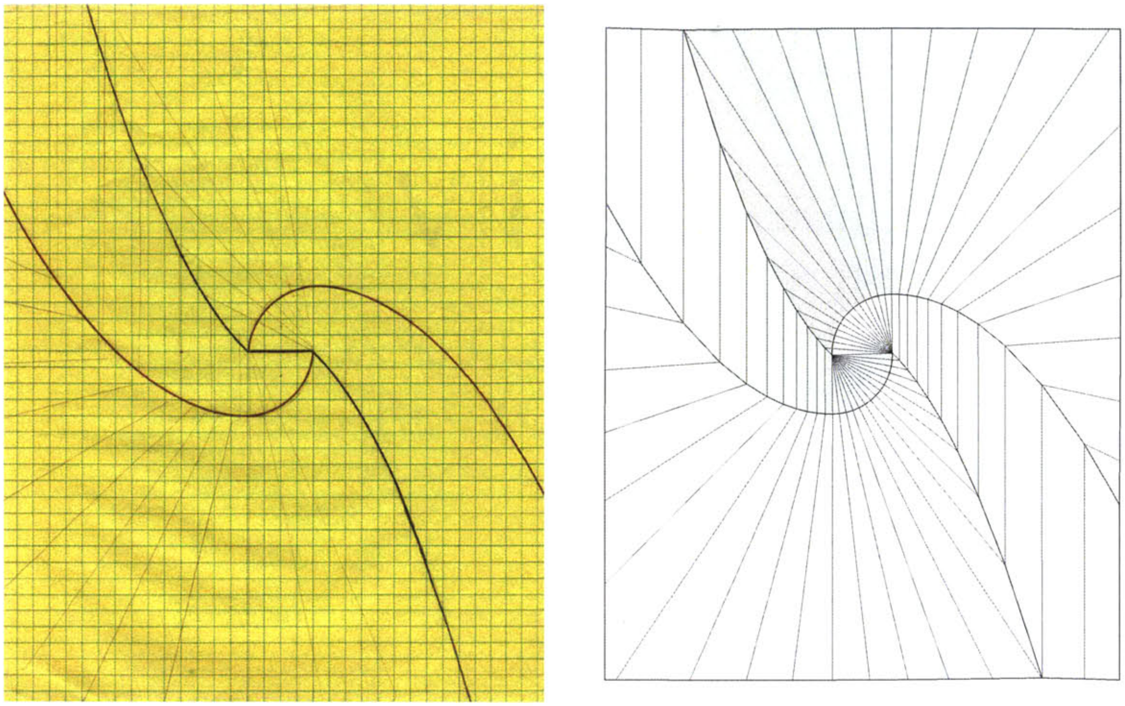


Fig 4.4.144 Paper model (undated, DAH [DK]), Simulated model [JH]

The following 2 designs use mixed splines made of arcs and parabolas. The tilings are similar to the previous design, but one of the parabolas in the prototile is extended with a circle arc.

Crease pattern and ruling analysis

2 prototiles create the above crease pattern (Fig 4.4.144). The rulings start at the foci of the parabolas and remain straight when crossing the arcs. The rulings between the *p* and *p'* are parallel, which is unusual. They get refracted to a general cone on either side.

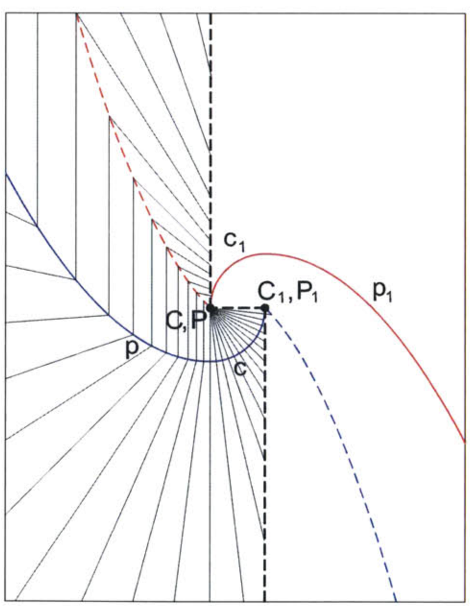


Fig 4.4.145 Crease pattern [DK]

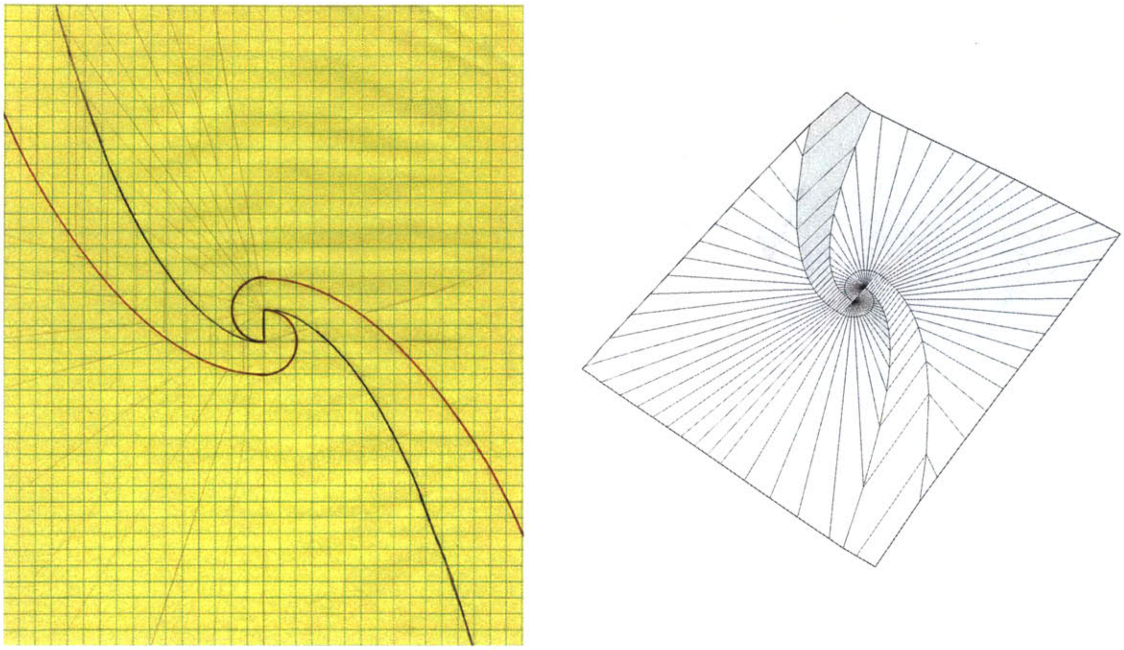


Fig 4.4.146 Paper model [DAH], Simulated model [JH]

Crease pattern and ruling analysis

The second example is also a rotational tiling with 2 prototiles (Fig 4.4.146). The parabolic mountain crease is extended with an arc. The straight crease connects the 2 foci vertically in this case.

The rulings behave in a very similar way to the previous example, but the model does not simulate well (Fig 4.4.146 right).

These example conclude the section on gadgets with parabolas and I discuss gadgets with hyperbolas in the following numbered section.

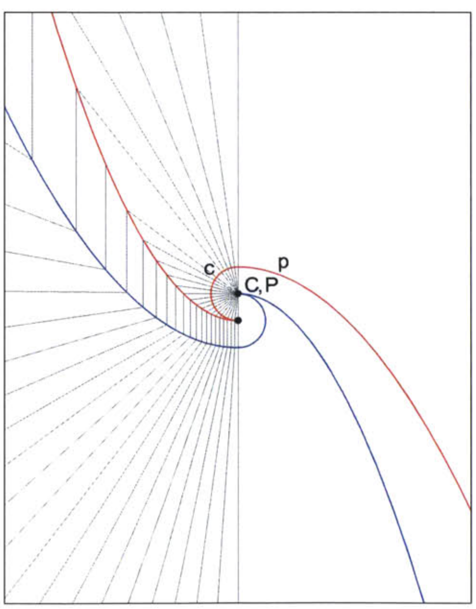


Fig 4.4.147 Crease pattern [DK]

4.5 Gadgets with hyperbolas

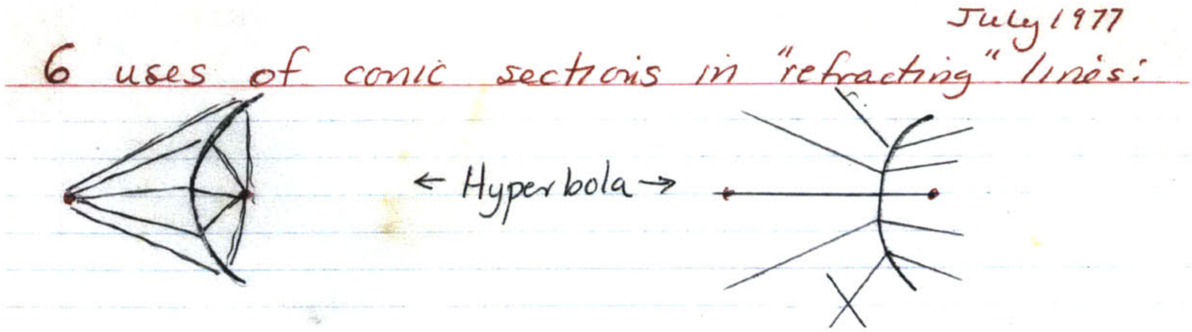


Fig 4.5.1 Index card (1977, [DAH] [DK])

This section investigates designs by Huffman that are based on hyperbolic curves, which are relatively rare in his repertoire. We can see the basis of the gadget on a detail of the same index card used in the introductions for the parabola and ellipse sections in (Fig 4.5.1). Huffman exploits ray refraction on a hyperbola in 2 ways. In the left diagram the rays start at the left focus and get reoriented toward the right focus, whereas in the right diagram the rays fall on extensions through the respective foci.

The gadgets redirect rulings such that they can only form conical surfaces, which limits their tiling possibilities (Fig 4.5.2). The tile boundaries of the left gadget usually fall on rulings. The right gadget finds use in pleated designs.

Huffman uses hyperbolas in the following design (Fig 4.5.3), which he draws with a red pen. The design seems incomplete at first as the left side of the page is blank, but the ruling analysis reveals that this area turns into a hardly curved conical surface.

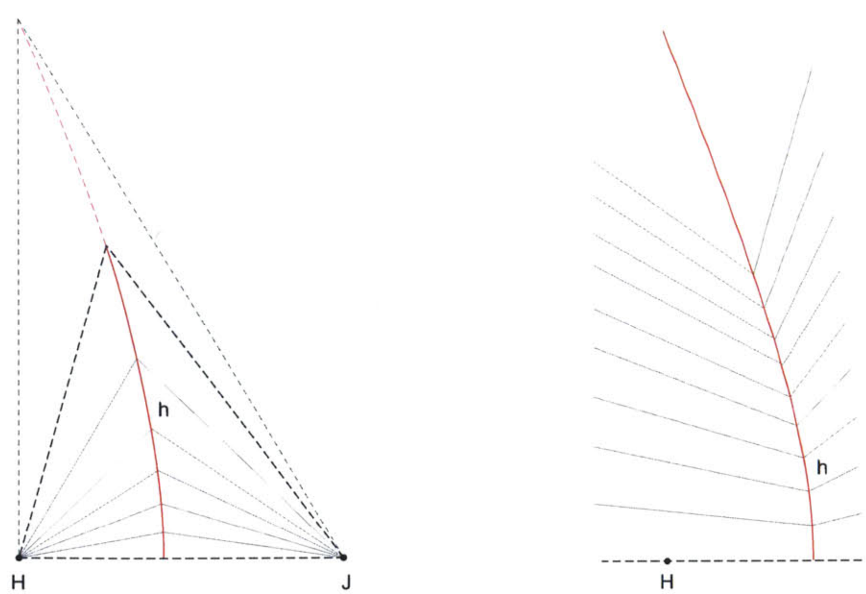


Fig 4.5.2 Gadgets with hyperbolas [DK]

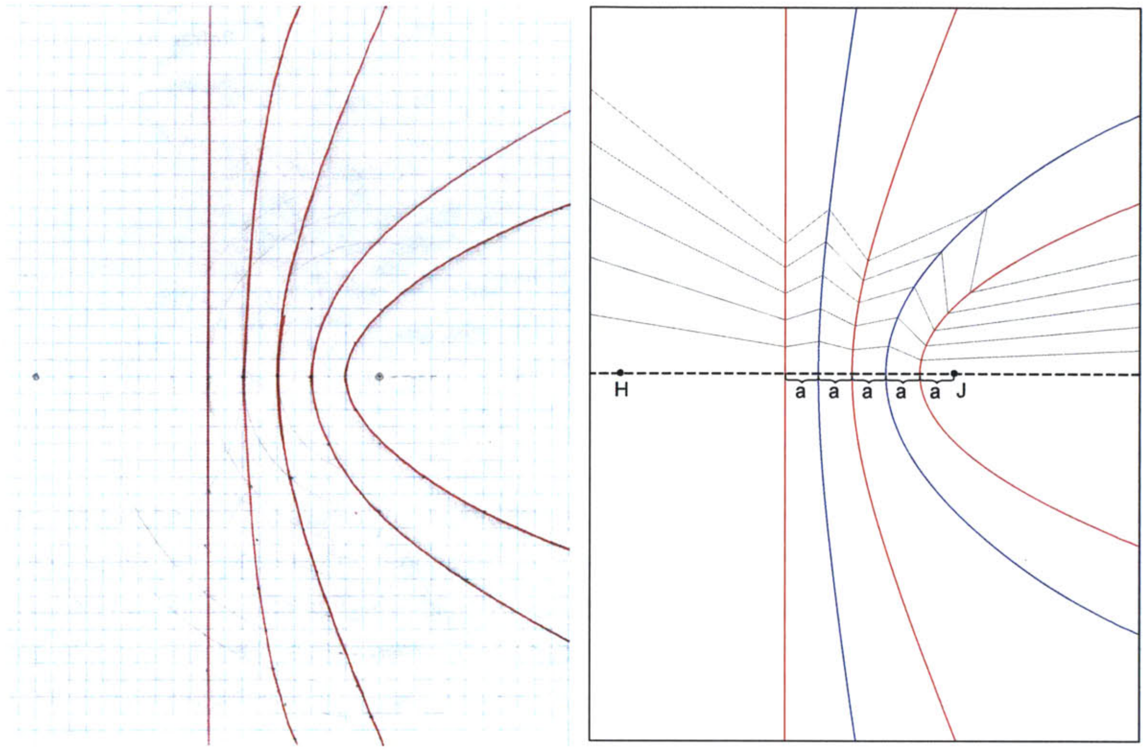


Fig 4.5.3 Paper model (undated, [DAH] [DK]), Crease pattern [DK]

Crease pattern and ruling analysis

Huffman pleats 4 hyperbolas (Fig 4.5.3) by starting with a straight mountain crease in the center and constructing the hyperbolas with concentric circles in pencil. He separates them equidistantly by *a* and alternates their mountain and valley assignments. The tiling consists of 2 prototiles that are the gadget. The rulings form 2 cones on the far left and far right of the crease pattern, which is consistent with the ruling directions of second gadget.

The model folds reasonably well during simulation and the straight crease folds with a large folding angle (Fig 4.5.4).

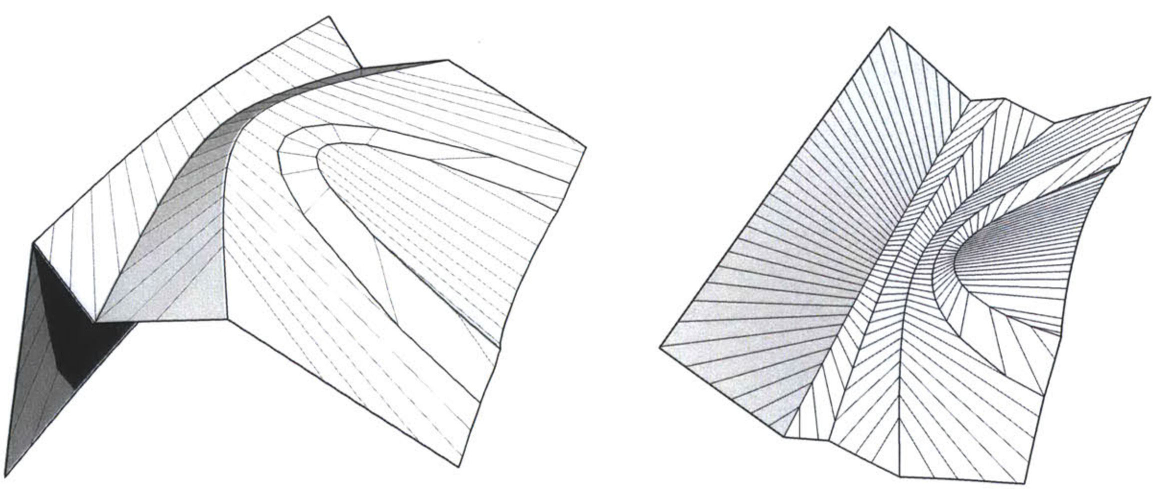


Fig 4.5.4 Simulated model, top and bottom view [JH]

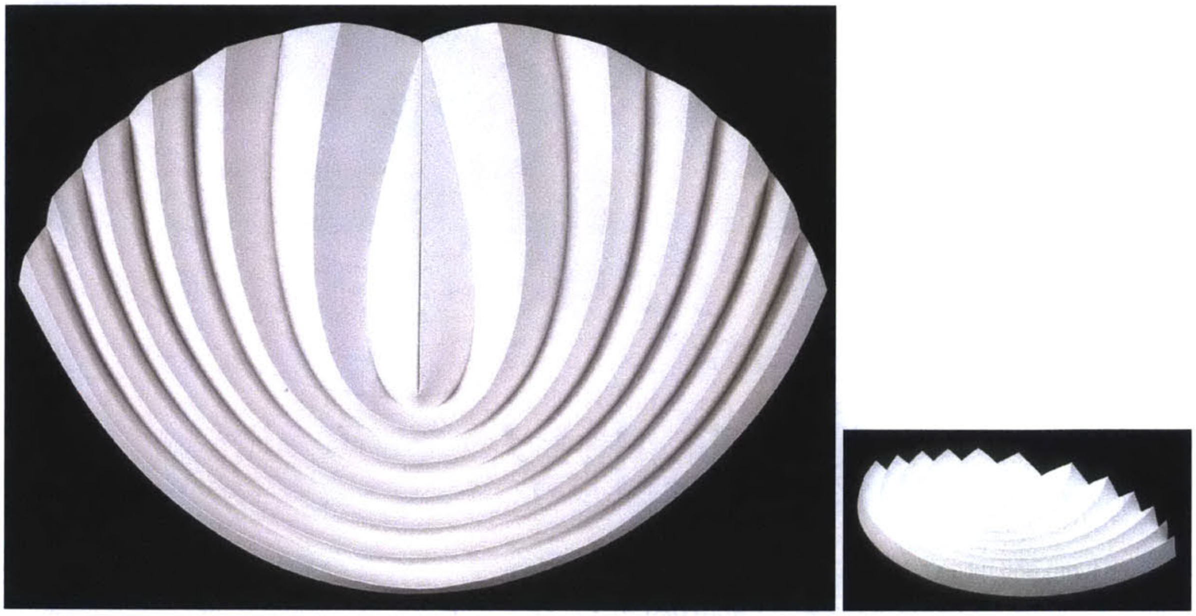


Fig 4.5.5 Vinyl model (1998, [DAH] [TG]), Identical model (1998, [DAH] [EAH])

The above design which the family calls 'Angel wings' is included in this section as my visual reconstruction assumes the use of pleated hyperbolas (Fig 4.5.5). Huffman appears to have no drawing, note or sketch model of the design beside the vinyl model, which is unusual. The design has strong similarities to a design with pleated parabolas, but is most likely not based on parabo-

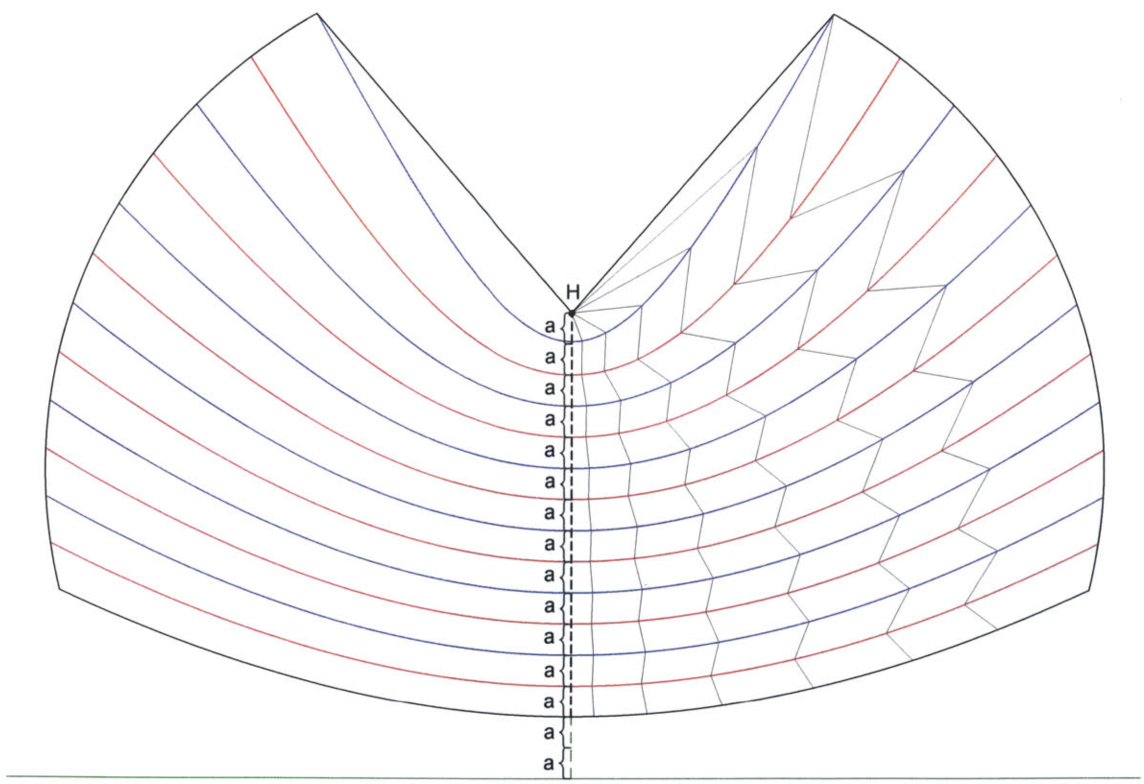


Fig 4.5.6 Crease pattern [DK]

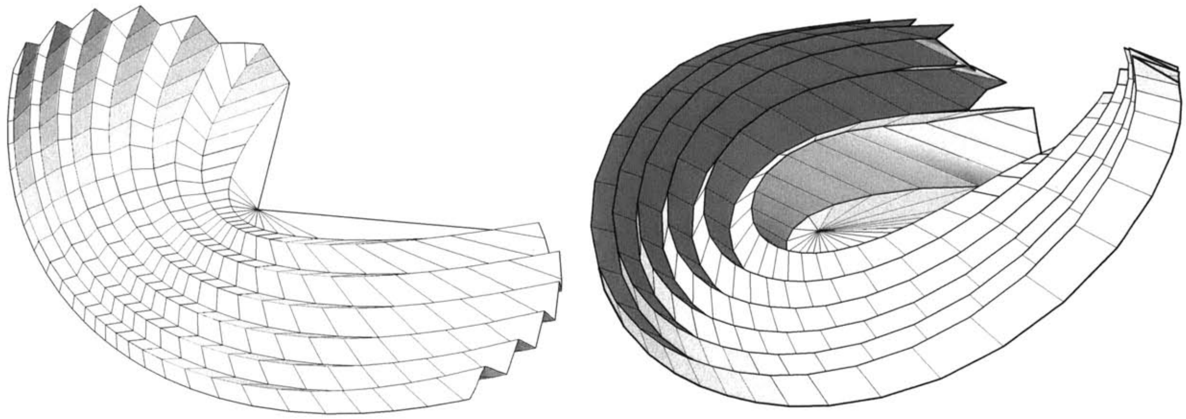


Fig 4.5.7 Simulated models [JH]

las (Fig 4.4.13).

Crease pattern and ruling analysis

Huffman pleats 12 hyperbolas with equal distances *a* and alternating mountain and valley assignments. He decides to omit the 2 curves closest to the hyperbola axis drawn in green. The outline of the cut-out consists of 3 parts and the v-shaped straight edges give a clue to which gadget he uses. The 2 straight edges coincide with rulings which remain straight in the folded model. The cut along the bottom curve appears to consist of the second hyperbolic curve of the set of pleats. The remaining cut that traverses the creases is most likely a circle arc.

The resulting tiling is similar to the previous example except that is uses the logic of the left gadget, but the last conical surface fans out.

Significant pressure must be applied to keep a paper model in the configuration Huffman uses. Simulation suggests that the model is stable in the configuration in (Fig 4.5.7) on the left, but stitching points together in software allows further folding.

Notes

Huffman mounts the model on a wooden plate and orients the model the way Tony Grant photographs it (Fig 4.5.5). The design by Ogawa may have served as inspiration for Huffman, but its crease pattern is significantly different (Fig 4.5.8).

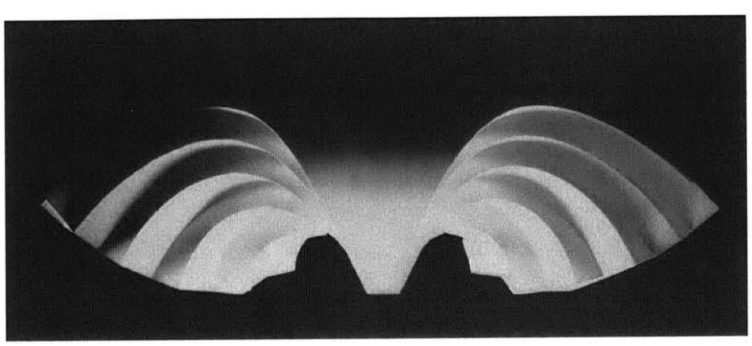


Fig 4.5.8 Figure in Forms of Paper (1971, H. Ogawa)

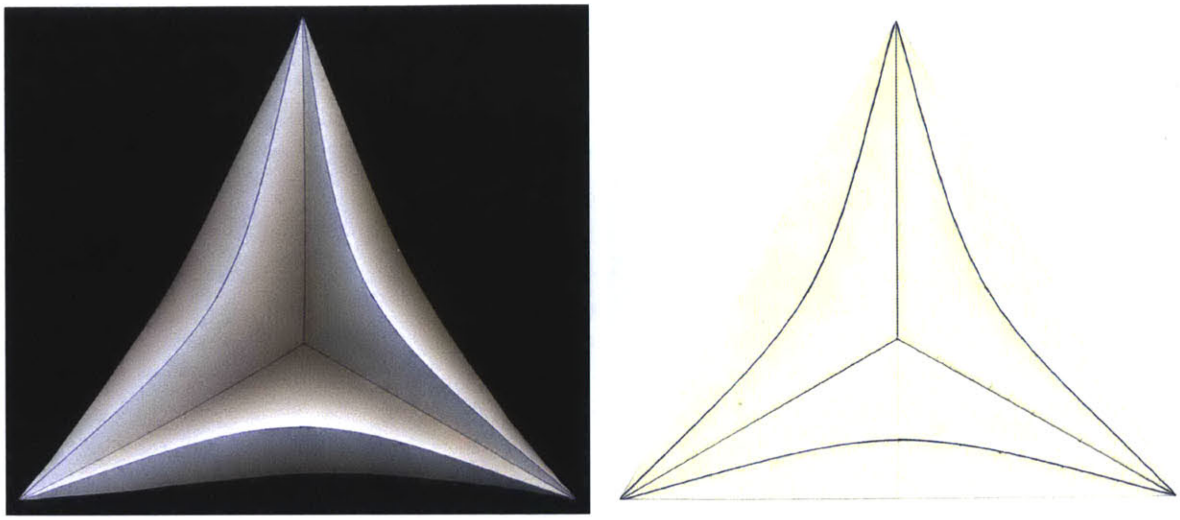


Fig 4.5.9 Paper model (undated, [DAH] [DK]), Identical model (undated, [DAH] [DK])

We now arrive at the last model in this section, which consists of a tiling with the left hyperbola gadget (Fig 4.5.2). Huffman derives the formula for a 3-pronged hyperbola in polar coordinates (Fig 4.5.10), but he decides to approximate the asymptotes in the design as the creases intersect in a single vertex.

Crease pattern and ruling analysis

The triangular configuration relies on 3 confocal hyperbolas (Fig 4.5.11). The crease pattern is comprised of 3 straight valley folds and 3 partial hyperbolas as mountains. The second foci of the

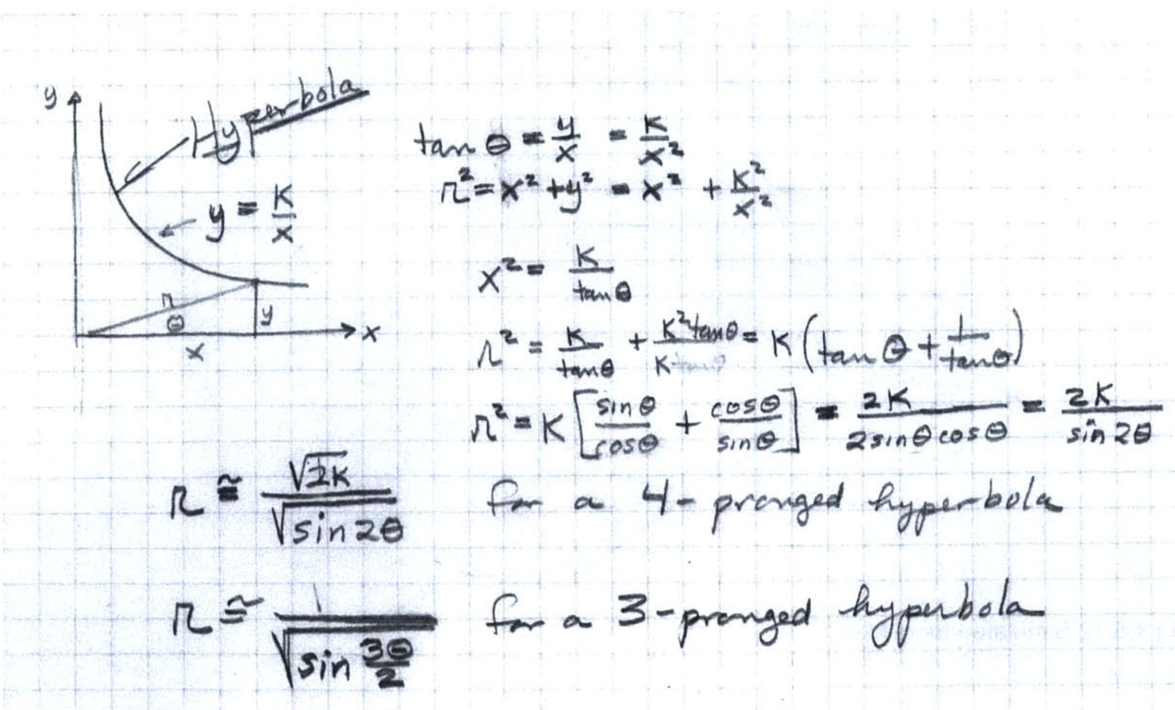


Fig 4.5.10 Note (undated, [DAH] [DK])

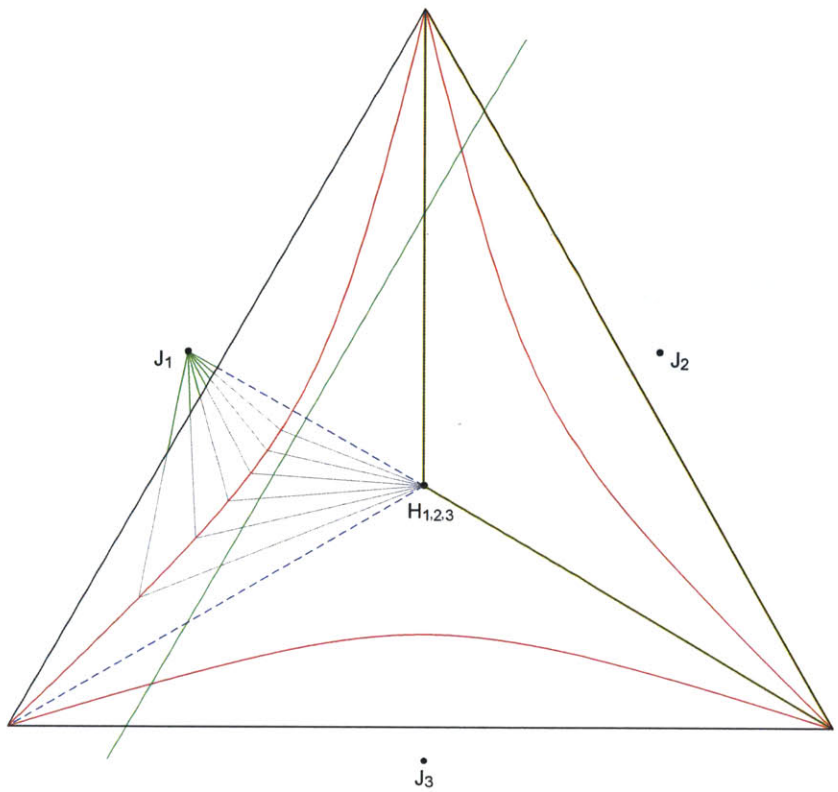


Fig 4.5.11 Crease pattern [DK]

hyperbolas J_1, J_2 and J_3 are not on the boundary of the crease pattern. The design tile consists of 2 prototiles.

The model simulates well (Fig 4.5.12).

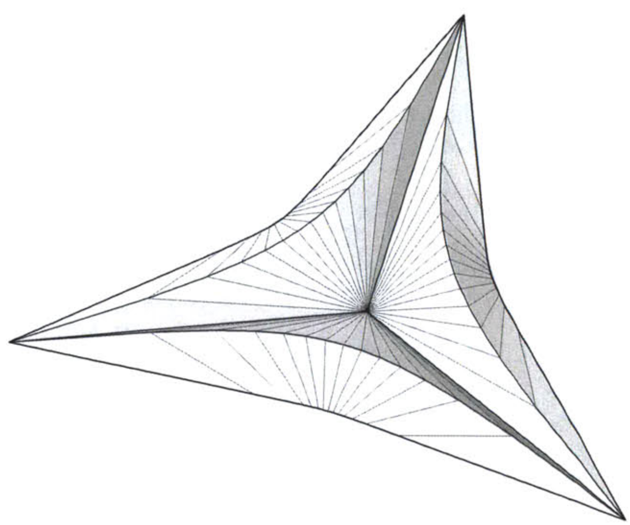


Fig 4.5.12 Simulated model [JH]

4.6 Gadgets with parabolas and ellipses

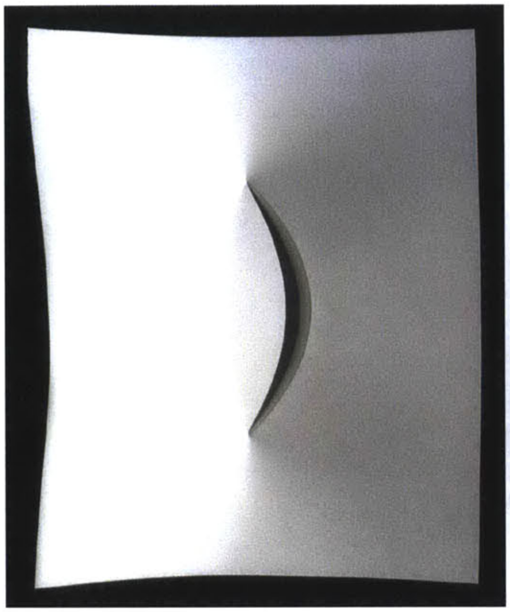


Fig 4.6.1 Vinyl model (undated, DAH [EAH]), Vinyl model (undated, DAH [EAH])

This section discusses designs with parabolas and ellipses that have a symmetry axis along the vertical and horizontal center lines. The first 2 vinyl models (Fig 4.6.1) can only be reconstructed visually as Huffman keeps no sketches. He most probably had parabolas and ellipses in mind similar to the crease pattern below (Fig 4.6.2). The gadget starts with horizontal rulings that refract at the parabola and then again at the confocal ellipse. It is unclear, if the necessary smooth transition at the top ruling exists. We can observe the conical area at the top and bottom of the photographs.

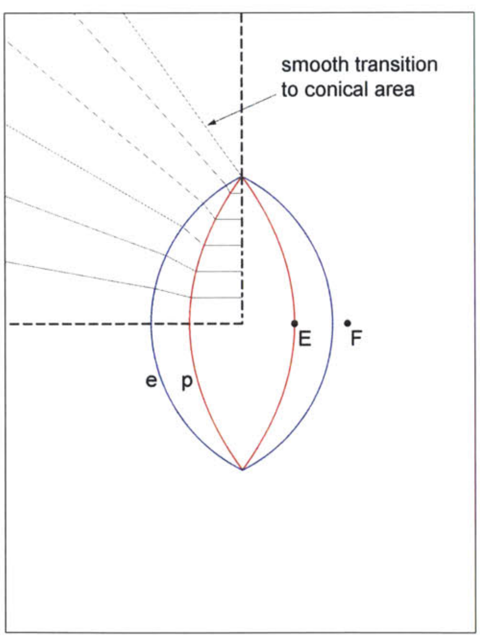


Fig 4.6.2 Crease pattern [DK]

Gadgets with parabolas, ellipses and line segments

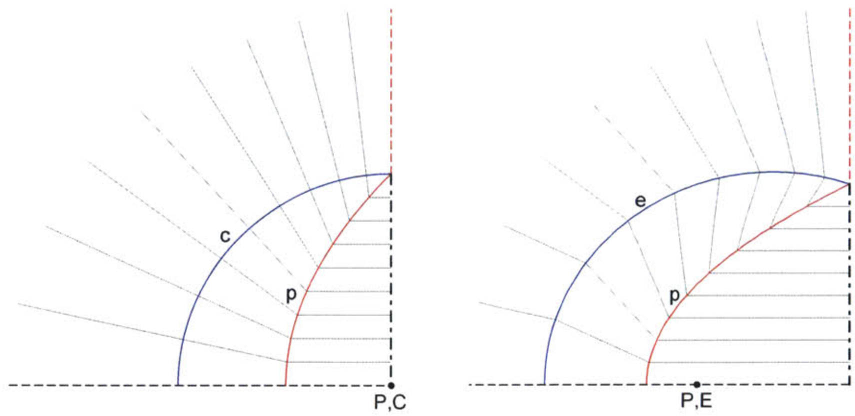


Fig 4.6.3 Gadgets [DK]

Huffman adds line segments to the previous gadget along the vertical center line for the examples in this section. The intersecting parabolas remain rotated with a horizontal axis of symmetry and circles or ellipses are added as outer creases. He assumes the rulings to be horizontal in the center and hence obtains a cylindrical surface. The central cylinder is common to all examples in this series.

The new line segment, used as mountain crease, gives the designs another characteristic visual feature, which renders them reminiscent of designs in 'Ellipses and tucking' (Fig 4.6.3). The 2 triangles in the center are replaced with the mentioned cylinder.

The gadgets

The left gadget (Fig 4.6.3) consists of a parabola and a confocal circle. The parallel rulings in the center produce a cylinder. Once they pass through the parabola, they form a general cone and keep their direction when passing through the circle.

The second gadget on the right (Fig 4.6.3) operates in a similar way, but the ellipse requires the rulings to change direction. The mountain and valley assignment is identical.

The straight mountain crease aligns with the rule line through the center of the circle or through one of the foci of the ellipse.

The first design uses the gadget with a circle and Huffman only makes a paper version (Fig 4.6.4). A single vertex for all foci defines this special case, which is followed by examples with ellipses.

Introducing nomenclature for a few distances (Fig 4.6.5 and 4.6.7) helps to distinguish Huffman's 5 variations (Fig 4.6.8). The distance between the focus of the parabola *P* and the intercept of the curve on the main axis *p* is called *a*. The distance from *P* to the vertex on the x-intercept is *l*. It is also half of the latus rectum, which is oriented sideways here. The distance between the two foci of the ellipse is *f*.

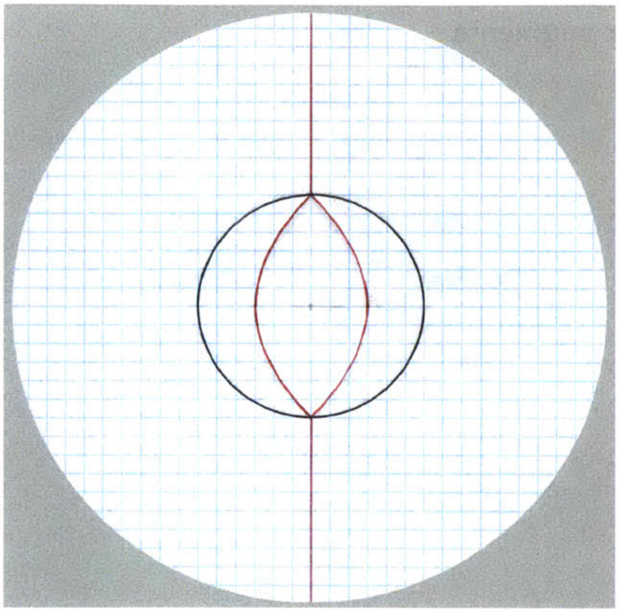
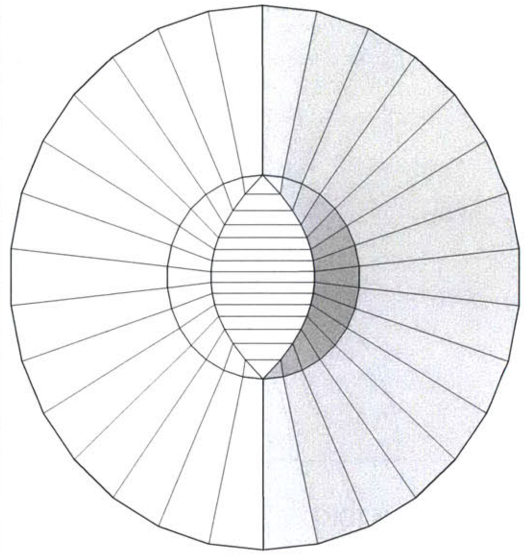
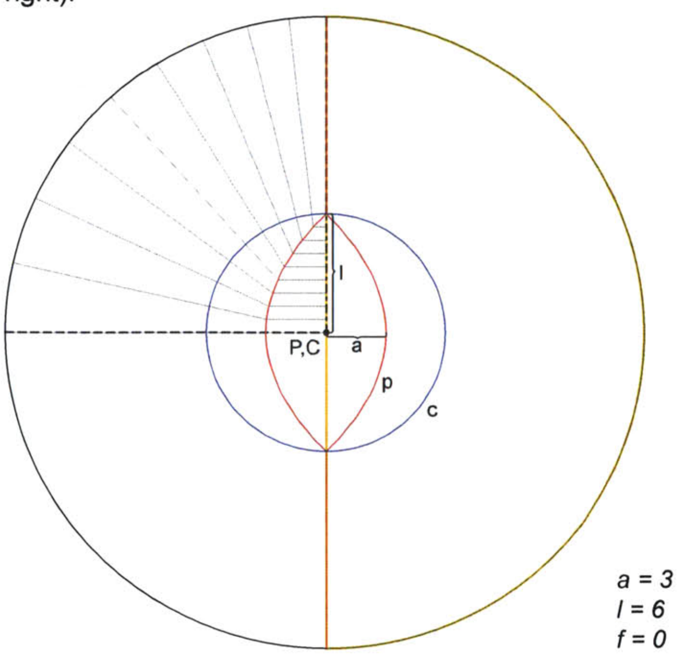



Fig 4.6.4 Paper model (undated, DAH [DK]), Simulated model [AH]

Crease pattern and ruling analysis

The parabola, drawn sideways, extends to its latus rectum and is a mountain crease (Fig 4.6.5). The confocal circle has the latus rectum of the parabola as its diameter and is a valley crease. The crease pattern consists of 4 prototiles and 2 design tiles. Huffman uses a circular cut-out for most examples in this series.

The rule lines on the vertical axis double as the tile edge. The model folds reasonably well during simulation, however, the straight mountain crease has a small folding angle (Fig 4.6.4 right).



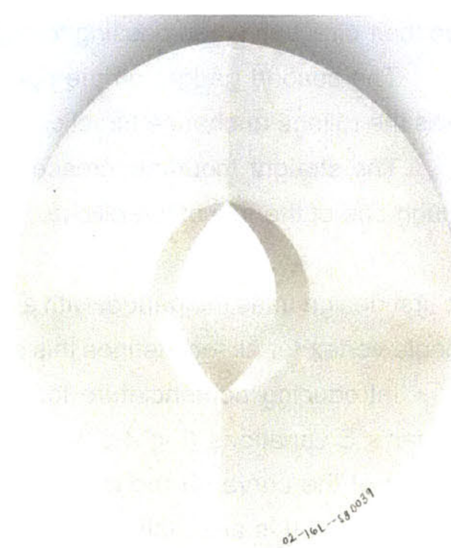


Fig 4.6.5 Crease pattern [DK], Paper reconstruction [AH]

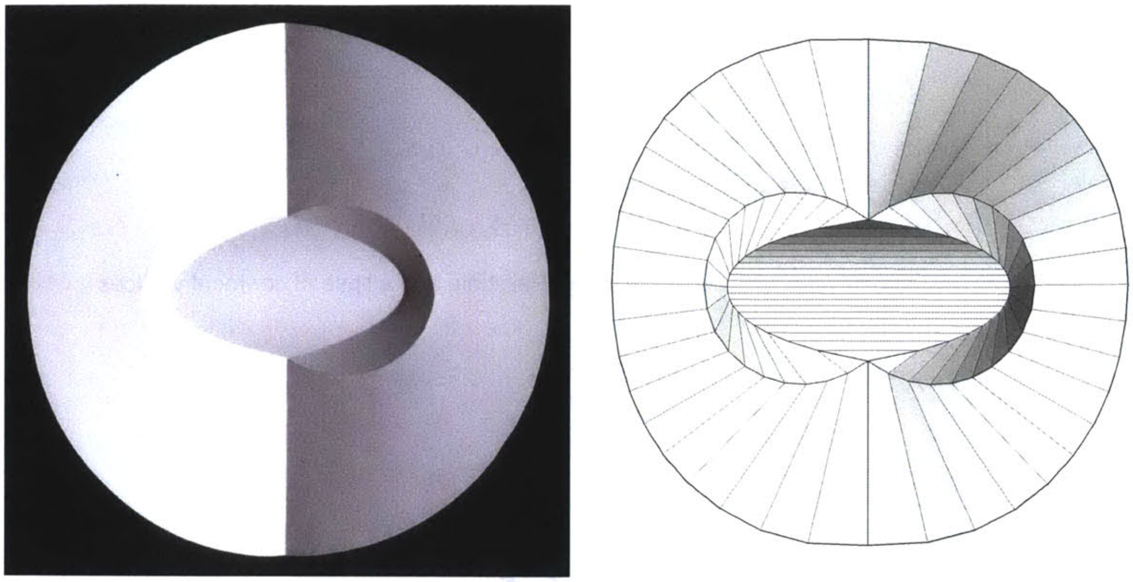


Fig 4.6.6 Vinyl model (undated, DAH [TG]), Simulated model [AH]

Huffman exchanges the circle with 2 ellipses and decides to make a make a vinyl model (Fig 4.6.6 left).

Crease pattern and ruling analysis

The parabolic mountain crease, drawn sideways, extends further than its latus rectum (Fig 4.6.7 right). The confocal elliptic valleys defined by *f* use a total of 3 vertices. The 4 prototiles and 2 design tiles have similar properties to the previous design and Huffman uses a circular cut-out again.

The rulings change direction twice, and the vertical rule lines double as tile edges again. The simulated model does not fold in the same way as the paper model as the folding angle along the vertical mountain crease is different (Fig 4.6.6 right).

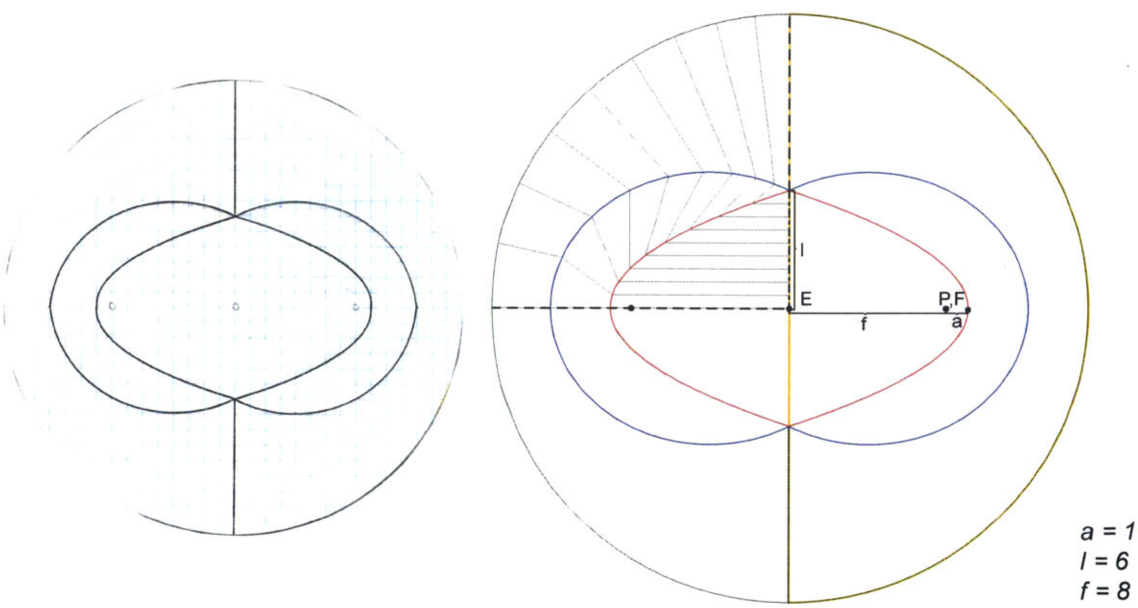
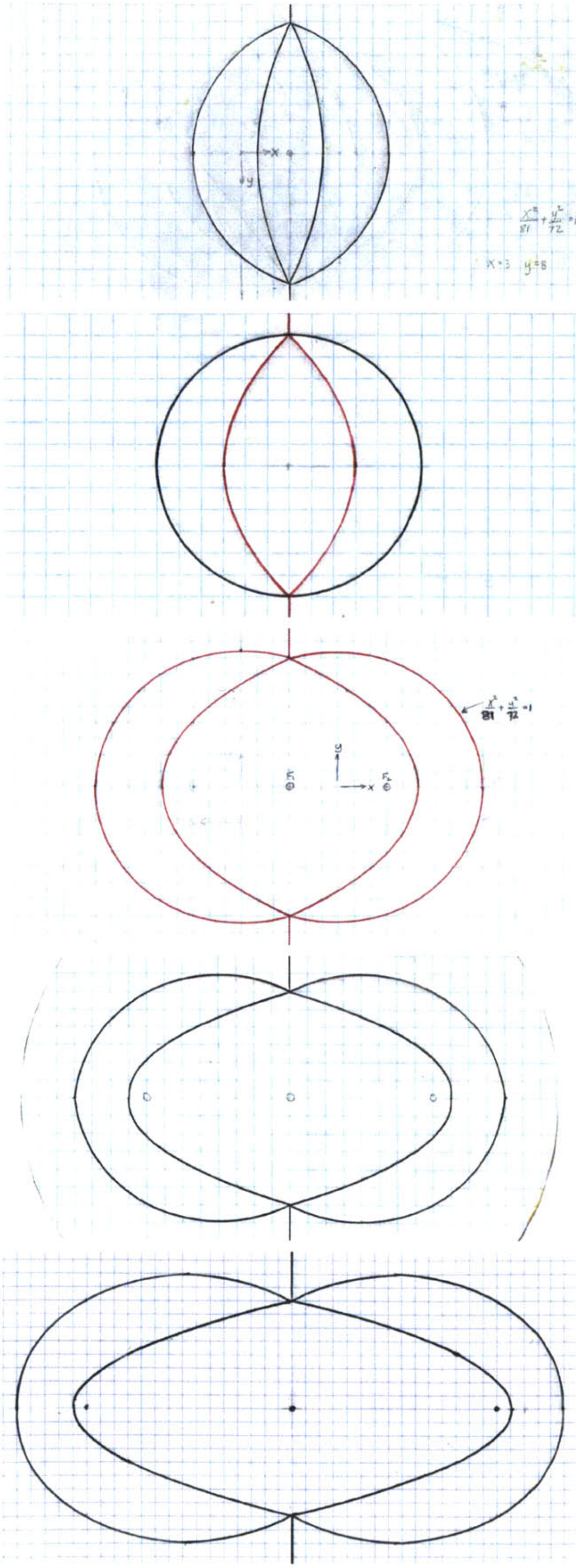


Fig 4.6.7 Paper model (undated, DAH [DK]), Crease pattern [DK]



a = 2 [2]
l = 8 [8]
f = -6 [-6]

Parabola and ellipse, 3 confocal vertices

a = 3 [4]
l = 6 [8]
f = 0 [0]

Parabola and circle, 1 confocal vertex
(Fig 4.6.3)

a = 2 [2]
l = 8 [8]
f = 6 [6]

Parabola and ellipse, 3 confocal vertices

a = 1 [4/3]
l = 6 [8]
f = 8 [32/3]

Parabola and ellipse, 3 confocal vertices
(Fig 4.6 5)

a = 1 [1]
l = 8 [8]
f = 15 [15]

Parabola and ellipse, 3 confocal vertices

Fig 4.6.8 Paper models (undated, DAH [DK])

4.7 Gadgets with parabolas, circles and ellipses

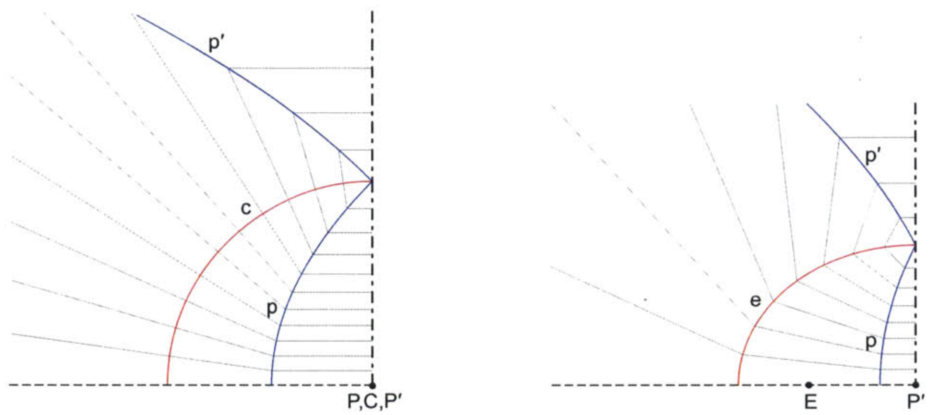


Fig 4.7.1 Gadgets [DK]

This section discusses gadgets with 3 curves. In some cases parabolas are combined with circles and in other cases they are combined with ellipses.

Huffman expands on the designs of the previous section and replaces the straight crease along the vertical axis with a pair of parabolas. The crease pattern of first design in this section is defined by a single circle and 2 parabolas (Fig 4.7.2 left). Later examples in the series are comprised of 2 circles and 2 parabolas.

Most simulations do not remain in a 3d configuration and revert to their initial flat state. I present paper reconstructions in this section in their place.

The gadgets

Huffman's initial assumption or base case regarding the gadget on the left consists of starting with a general cylinder in the center (Fig 4.7.1 left), similar to the gadget in the previous chapter. The inner parabola *p* redirects the rulings such that they form a conical surface. They remain uninterrupted when traversing the circle *c*, but change direction at the new parabola *p'* to form a new cylinder.

The gadget on the right consists of 2 parabolas and a confocal ellipse instead of the circle (Fig 4.7.1 right). The rule lines behave mostly the same but change direction when crossing the ellipse *e*.

The following examples will focus on parabolas that are combined with circles.

Gadgets with parabolas and circles

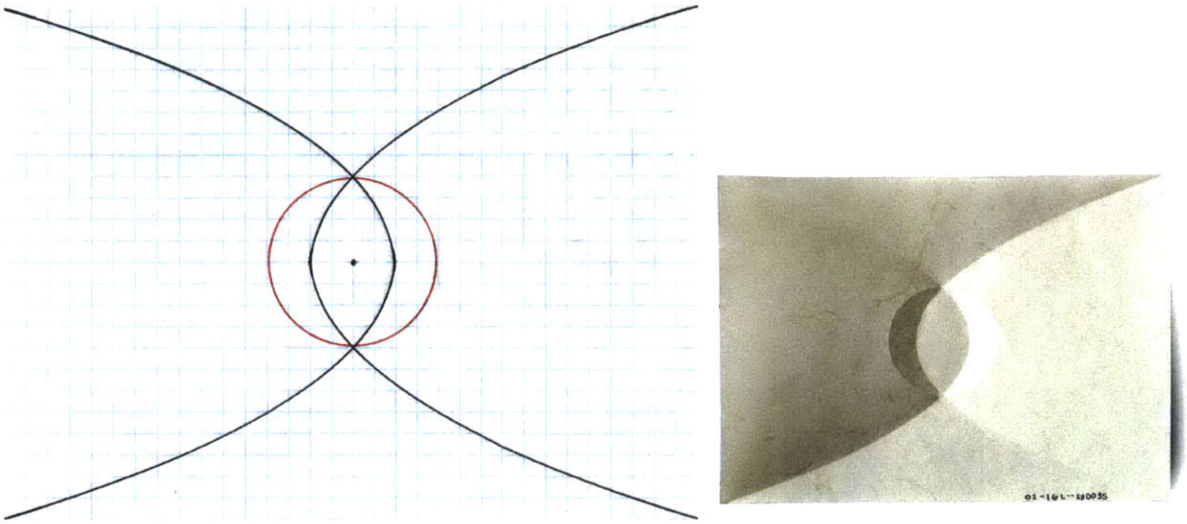


Fig 4.7.2 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The nomenclature for distances in the crease patterns (Fig 4.7.3, 4.7.6 and 4.7.8) has to expand to describe the new curves. The distance between the parabola focus *P* and the vertex on its axis of symmetry is called *a*. Half of the latus rectum is *l*, which is also the radius of the circle in the first example. The distance between *P'* and the vertex on its axis of symmetry is called *a'*. The distance from *P* to the center of the crease pattern, in some cases 0, is called *d*. Distances in square brackets are normalized to *l* = *12*, which allows for easy comparison between Huffman's variations in this series (Fig 4.7.11).

Crease pattern and ruling analysis

The gadget consists of 2 partial parabolas based on a similar curve and 1 circle, which all share

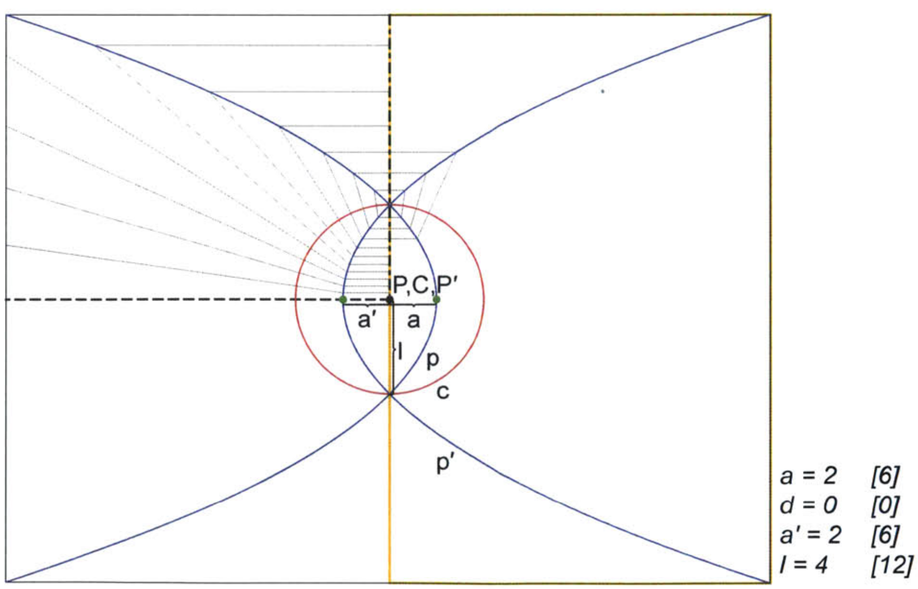
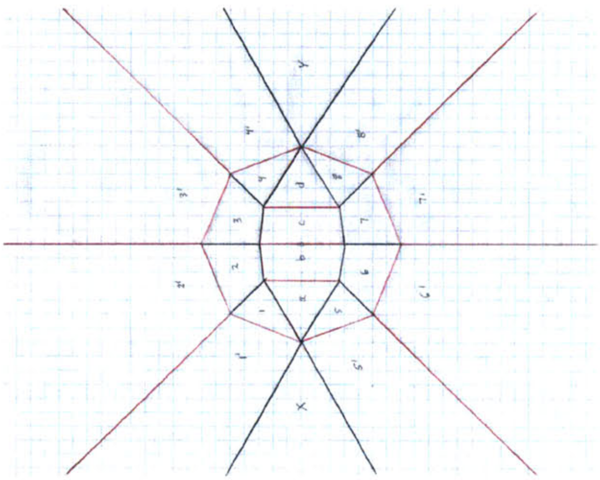


Fig 4.7.3 Crease pattern [DK]

the same vertex as focus (Fig 4.7.3). The prototile, also the gadget in this case, refracts the rulings twice to form the above mentioned cylinders.

The model shows curvature inconsistencies during simulation and quickly resumes to its flat configuration. The paper reconstruction provides visual feedback (Fig 4.7.2 right).

Huffman draws a coarse discrete version of the design, which folds well during simulation (Fig 4.7.4).



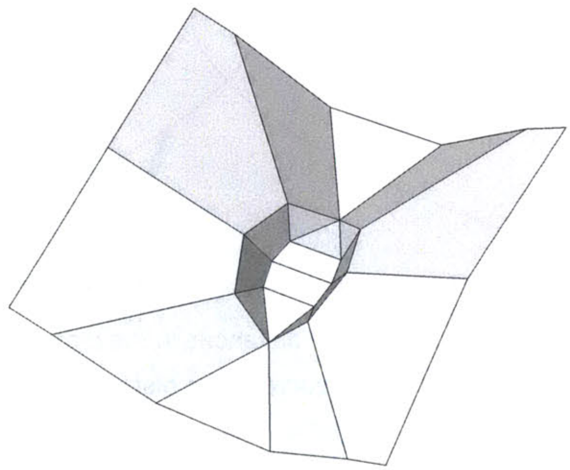


Fig 4.7.4 Paper model (undated, DAH [DK]), Simulated model [PC]

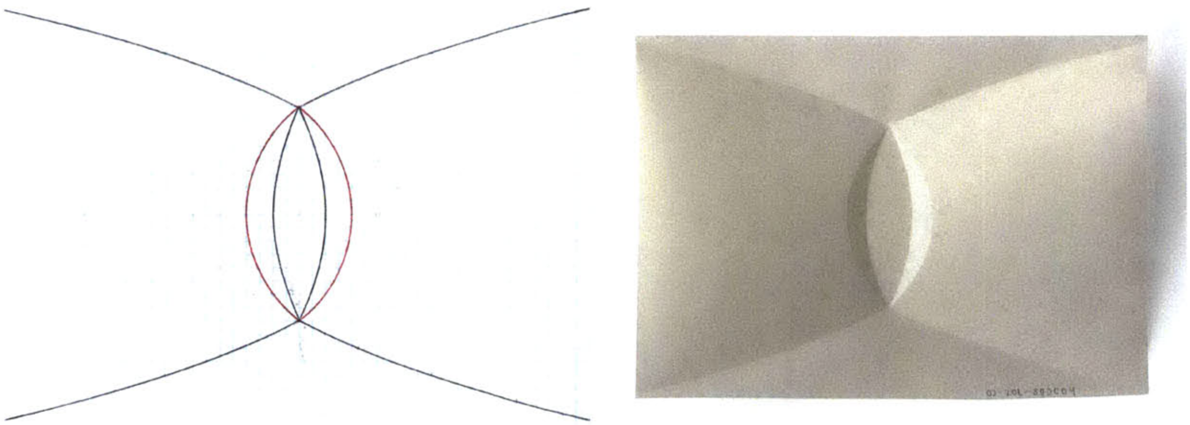


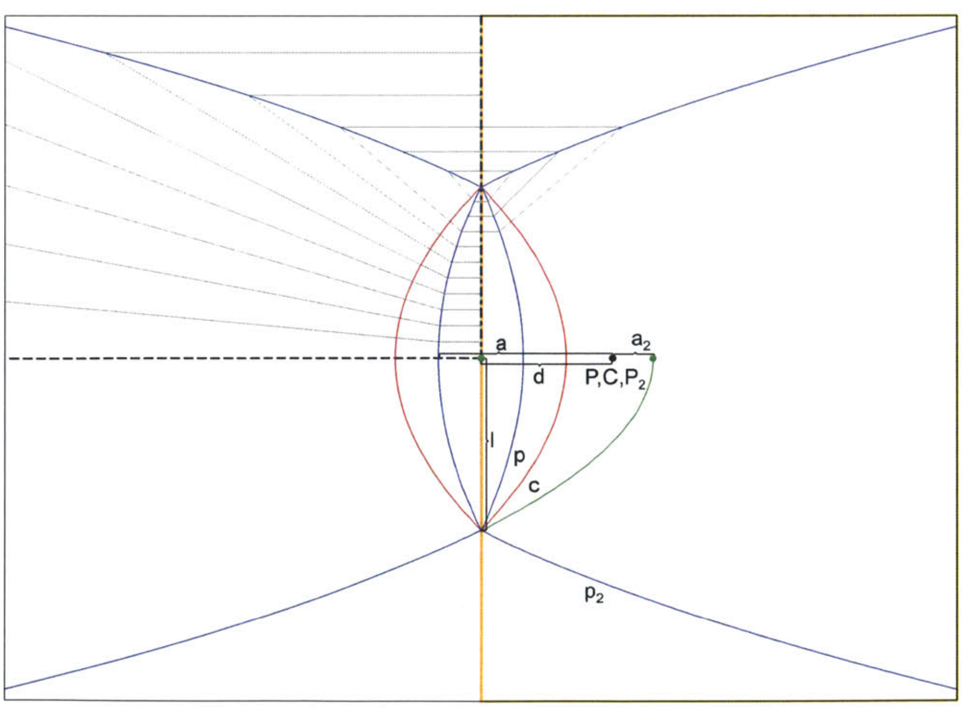
Fig 4.7.5 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The above paper model (Fig 4.7.5) consists of different partial parabolas. The resulting design has short distances between the curves on the horizontal axis of symmetry.

Crease pattern and ruling analysis

The gadget consists of 2 different parabolas and 1 circle, which all share 1 vertex as focus on the opposite side of the vertical axis of symmetry of the crease pattern (Fig 4.7.6). The distances *a*, *d*, a_2 and *l* locate all curves of the 4 prototiles. The rulings refract twice in a cyclic way and form the 2 cylinders that visually characterize the series.

The paper reconstruction shows the folded configuration of Huffman's crease pattern (Fig 4.7.5 right).



a = 8 [12]
d = -6 [-9]
a_2 = 2 [3]
l = 8 [12]

Fig 4.7.6 Crease pattern [DK]

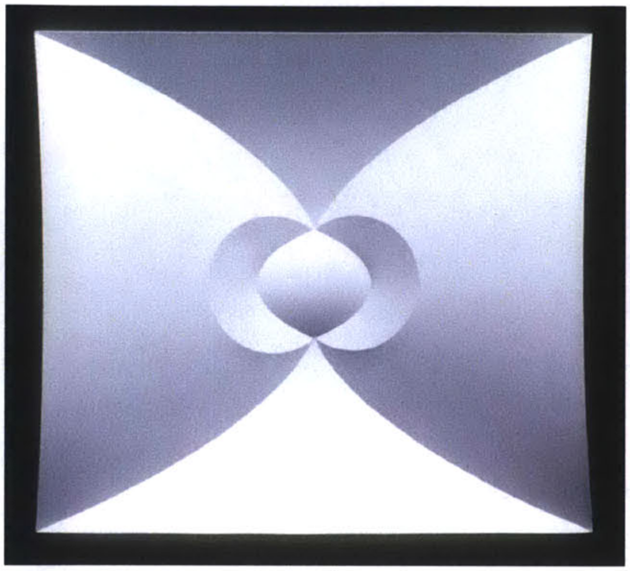


Fig 4.7.7 Vinyl model (1998, DAH [DAH])

Huffman makes the next 2 examples in vinyl and draws identical crease patterns on several kinds of graph paper. Here, 2 parabolas and 2 circles share one vertex (Fig 4.7.7).

Crease pattern and ruling analysis

The gadget consists of 2 different parabolas and 1 circle (Fig 4.7.8). The curves share 1 confocal vertex on the same side of the vertical axis of symmetry of the crease pattern (Fig 4.7.8). The distances a_2 and l are identical. The rulings are cyclic around the degree 6 vertices.

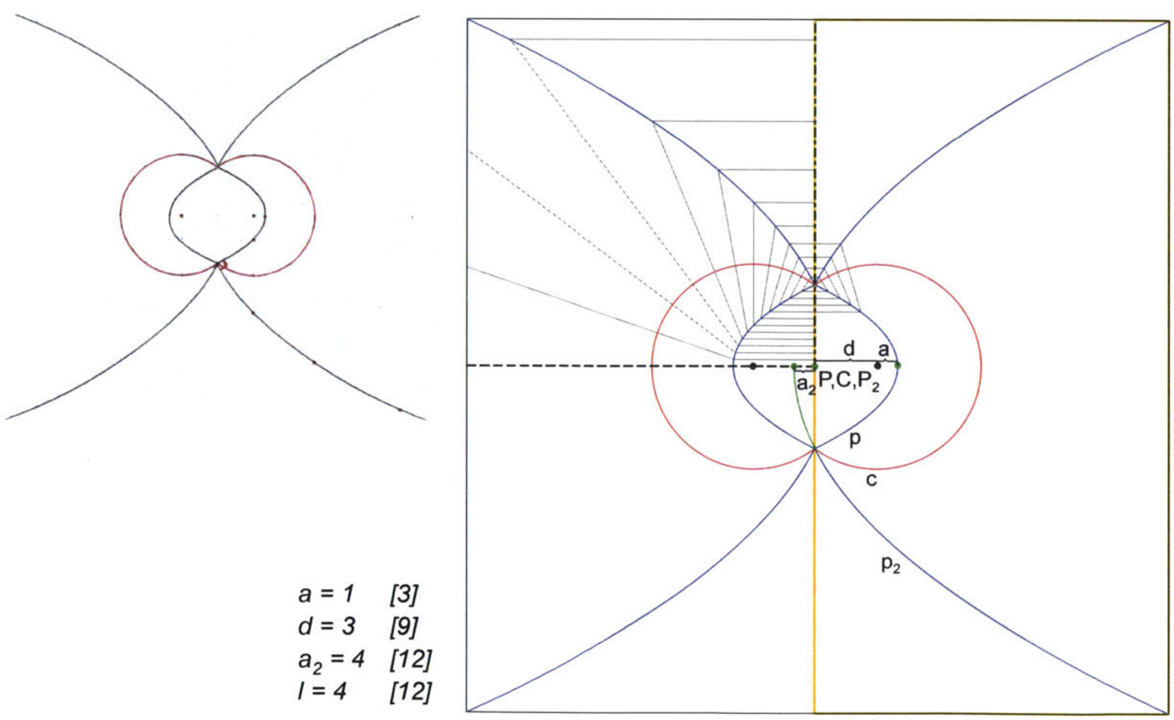


Fig 4.7.8 Paper model (undated, DAH [DK]), Crease pattern [DK]

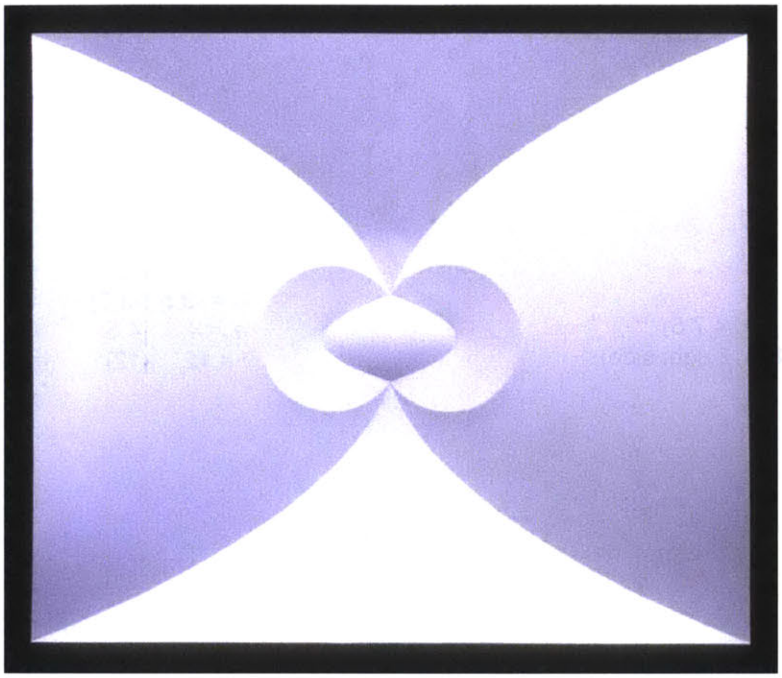


Fig 4.7.9 Vinyl model (undated, DAH [EAH])

The second design made in vinyl consists of very similar curves that create a smaller cylinder at the center. The parabolas and circles in the above photograph of the design share foci on the same side of the vertical axis of symmetry (Fig 4.7.9).

Crease pattern and ruling analysis

The relationships of all curves to their foci (Fig 4.7.10) resemble the previous design. The distances a_2 and l are not identical in this case. The rulings are cyclic similar to the previous examples.

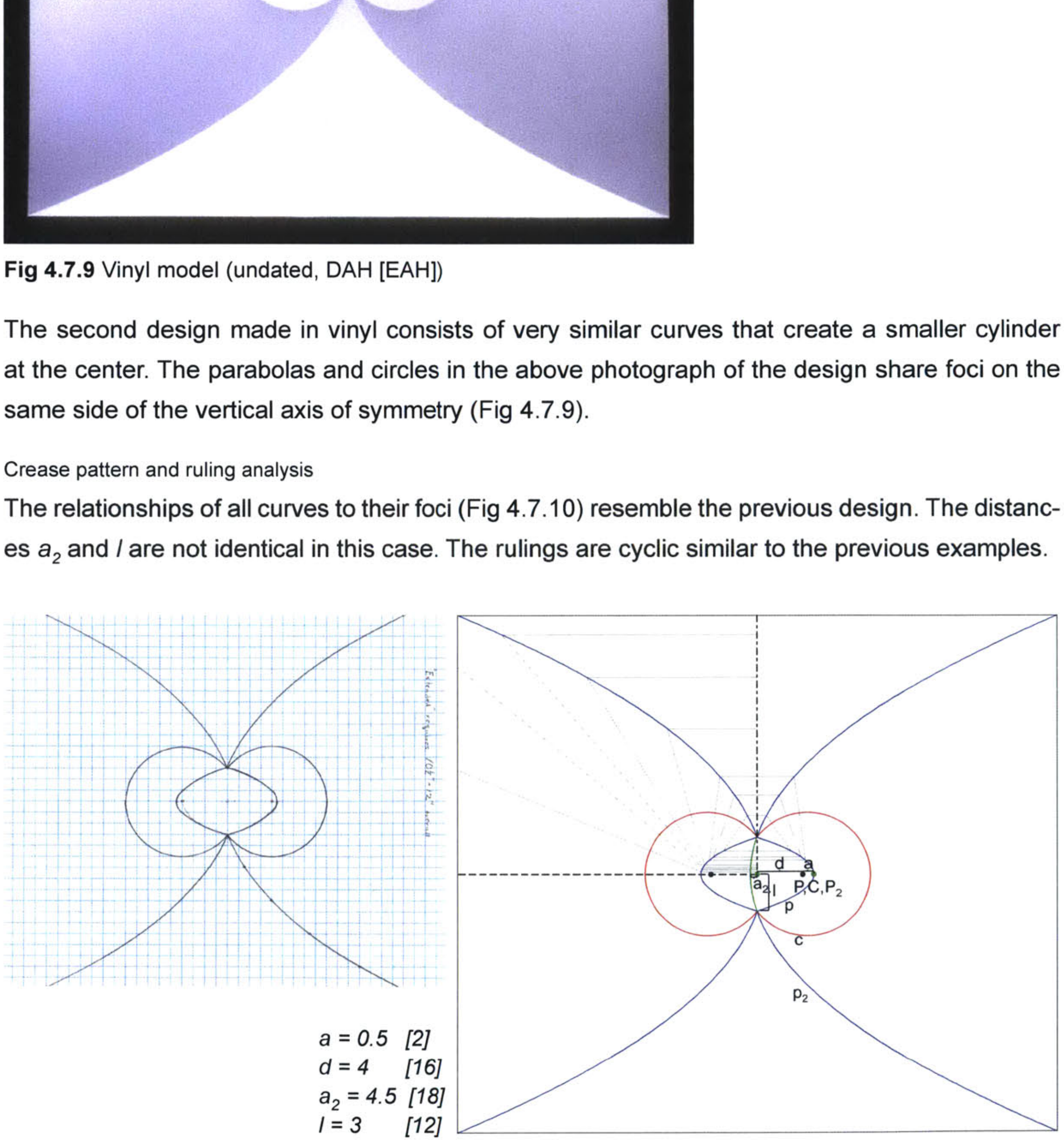
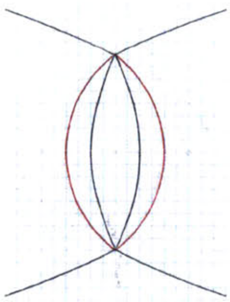
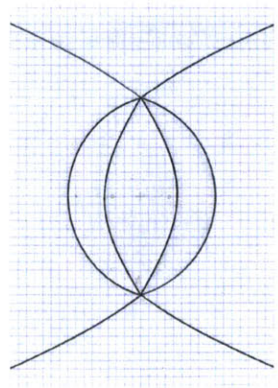


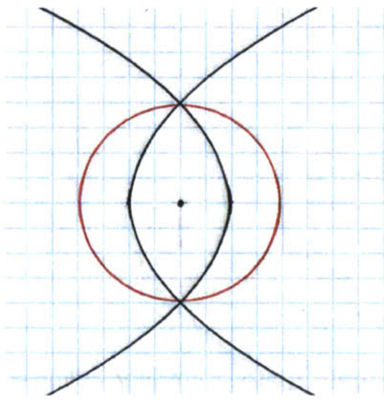
Fig 4.7.10 Paper model (undated, DAH [DK]), Crease pattern [DK]



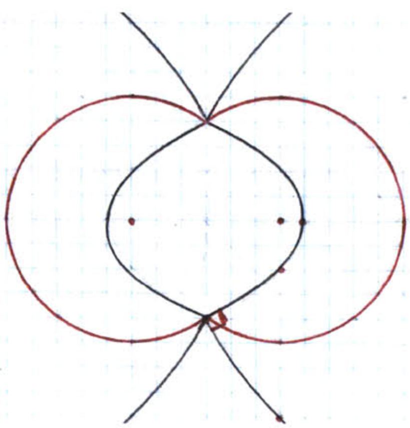
$a = 8$ [12]
$d = -6$ [-9]
$a_2 = 2$ [3]
$l = 8$ [12] (Fig 4.7.5)
Same as (Fig 4.7.12 right side)



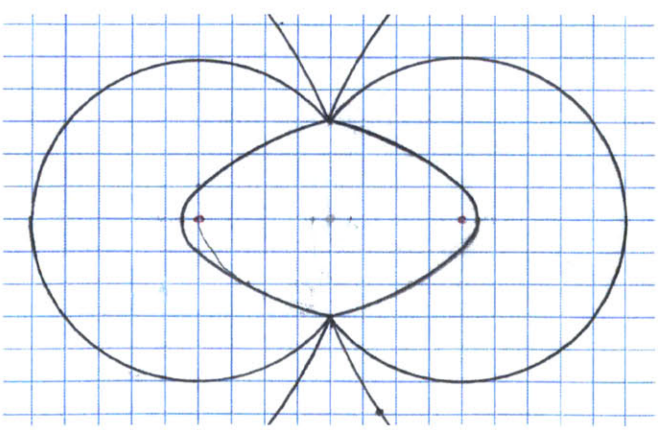
$a = 8$ [8]
$d = -3.5$ [-3.5]
$a_2 = 4.5$ [4.5]
$l = 12$ [12]



$a = 2$ [6]
$d = 0$ [0]
$a_2 = 2$ [6]
$l = 4$ [12] (Fig 4.7.2)



$a = 1$ [3]
$d = 3$ [9]
$a_2 = 4$ [12]
$l = 4$ [12] (Fig 4.7.7)
Same as (Fig 4.7.12 left side)



$a = 0.5$ [2]
$d = 4$ [16]
$a_2 = 4.5$ [18]
$l = 3$ [12] (Fig 4.7.9)

Fig 4.7.11 Paper models with parametric values (undated, DAH [DK])

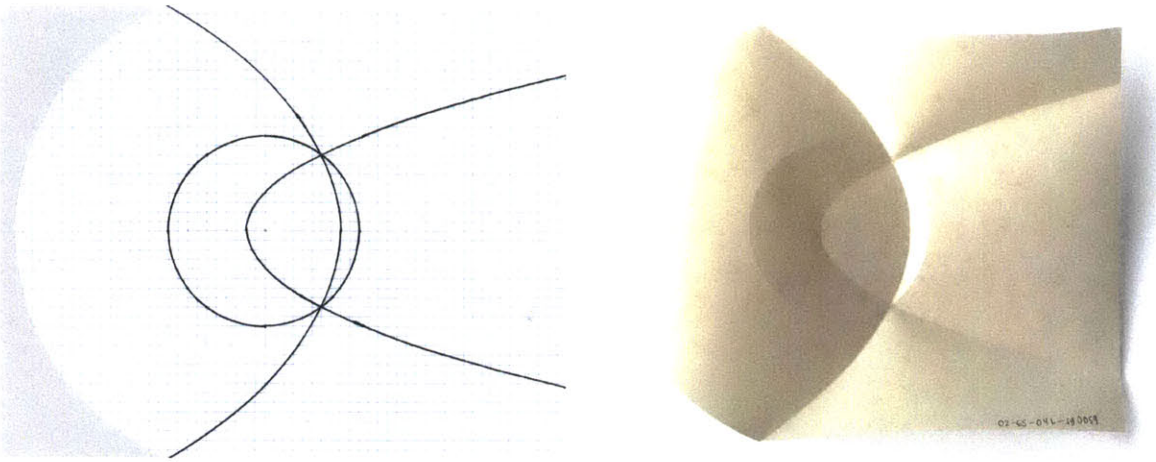


Fig 4.7.12 Paper model (undated, DAH [DK]), Paper model [AH]

Huffman makes a design in this series that is not symmetrical along the vertical (Fig 4.7.12). He cuts the left half of the design with an arc.

Crease pattern and ruling analysis

The parabolas and the circle are all confocal (Fig 4.7.13). The relationships of the parabolas to the circle resemble the first design of this series (Fig 4.7.1) as 1 circle and 2 parabolas define the crease pattern, However, in this case the 2 parabolas are different, which results in an asymmetrical design. The distance *o* defines the offset to the center of the crease pattern. The rulings remain cyclic.

The paper reconstruction highlights the asymmetrical nature of the crease pattern (Fig 4.7.12 right).

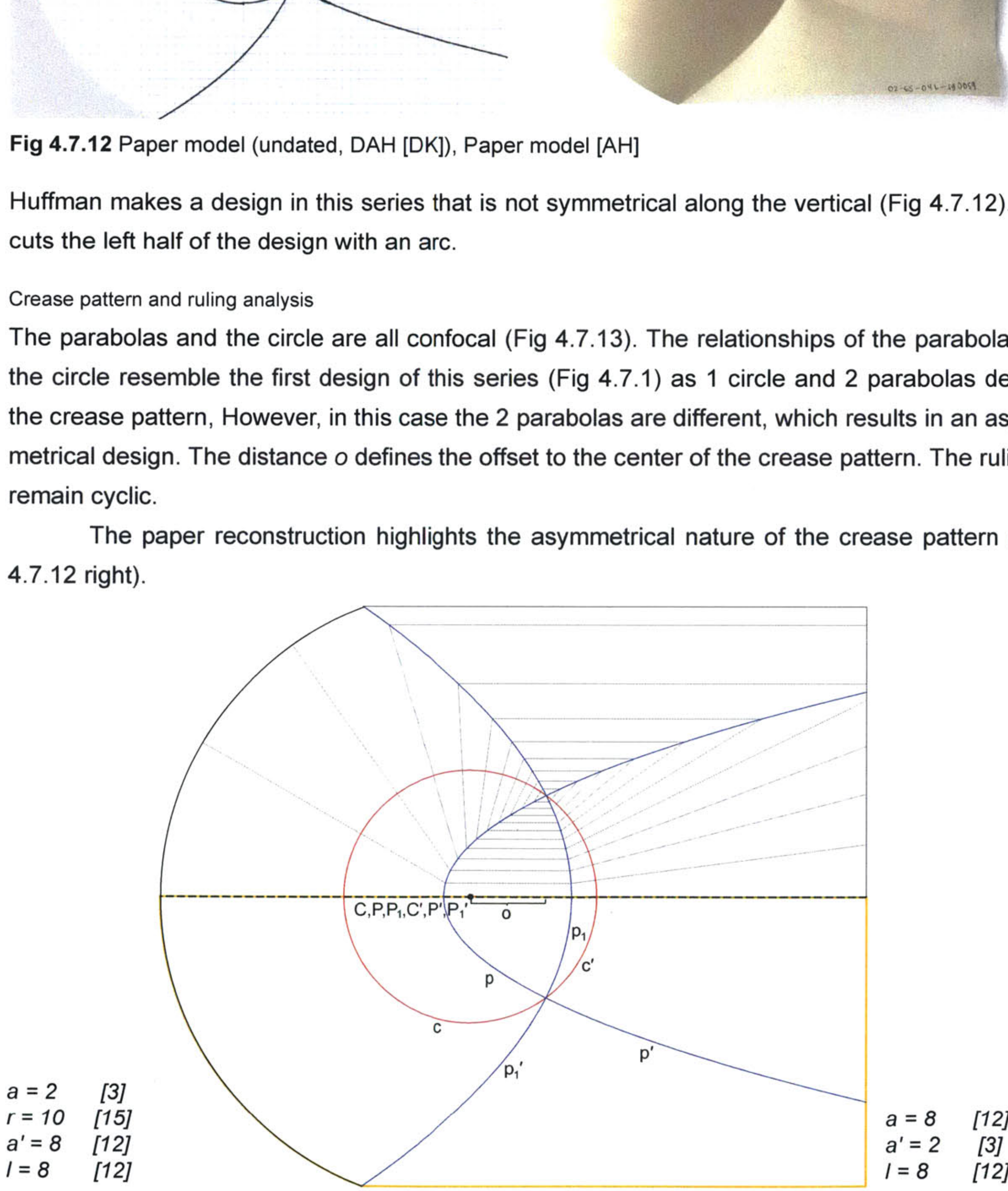


Fig 4.7.13 Crease pattern [DK]

Gadgets with parabolas and ellipses

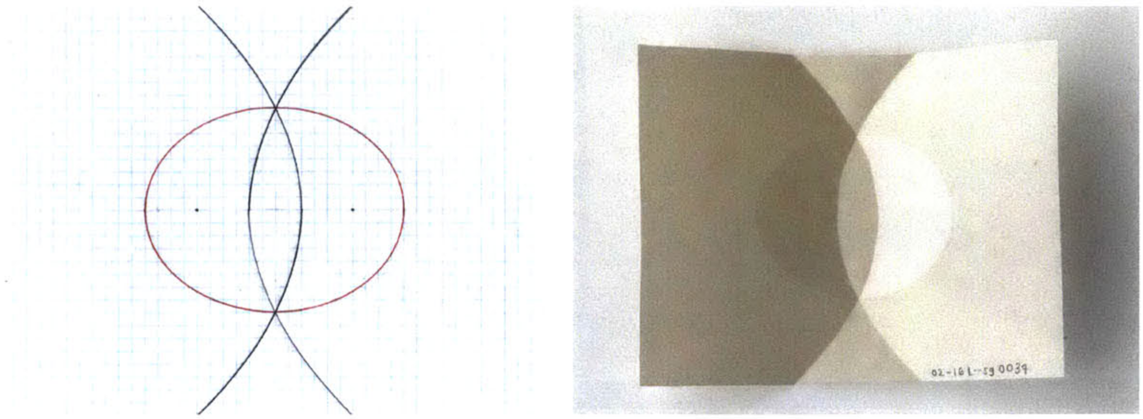


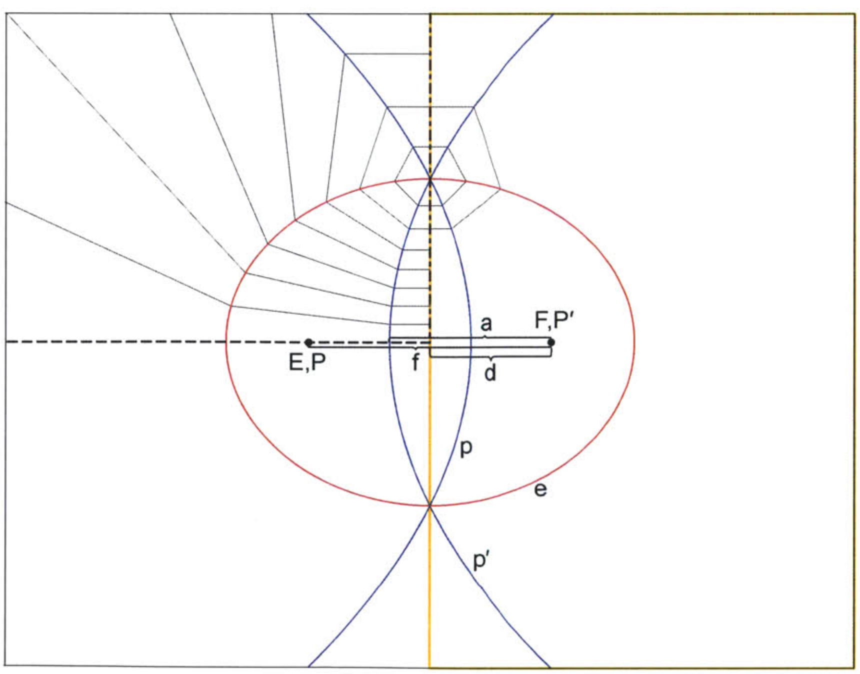
Fig 4.7.14 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman continues to alter the previous gadget and introduces ellipses instead of circles in the following section. A single ellipse and 2 parabolas define the special case of the above design (Fig 4.7.14). 2 ellipses and 2 parabolas create more general cases later in this series. Huffman decides to make only 1 model in vinyl.

Crease pattern and ruling analysis

The first design consists of 2 identical, mirrored parabolas and 1 ellipse with shared foci (Fig 4.7.15). The gadget that doubles as the prototile refracts the rulings 3 times to form cylinders similar to previous examples.

The rulings are cyclic and the paper reconstruction displays folding angles of varying sizes (Fig 4.7.14 right).



a = 8 [18]
d = -6 [-9]
f = 12 [18]
l = 8 [12]

Fig 4.7.15 Crease pattern [DK]

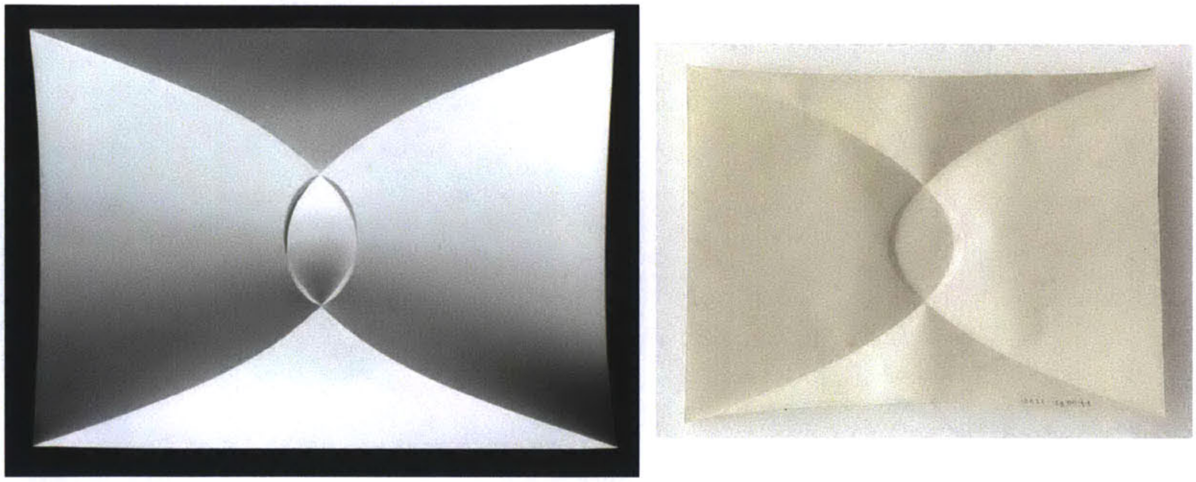


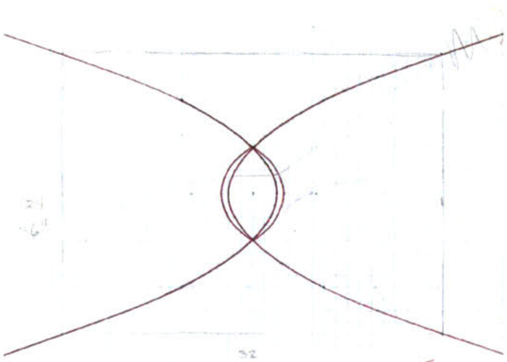
Fig 4.7.16 Vinyl model (undated, DAH [DAH]), Paper reconstruction [AH]

The nomenclature to describe the crease patterns (Fig 4.7.15, 4.7.17 and 4.7.18) includes 3 main parameters. The distance between the first parabola focus *P* and the intersection of the curve on its axis of symmetry is *a*. The distance from *P* to the center of the crease pattern is called *d*. Later examples include *a'* and *d'* for the second parabola. The foci of the ellipses *E* and *F* are separated by the distance *f*. Values in square brackets are normalized for better comparison (Fig 4.7.18).

Crease pattern and ruling analysis

The second example, made of vinyl, uses 2 identical, mirrored parabolas and 2 different ellipses. They share 3 vertices as foci. The prototile, again the gadget in this case, refracts the rulings in a similar way to the previous examples.

The rulings are cyclic. The paper reconstruction has a less dramatic mountain fold (Fig 4.7.16 right) and is cropped according to the pencil marks in Huffman's drawing (Fig 4.7.17 left).



a = 2 [6]
d = 0 [0]
f = 5.25 [15.75]
l=4 [12]

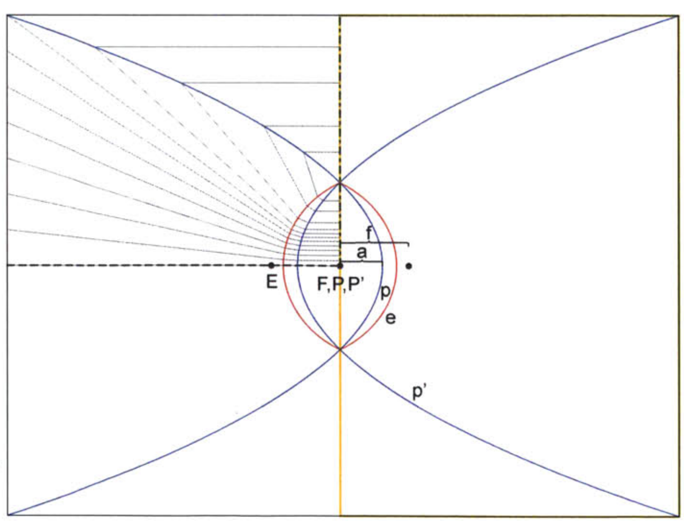
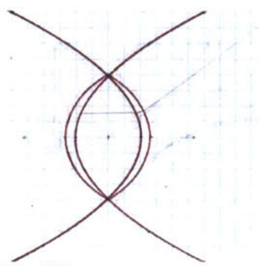


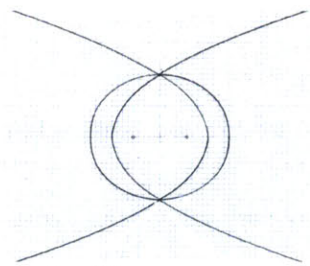
Fig 4.7.17 Paper model (undated, DAH [DK]), Crease pattern [DK]

The variations consist of 7 examples that use different numbers of confocal vertices. The first row positions the parabola foci on the vertical *d = 0*. The middle row uses 2 confocal vertices throughout and the third row consists of examples with 4 vertices all of which are used for 2 foci each (Fig 4.7.18).

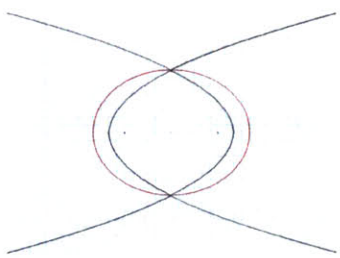


a = 2 [6]
d = 0 [0]
f = 5.25 [15.75]
l = 3 [12] *(Fig 4.7.17)*

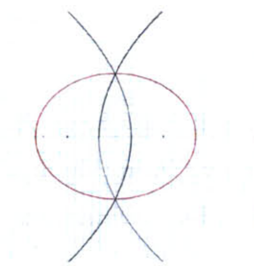
3 vertices || 1 confocal



a = 4 [4]
d = 5 [5]
f = 10 [10]
l = 12 [12]

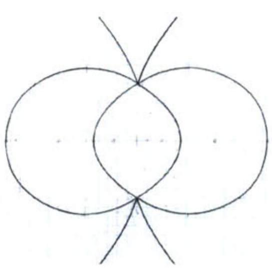


a = 2 [3]
d = 6 [9]
f = 12 [18]
l = 12 [12]

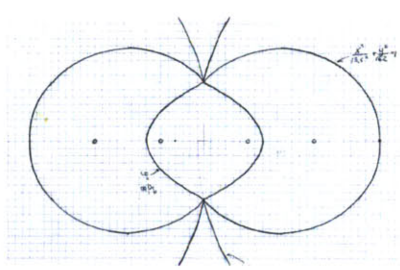


a = 8 [18]
d = -6 [-9]
f = 12 [18]
l = 8 [12] *(Fig 4.7.15)*

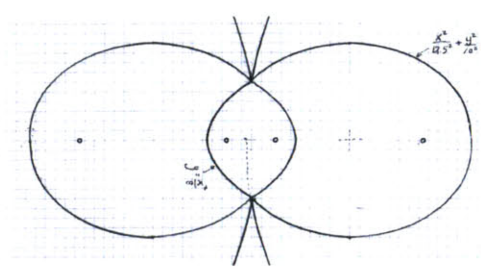
2 vertices || 2 confocal



a = 2 [4]
d = 2.5 [5]
f = 5.25 [11]
a_2*= 9 [18]*
l = 6 [12]
d_2*= 8 [16]*



a = 2 [3]
d = 6 [9]
f = 9 [13.5]
a' = 7 [10.5]
l = 8 [12]
d_2*= 15 [22.5]*



a = 2 [4]
d = 2.5 [5]
f = 15 [30]
a' = 18 [36]
l = 6 [12]
d_2*= 17.5 [35]*

4 vertices || 4 confocal

Fig 4.7.18 Paper models and parametric values (undated, DAH [DK])

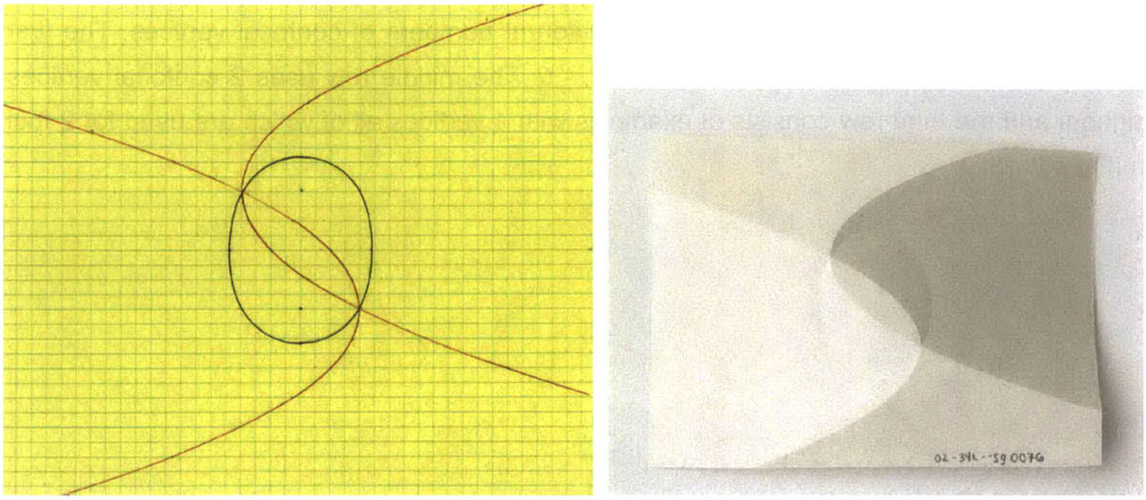


Fig 4.7.19 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman designs a crease pattern that is included in this section as it consists of similar curves with similar relationships, but the single ellipse in the center is turned 90°. This design provides the central cylinder, but the 2 parabolas do not align along the horizontal (Fig 4.7.19). Huffman only makes a paper model of this design.

Crease pattern and ruling analysis

The design consists of 2 identical, mirrored and moved parabolas, and 1 ellipse with shared foci similar to previous examples (Fig 4.7.20).

The rulings refract 6 times within the regions of the 2 prototiles, but their path is not cyclic. They converge slowly toward the degree 4 vertex and also very slowly converge toward the horizontal center line, which causes issues in terms of drawing a discrete crease pattern for simulation (Fig 4.7.20 right). The paper reconstruction has folding angles with varying sizes (Fig 4.7.19 right).

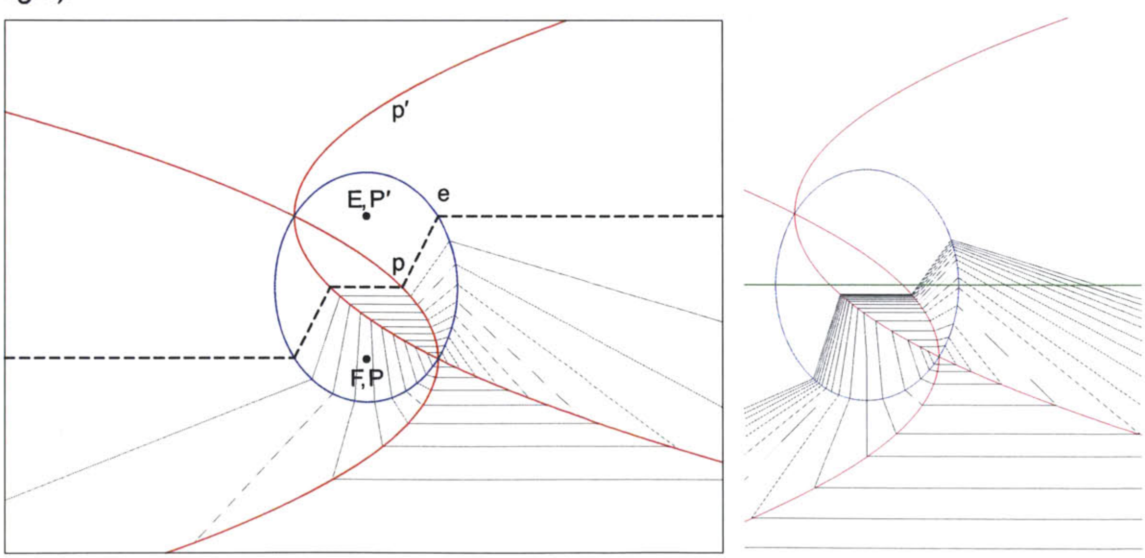


Fig 4.7.20 Crease pattern [DK], Diagram of slowly converging rulings [PC]

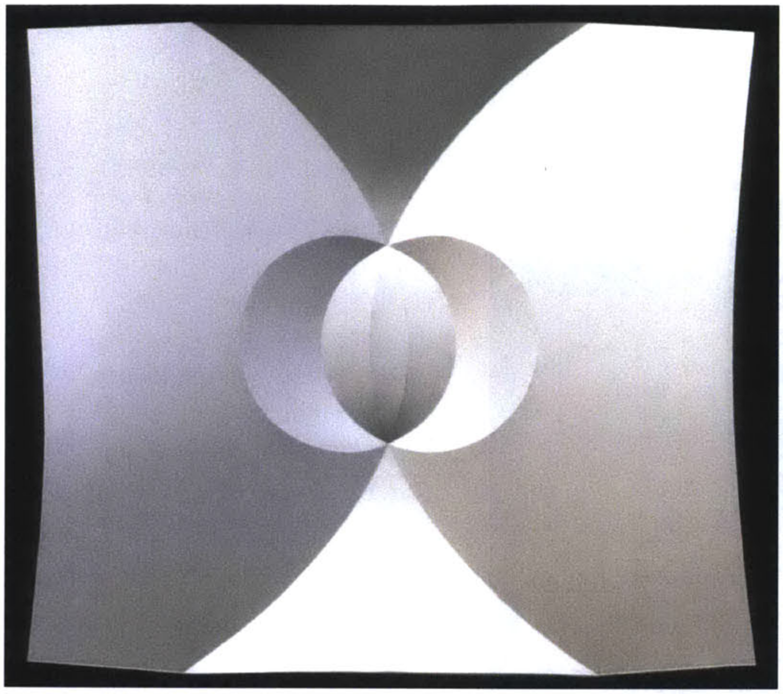


Fig 4.7.21 Vinyl model (undated, DAH [EAH])

Huffman combines a total of 10 curved creases in the above vinyl model (Fig 4.7.21) that uses 2 different parabolas and 2 ellipses. The resulting design has 2 more creases than previous examples.

Crease pattern and ruling analysis

The gadget consists of 2 parabolas and 2 ellipses, which share 3 vertices as foci (Fig 4.7.22 right). The rulings refract 4 times to form the 2 characteristic cylinders of the series. The rulings remain cyclic.

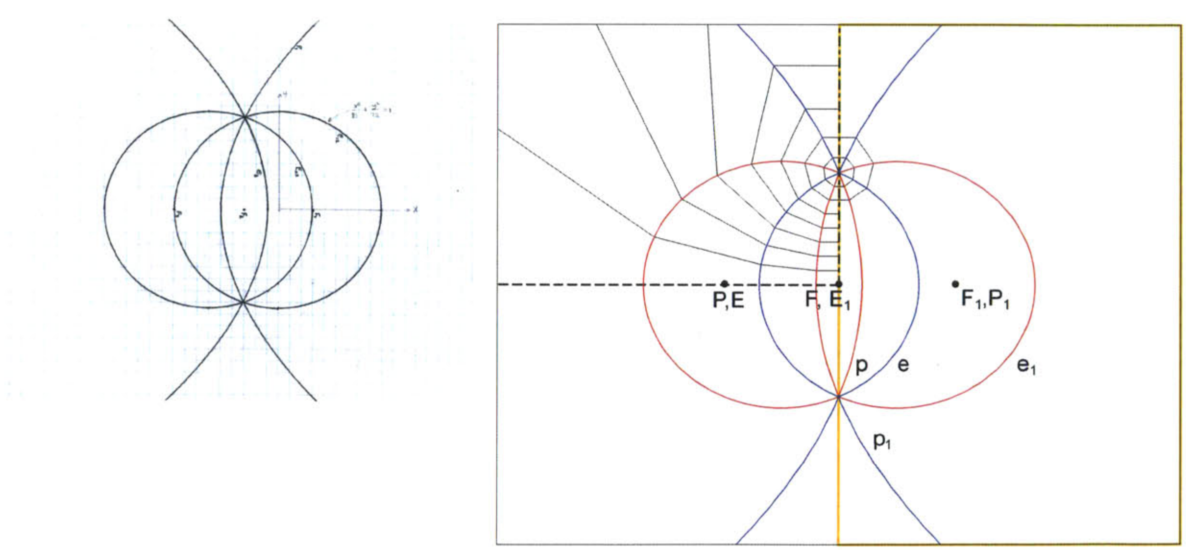


Fig 4.7.22 Crease pattern [DAH], Crease pattern [DK]

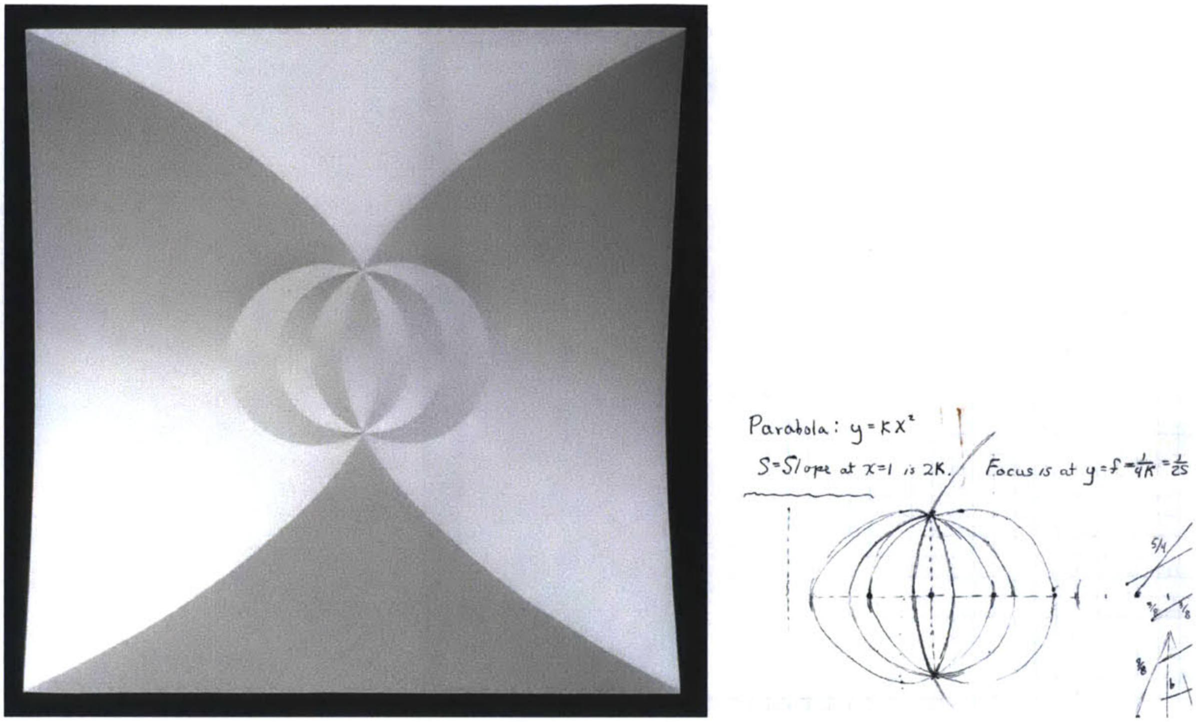


Fig 4.7.23 Vinyl model (undated, DAH [EAH]), Sketch (undated, DAH [DK])

The above design (Fig 4.7.23) uses 2 different parabolas and 3 ellipses. Huffman draws a sketch, in which he contemplates on a similar design with an arrangement of 12 individual curved creases.

Crease pattern and ruling analysis

The gadget consists of 2 partial parabolas and 3 partial ellipses, which share 4 vertices as foci (Fig 4.7.24 right). The rulings refract 5 times to form the 2 central cylinders of the series. The rulings assume a cyclic path.

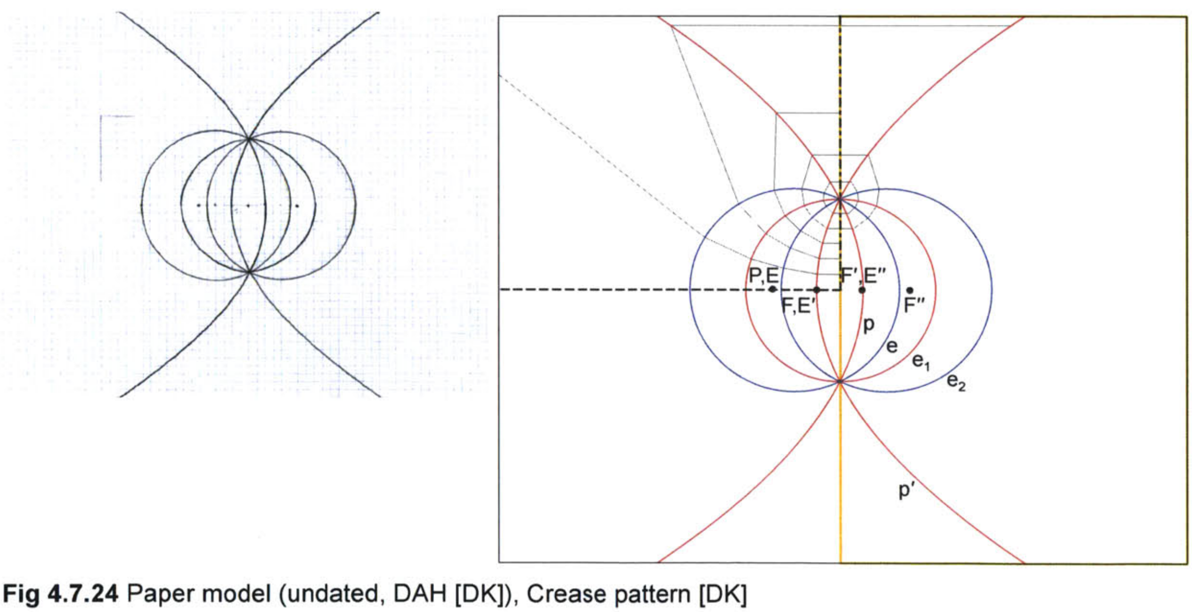


Fig 4.7.24 Paper model (undated, DAH [DK]), Crease pattern [DK]

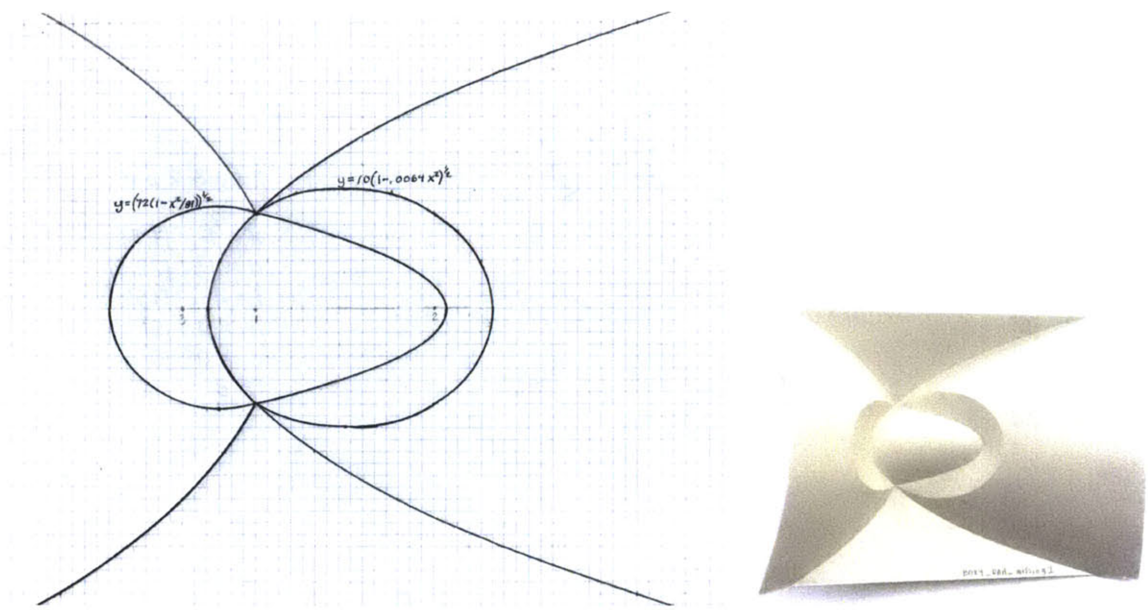


Fig 4.7.25 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman creates less symmetrical designs and even combines circles and ellipses. The previously established definitions for the parameters *a*, *a'*, *d*, *d'* and *f* apply and help when comparing the designs in this series (Fig 4.7.27). The crease patterns use circles on the left and ellipses on the right. They complete Huffman's previous variations (Fig 4.7.16).

Crease pattern and ruling analysis

The gadget follows all main characteristic of the previous designs including both cylinders. The rulings remain cyclic. The paper reconstruction highlights the asymmetry (Fig 4.7.25 right).

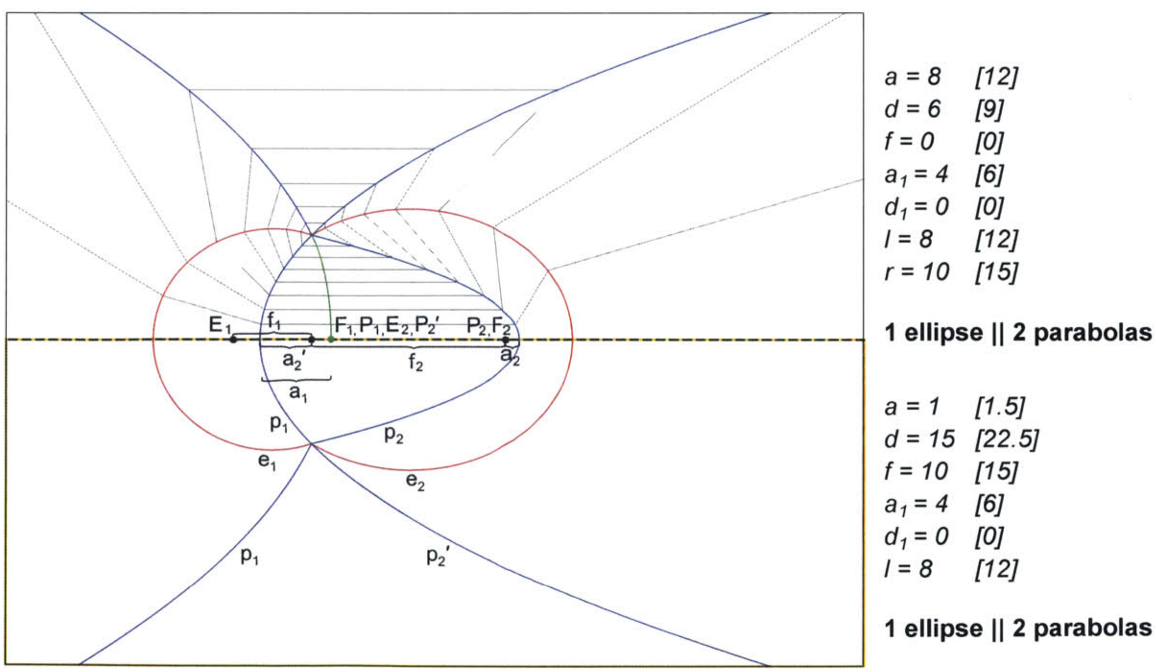
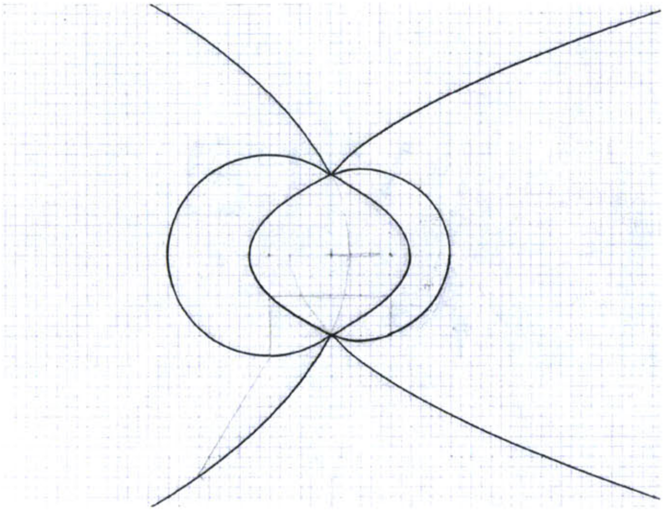


Fig 4.7.26 Crease pattern [DK]

1 cir & 2 par:

a = 8 [12]
d = 6 [9]
r = 10 [15]
a_1 = 4 [6]
d_1 = 0 [0]
l = 8 [12]
f = 0 [0]

(Fig 4.7.7)



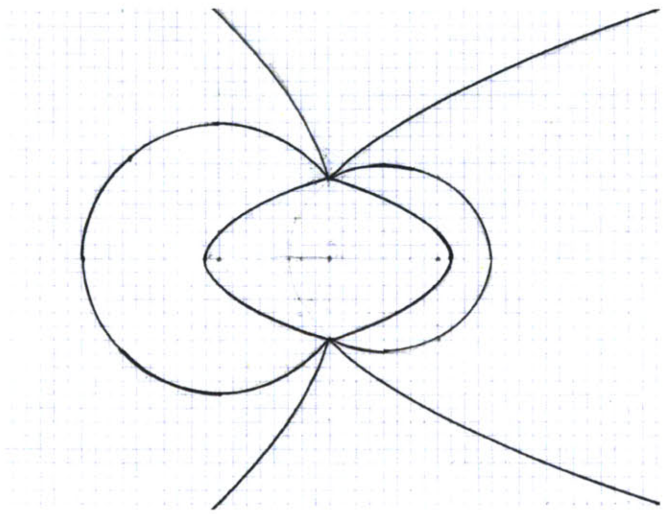
1 ell & 2 par

a = 1 [1.5]
d = 15 [22.5]
f = 4 [15]
a_1 = 4 [6]
d_1 = 0 [0]
l = 8 [12]

1 cir & 2 par:

a = 1 [2]
d = 8 [16]
r = 10 [10]
a_1 = 1 [1]
d_1 = 8 [16]
l = 6 [12]
f = 0 [0]

(Fig 4.7.9)

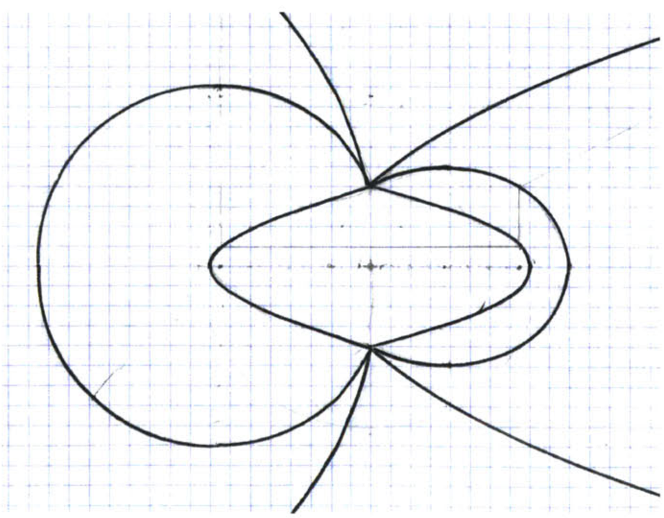


1 ell & 2 par

a = 1 [2]
d = 8 [16]
f = 8 [16]
a_1= 3 [6]
d_1 = 0 [0]
l = 6 [12]

1 cir & 2 par:

a = 0.5 [1.5]
d = 7.5 [22.5]
r = 9 [27]
a_1 = 8 [24]
d_1 = 7.5 [22.5]
l = 4 [12]
f = 0 [0]



1 ell & 2 par

a = 2 [6]
d = 7.5 [22.5]
f = 7.5 [22.5]
a_1 = 0.5 [1.5]
d_1= 0 [0]
l = 4 [12]

Fig 4.7.27 Paper models and parametric values (undated, DAH [DK])

4.8 Gadgets with ellipses and hyperbolas

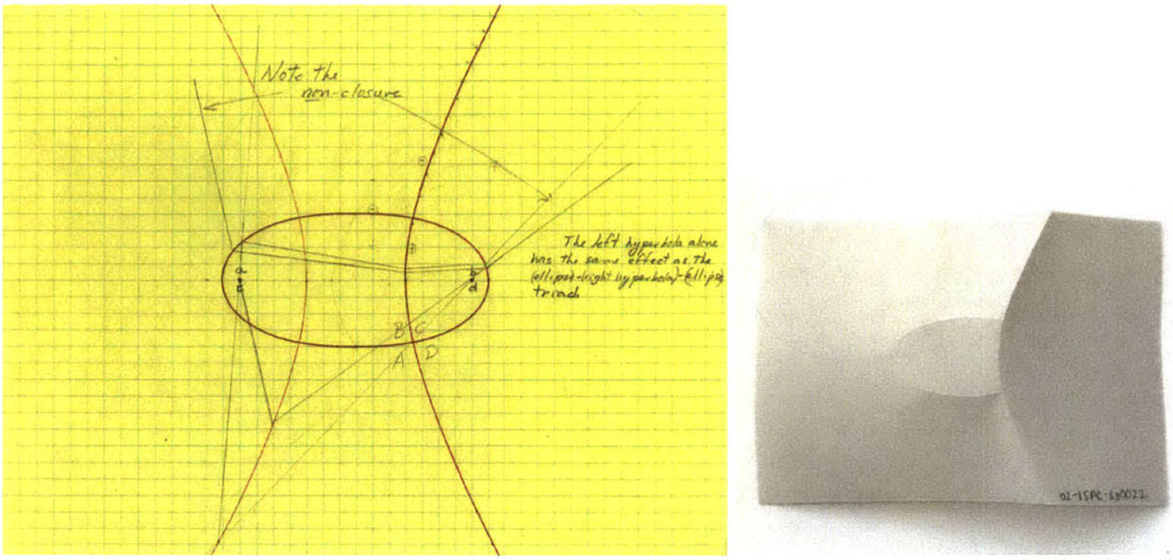


Fig 4.8.1 Drawing (undated, DAH [DK]), Paper reconstruction [AH]

Huffman designs with ellipses and hyperbolas regarding the following examples. He shows that the above confocal configuration (Fig 4.8.1) of an ellipse and a hyperbola refract the rulings in a similar way as the left hyperbola does on its own. The design has similarities to the first design with ellipses (Fig 4.3.3).

Crease pattern and ruling analysis

The gadget consists of 2 partial ellipses with alternating mountains and valleys and 2 partial, confocal hyperbolas as mountains (Fig 4.8.2). The rulings have to assume the shown configuration within the area of the ellipse as they would otherwise form 2 cones with apices in *E* and *F*. The rulings, if drawn as connected cycles, converge very slowly toward the degree 4 vertex, which makes simulation difficult (Fig 4.8.2 right). The paper reconstruction displays large folding angles along the hyperbola (Fig 4.8.1 right).

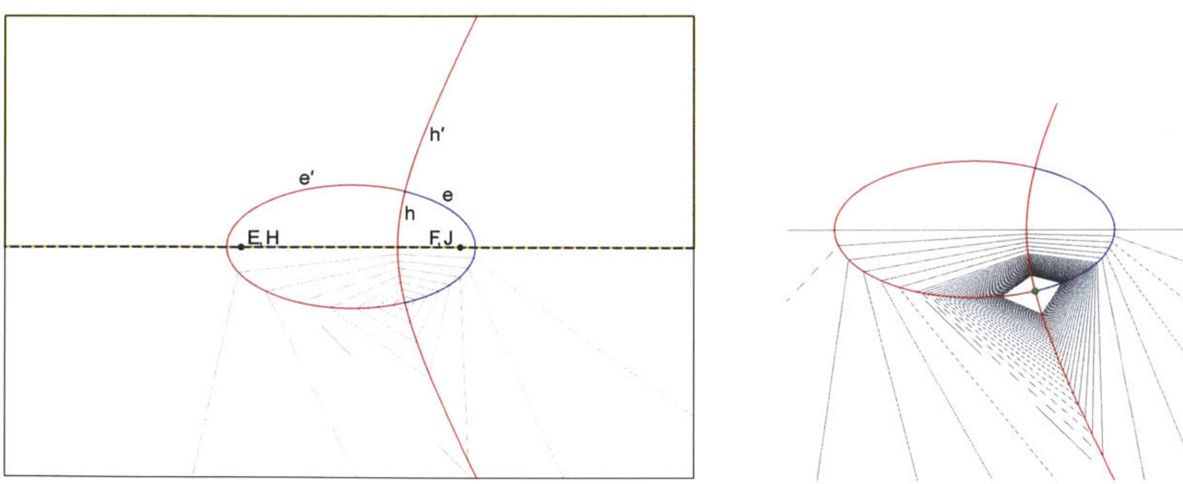


Fig 4.8.2 Crease pattern [DK], Diagram of converging rulings [PC]

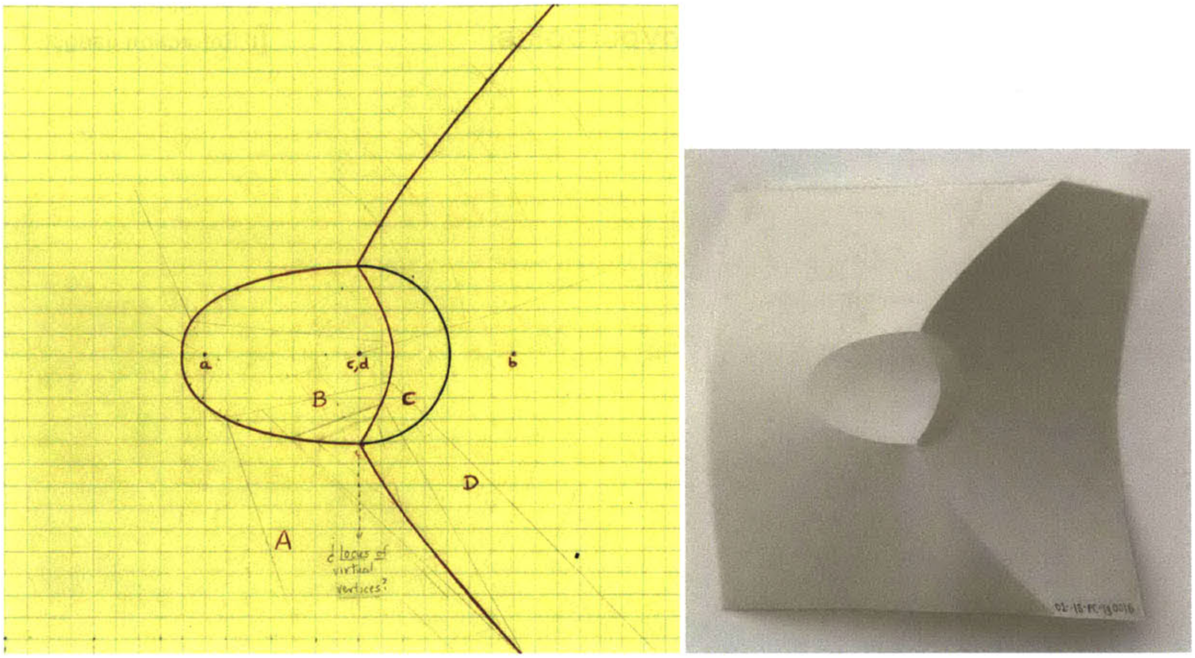


Fig 4.8.3 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman alters the design by mirroring the inner hyperbola and replacing the partial ellipse on the right with a circle. The pencil note, which does not resemble Huffman's handwriting, mentions a 'virtual vertex' (Fig 4.8.3). The note might relate to slowly converging rulings.

Crease pattern and ruling analysis

The gadget, which is also the prototile, consists of 2 partial ellipses, an arc, and confocal hyperbolas (Fig 4.8.4). The hyperbolas are similar, but mirrored. The rulings behave in a similar way to the previous example and refract in cycles with very small increments, which makes simulation difficult. The paper reconstruction folds well (Fig 4.8.3 right).

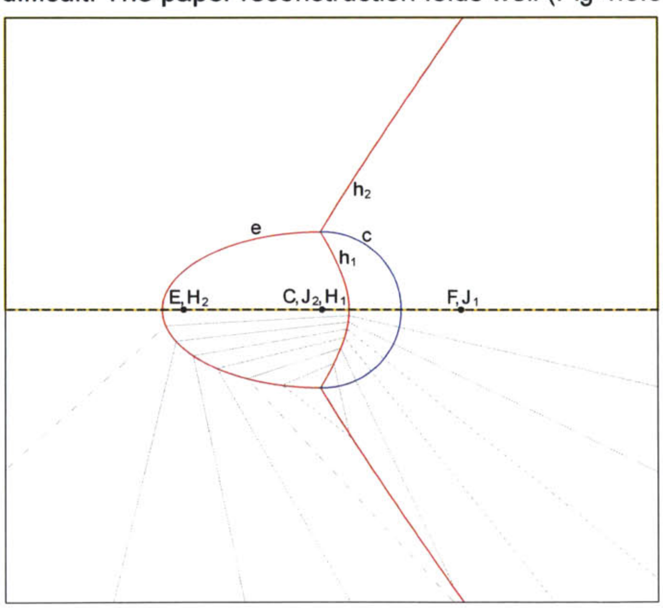


Fig 4.8.4 Crease pattern [DK]

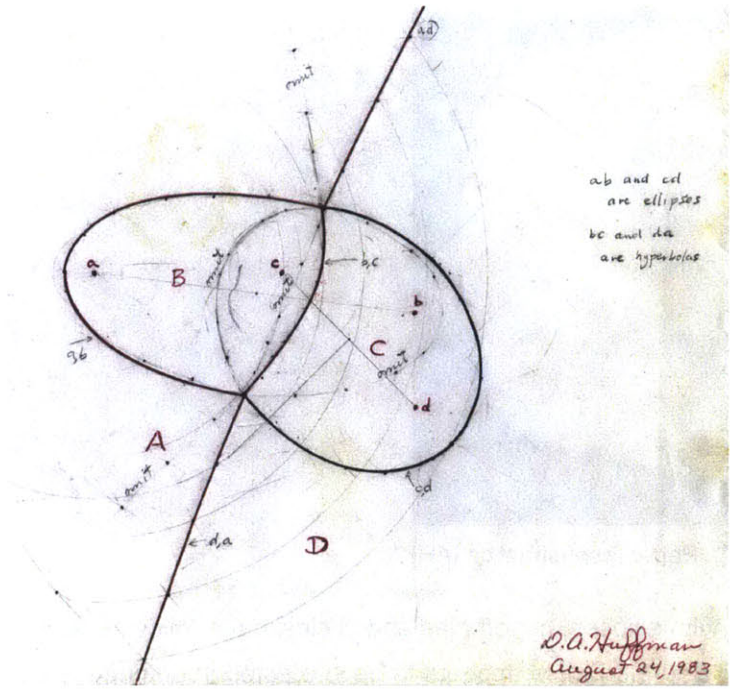


Fig 4.8.5 Drawing (1983, DAH [DK])

The ellipses in the above design are rotated and the hyperbolas use different sets of foci (Fig 4.8.5). The crease pattern consists of 2 partial ellipses and 3 partial hyperbolas, but the relationship between the foci is not clear. Huffman denotes that the marked foci *a*,*b* and *c*,*d* belong to the 2 ellipses and *b*,*c* and *a*,*d* to the hyperbolas. It appears that *a*,*c* align along a horizontal line and *b*,*d* along a vertical line.

The sketch below (Fig 4.8.6) shows a cyclic ruling path for a design that is similar to the above drawing.

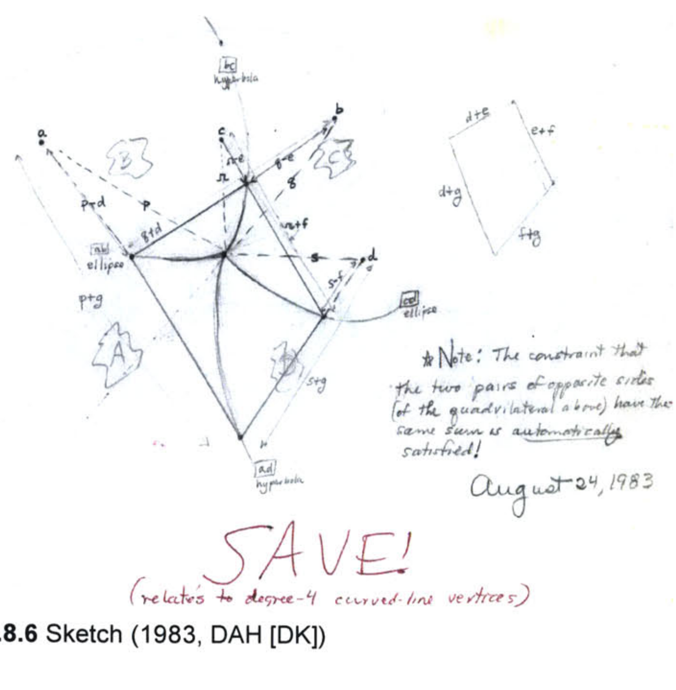


Fig 4.8.6 Sketch (1983, DAH [DK])

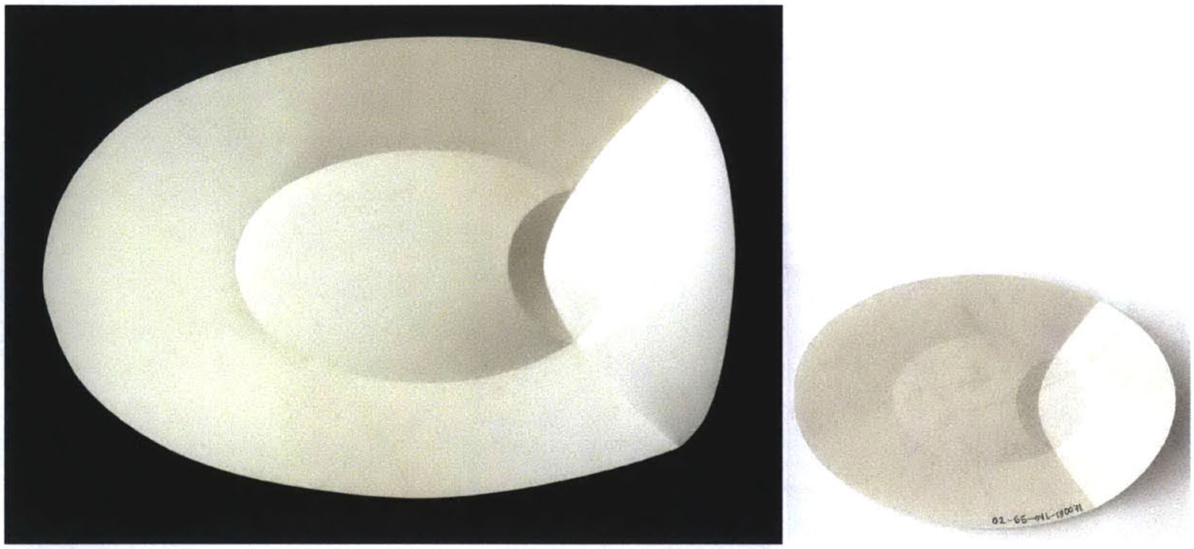


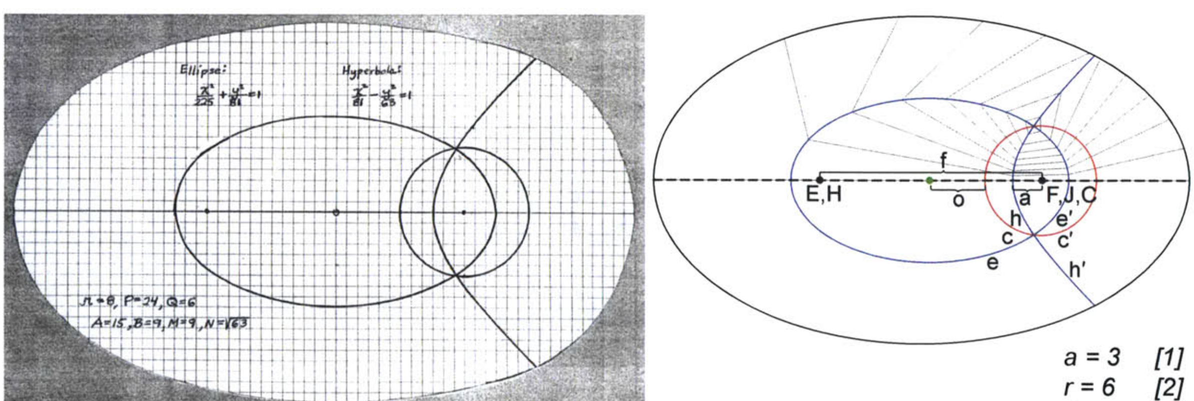
Fig 4.8.7 Vinyl model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman continues to explore designs with ellipses, hyperbolas and circles in the following series that begins with the above vinyl model (Fig 4.8.7). He appears to be interested in cutting the designs with ellipses and circles, which gives the series its distinct visual character. The rulings for all designs are not cyclic, but have a tendency to spiral towards a limit, which makes simulation difficult.

Crease pattern and ruling analysis

The gadget consists of 2 partial circles as mountains, 2 partial ellipses as valleys and 2 partial hyperbolas as valleys (Fig 4.8.8 right). The 6 creases are defined by 3 curves, and the notation of *h* and *h'*, for example, help in identifying individual but related creases.

The rulings have to assume the depicted configuration in the area between the ellipse and the hyperbola in order to not create a cone with an apex in *F*, *J*, *C*. The following crease pattern shows the parametric distances of the series.



a = 3 *[1]*
r = 6 *[2]*
f = 24 *[3]*
d = 6 *[2]*

Fig 4.8.8 Photo copy of drawing (undated, DAH [DK]), Crease pattern [DK]

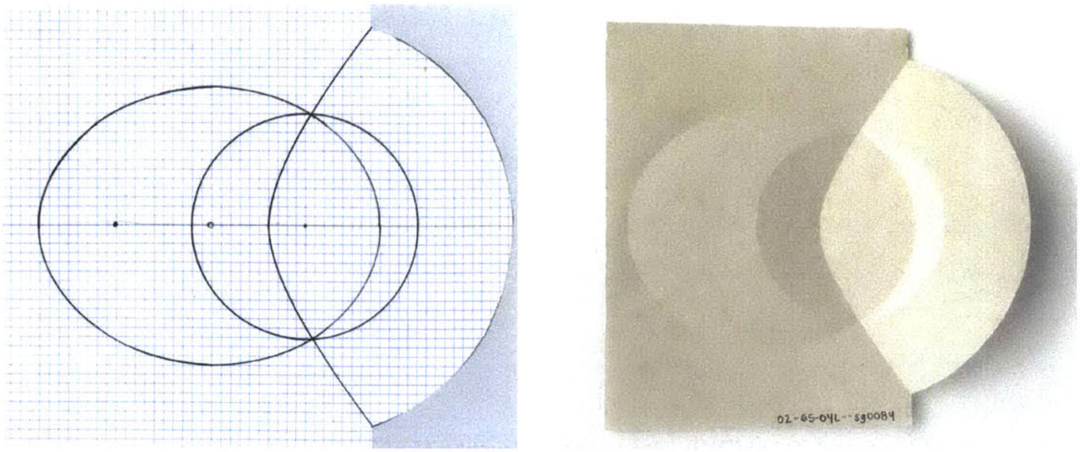
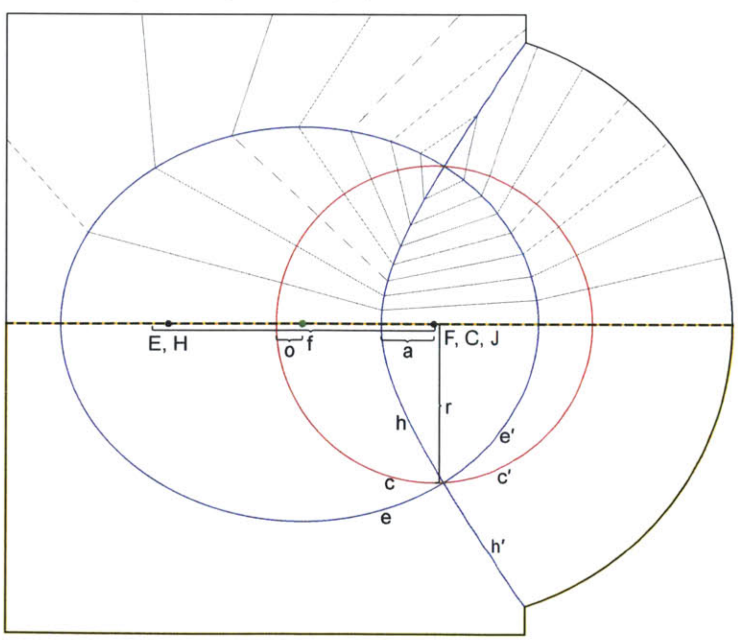


Fig 4.8.9 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The second design in the series consists of a similar set of curves, a circle, an ellipse, and a hyperbola. All curves share 2 vertices as their foci. Huffman cuts out only one half of the paper model (Fig 4.8.9).

Crease pattern and ruling analysis

The gadget consists of 2 partial circles as mountains, 2 partial ellipses as valleys, and 2 partial hyperbolas as valleys (Fig 4.8.10). The 6 creases are again defined by 3 curves. The distance between *J* and the vertex of the hyperbola on the axis of symmetry is *a*. The radius of the circle is *r* and the distance between the foci of the ellipse is *f*. The left vertex of the circle on the horizontal axis is positioned to the center of the crease pattern with an offset of *o*, which is negative in this case. The rulings behave similar to the first design and the paper reconstruction folds with varying folding angles (Fig 4.8.9 right).



a = 4 [1]
r = 12 [3]
f = 20 [5]
o = -2 [-0.5]

Fig 4.8.10 Crease pattern [DK]

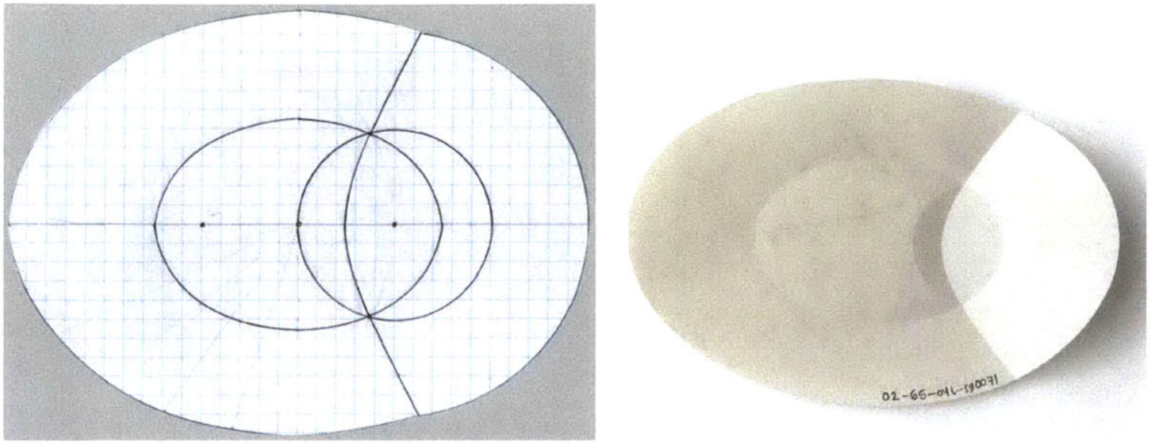


Fig 4.8.11 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The above iteration of the series locates the circle such that *o* is 0. It consists of a similar set of curves and Huffman cuts out the entire paper model with a circle and an ellipse (Fig 4.8.11).

Crease pattern and ruling analysis

The gadget changes very little in comparison to the previous model (Fig 4.8.12). Mountain and valley assignments have been kept the same in order to better compare the designs in the series. The distances *a = 2.5* and *r = 5* locate all curves such that the distances between the circle and both inner curves is the same, which gives the model a visual balance. The rulings refract in similar ways to before and the paper reconstruction folds with varying folding angles (Fig 4.8.11 right).

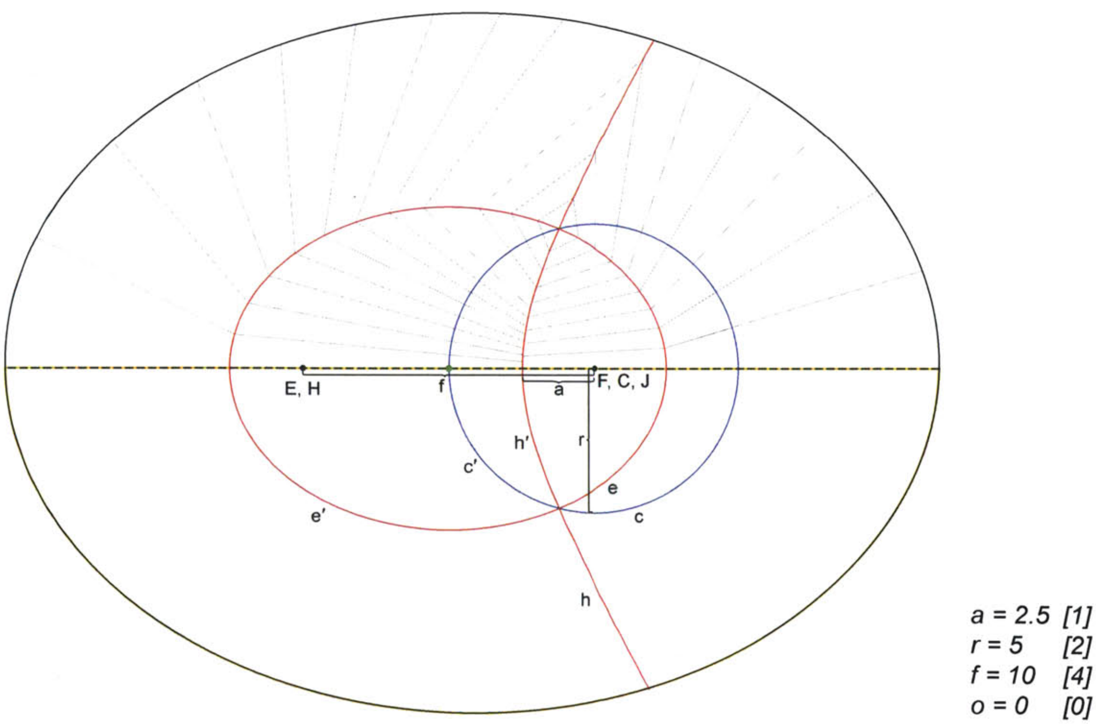


Fig 4.8.12 Crease pattern [DK]

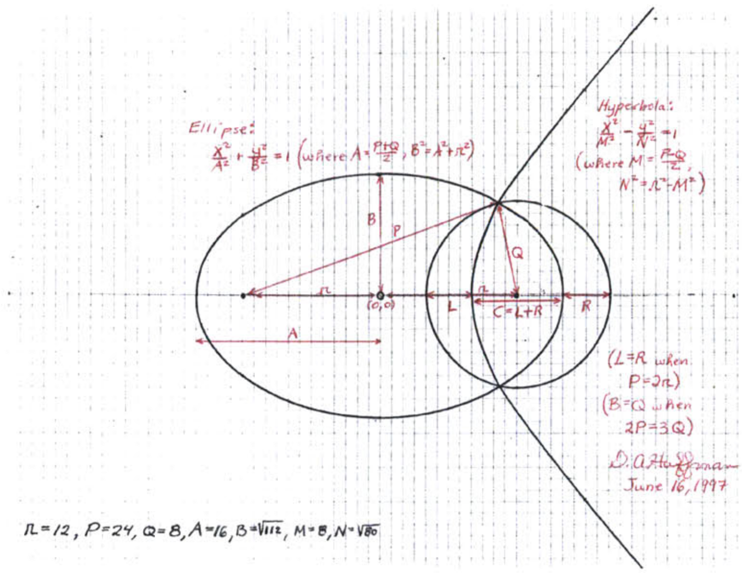
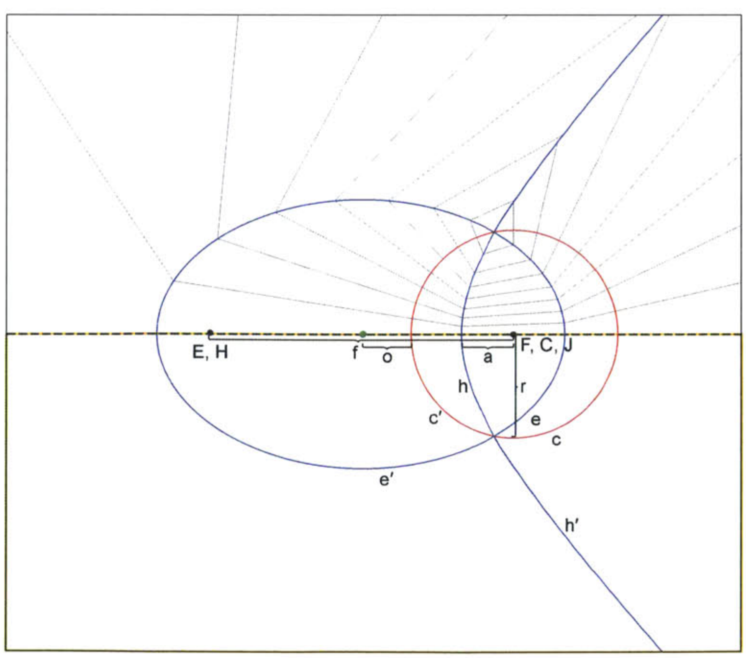


Fig 4.8.13 Paper model (1997, DAH [DK]), Paper reconstruction [AH]

Huffman makes many drawings of the above design and dates a photocopied version (Fig 4.8.13). He notes that the provided values for *P*, *Q*, *B* and *L* and *R* can produce certain symmetries.

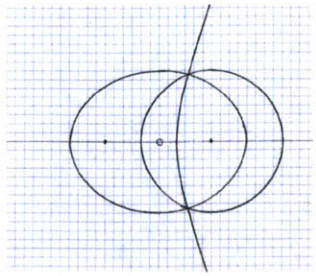
Crease pattern and ruling analysis

The gadget is fairly consistent to the previous design, including mountain and valley assignments (Fig 4.8.14). The symmetries Huffman points out can be seen in the parameter values for *a*, *r*, *f*, and *o*. The rulings behave similarly to all examples in the series. The paper reconstruction folds with varying folding angles (Fig 4.8.13 right). The series can be compared in the next figure that includes all parameter values and normalized values to *a* = *1* (Fig 4.8.13).

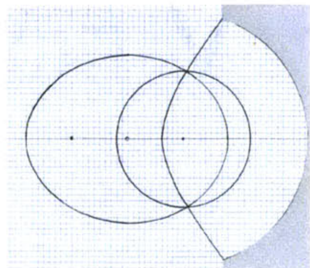


a = *4* *[1]*
r = *8* *[2]*
f = *24* *[3]*
o = *4* *[1]*

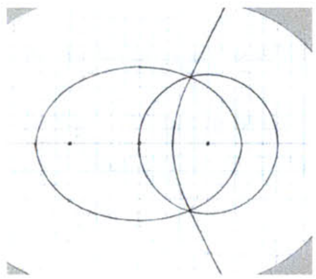
Fig 4.8.14 Crease pattern [DK]



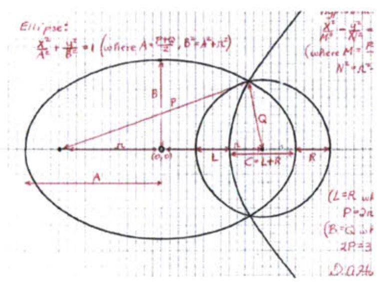
a = 4 [1]
r = 8 [2]
f = 12 [3]
o = -2 [-0.5]



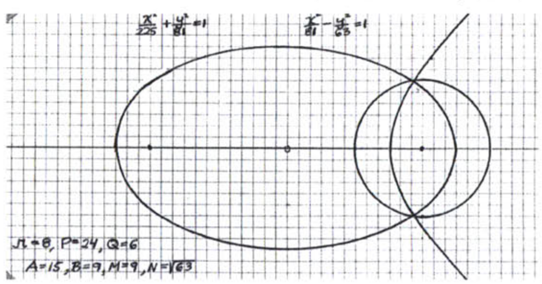
a = 4 [1]
r = 12 [3]
f = 20 [5]
o = -2 [-0.5] (Fig 4.8.9)



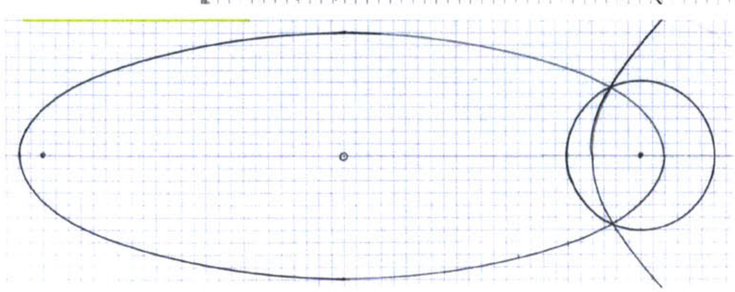
a = 2.5 [1]
r = 5 [2]
f = 10 [4]
o= 0 [0] (Fig 4.8.11)



a = 4 [1]
r = 8 [2]
f = 24 [0]
o = 4 [1] (Fig 4.8.13)



a = 3 [1]
r = 6 [2]
f = 24 [3]
o = 6 [2] (Fig 4.8.8)



a = 4 [1]
r = 6 [1.5]
f = 48 [12]
o = 18 [4.5]

Fig 4.8.15 Paper models with parameter values (most undated, DAH [DK])

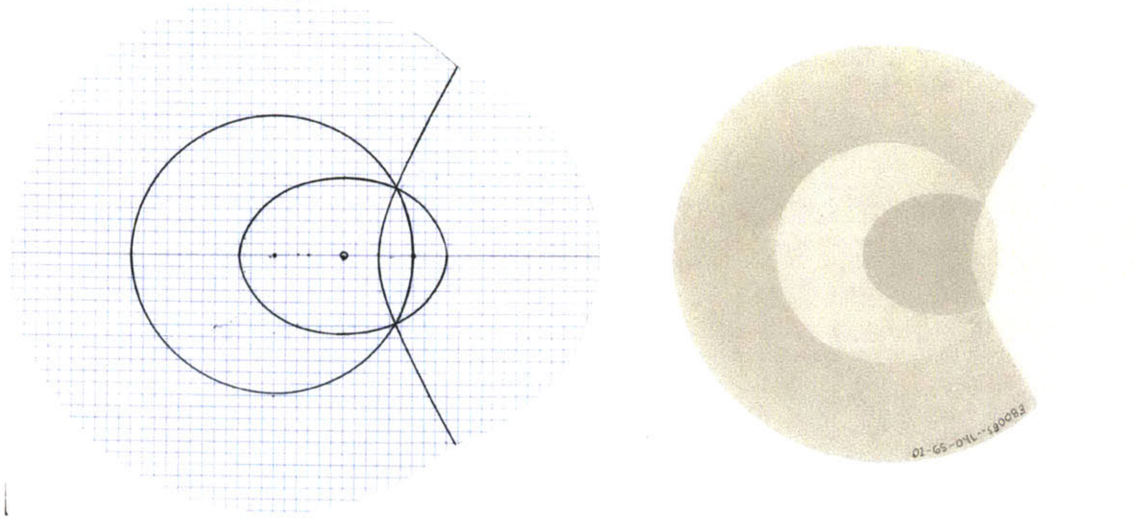


Fig 4.8.16 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The above paper model falls out of the series as the ellipse and circle swap roles (Fig 4.8.16). Huffman uses 2 curves to cut out the design.

Crease pattern and ruling analysis

The gadget is remarkably similar to the previous series and consists of similar pairs of partial circles, ellipses and hyperbolas (Fig 4.8.17). The rulings also behave in a similar way and the paper reconstruction displays a similar variety of folding angles (Fig 4.8.16 right).

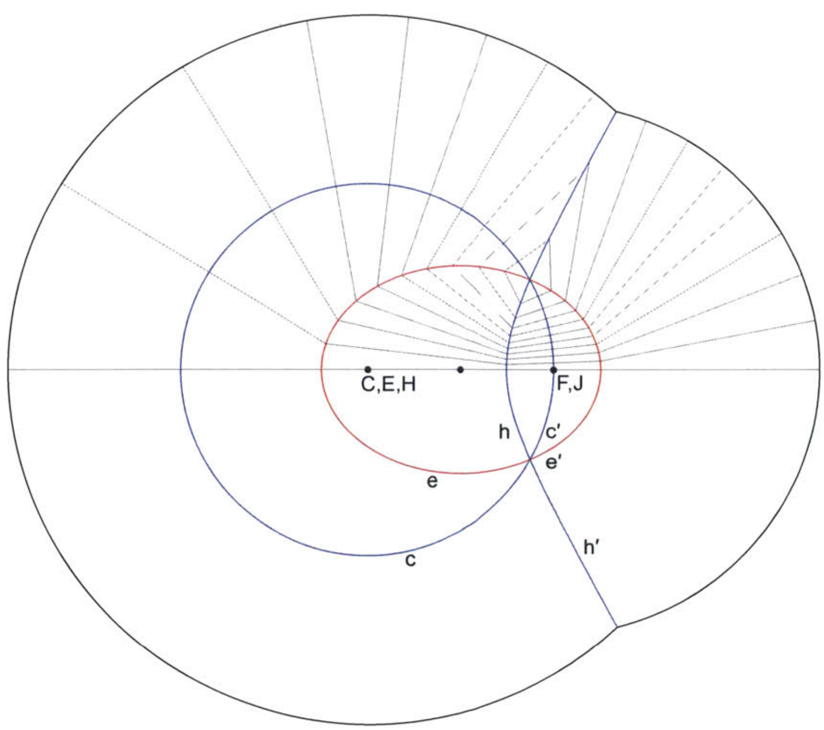


Fig 4.8.17 Crease pattern [DK]

Gadgets with ellipses, hyperbolas and line segments

Regarding the short series in this section Huffman combines ellipses and hyperbolas with line segments in his tilings. He only creates a few models in paper and one in vinyl. The first design (Fig 4.8.19) is reminiscent of his '4-lobed, cloverleaf design' in the chapter 'Gadgets with ellipses and tucking' (Fig 4.3.14).

The gadget

Huffman uses an ellipse and a hyperbola to control the rulings such that they intersect in *J, F* to form a cone in the center (Fig 4.8.18). The rule line between *H* and *E* can create a smooth transition between the 2 adjacent conical surfaces or can, alternatively, become a crease. The edges along *E*, *F* and to the left of *H* are creases, but their extensions consist of smooth transitions in the first examples.

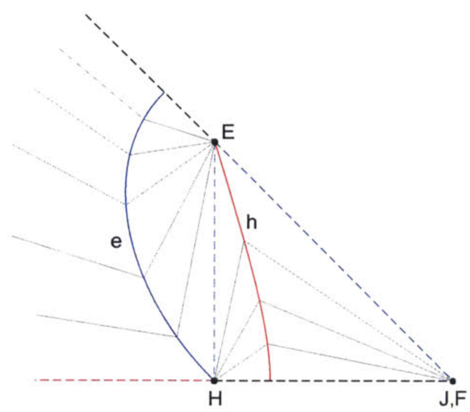


Fig 4.8.18 Gadget [DK]

Crease pattern and ruling analysis

Following the gadget, the ellipse and the hyperbola share a focus in the center of the crease pattern (Fig 4.8.20). The foci *E* and *H* are aligned along a vertical line. The tiling consists of 8 triangular prototiles and Huffman uses a square as boundary for the design.

The rulings start in the center of the crease pattern and get refracted along the hyperbola towards *H*. The vertical ruling between *E* and *H* doubles as a smooth transition between the 2 conical surfaces. The second part of the gadget refracts the ruling along the ellipse such that they form conical surfaces.

The simulated model folds very little before the ruling between *E* and *H* folds into a crease with a much larger folding angle than its neighbors. The paper reconstruction gives a better idea of the conical transition (Fig 4.8.19 right).

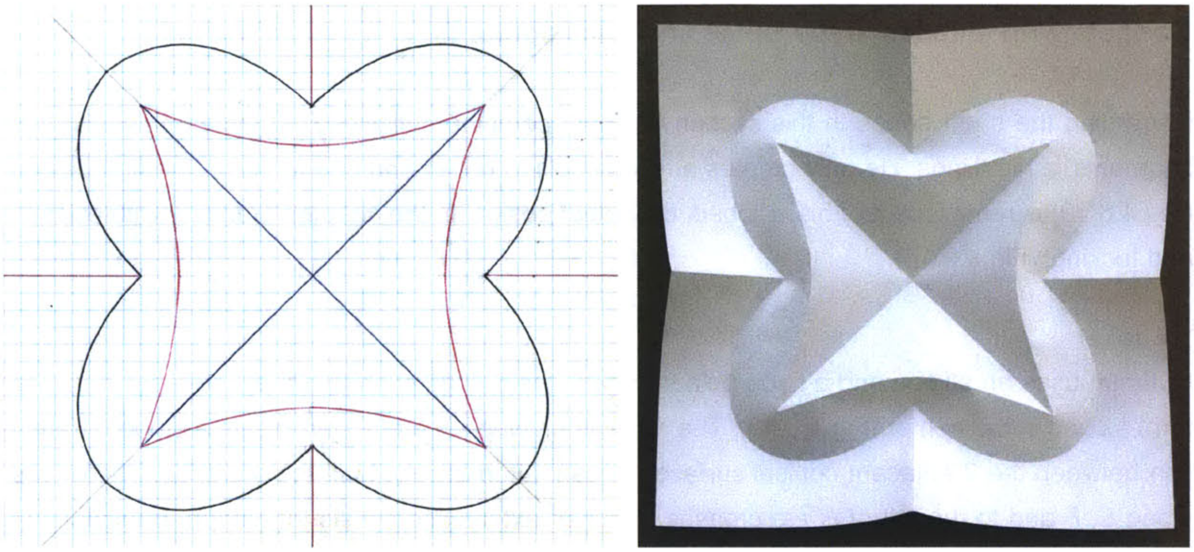


Fig 4.8.19 Paper model (undated, DAH [DK]), Paper reconstruction [JH]

Notes

The mountain and valley assignments turn the outer partial cones into convex surfaces, which gives this design a different appearance than the previously mentioned '4-lobed, cloverleaf design'.

In the second design Huffman adds a crease in the gadget (Fig 4.8.21), namely the one that is used as a smooth transition in the first design.

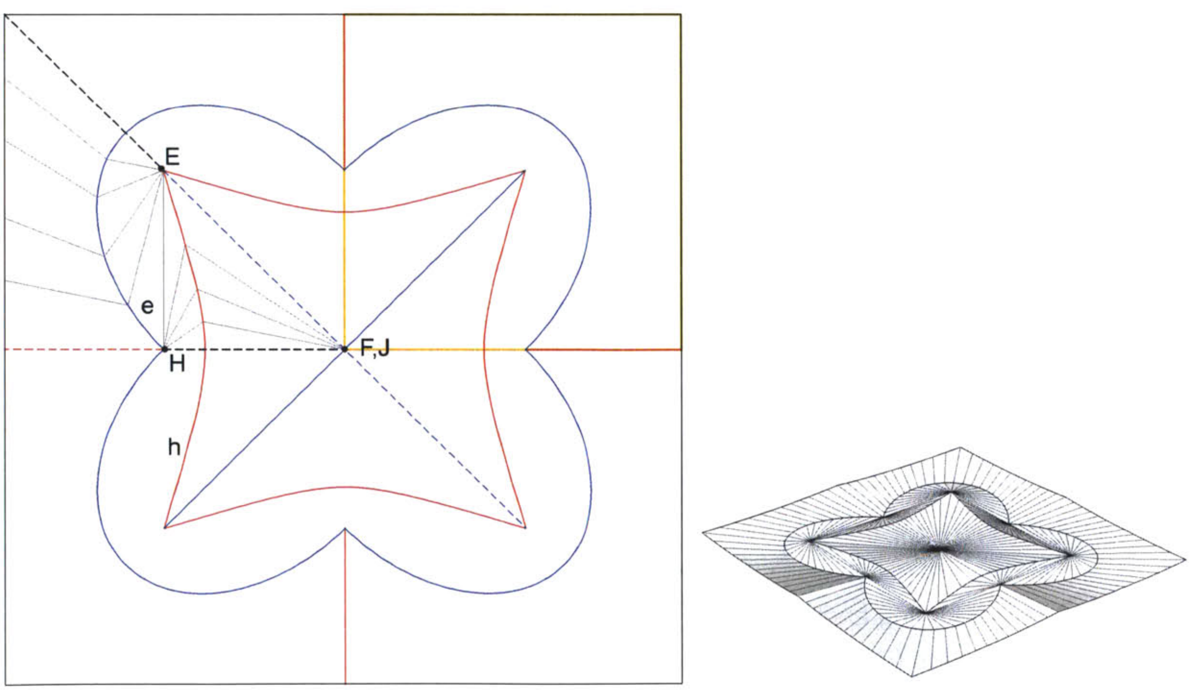
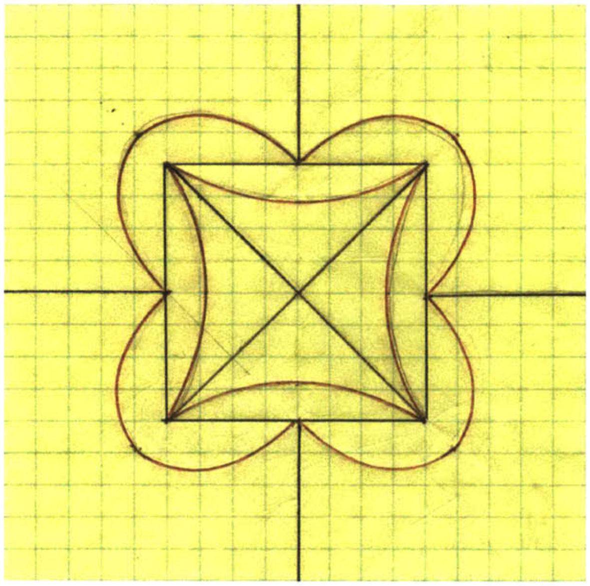


Fig 4.8.20 Crease pattern [DK], Simulated model [PC]



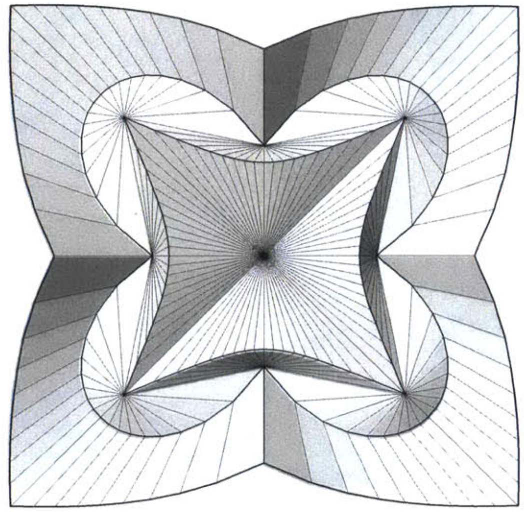


Fig 4.8.21 Paper model (undated, DAH [DK]), Simulated model [JH]

Crease pattern and ruling analysis

The gadget consists of a very similar triangular prototile (Fig 4.8.22). The ellipses and the hyperbolas share the same foci with similar alignments. The new valley crease between *E* and *H* forces the ellipse and the hyperbola into mountain creases. The rulings follow a very similar path to the previous design.

The altered mountain and valley assignments give this example its distinct visual feature. The model folds well during simulation (Fig 4.8.21 right).

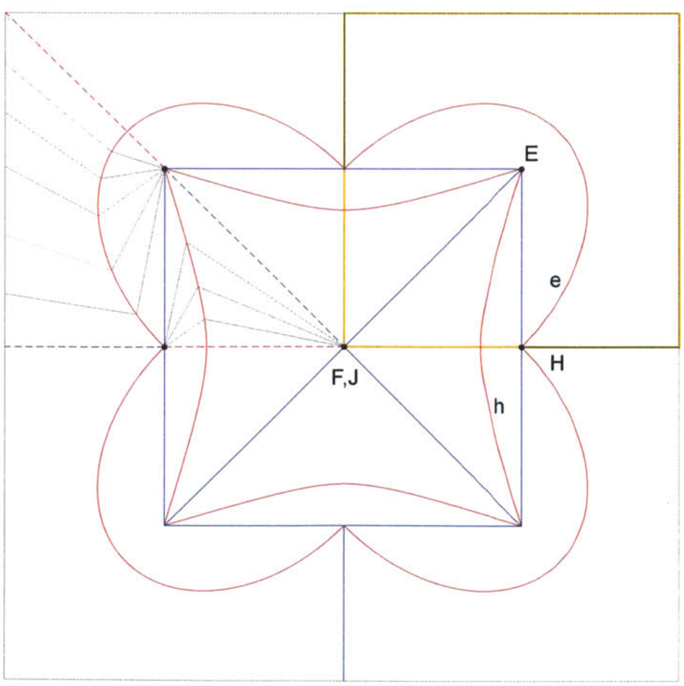


Fig 4.8.22 Crease pattern [DK]

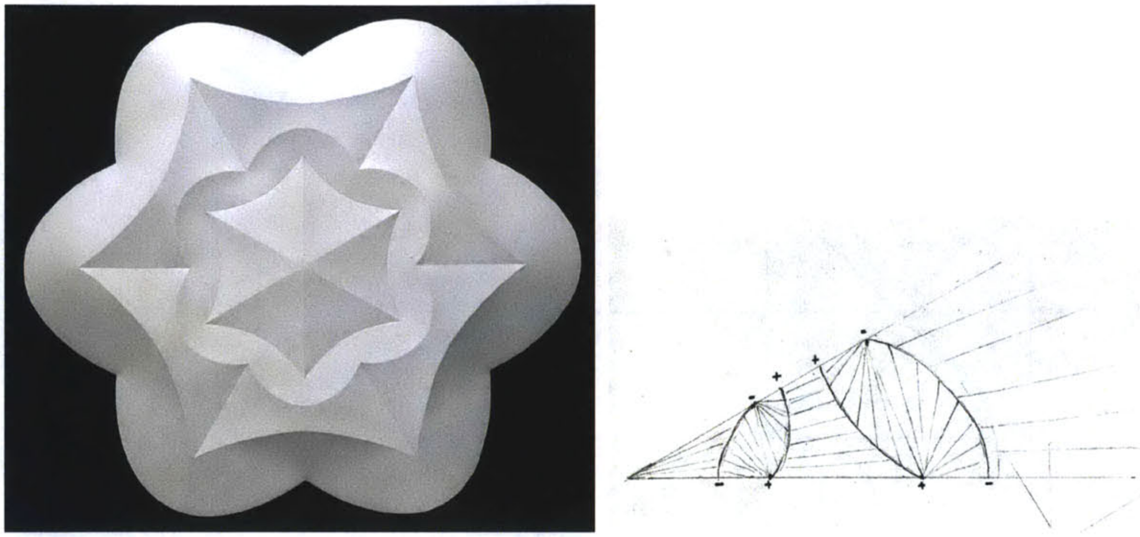


Fig 4.8.23 Vinyl model (undated, DAH [TG]), Sketch (undated, DAH [DK])

Huffman designs a finite tiling that appears unrelated at first (Fig 4.8.23). Closer inspection shows that ellipses and hyperbolas play similar roles in the crease pattern.

Crease pattern and ruling analysis

Following the sketch (Fig 4.8.23 right) 2 ellipses and 2 hyperbolas with confocal foci create the gadget. Huffman uses the second ellipse as boundary to cut out the model. The crease pattern consists of 12 prototiles (Fig 4.8.24). The simulated model hardly folds.

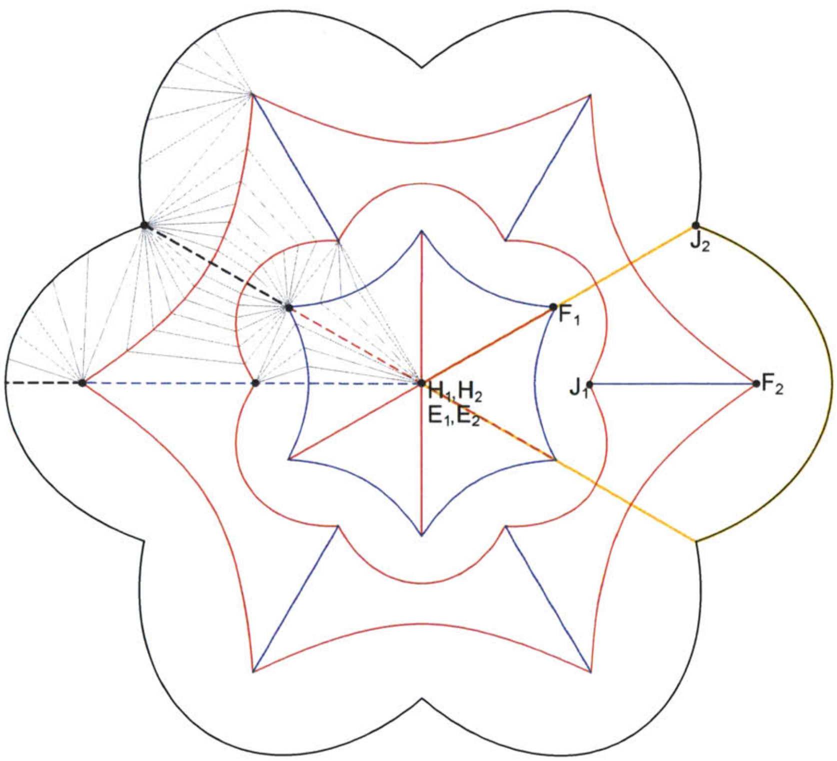


Fig 4.8.24 Crease pattern [DK]

Gadgets with elliptic and hyperbolic splines

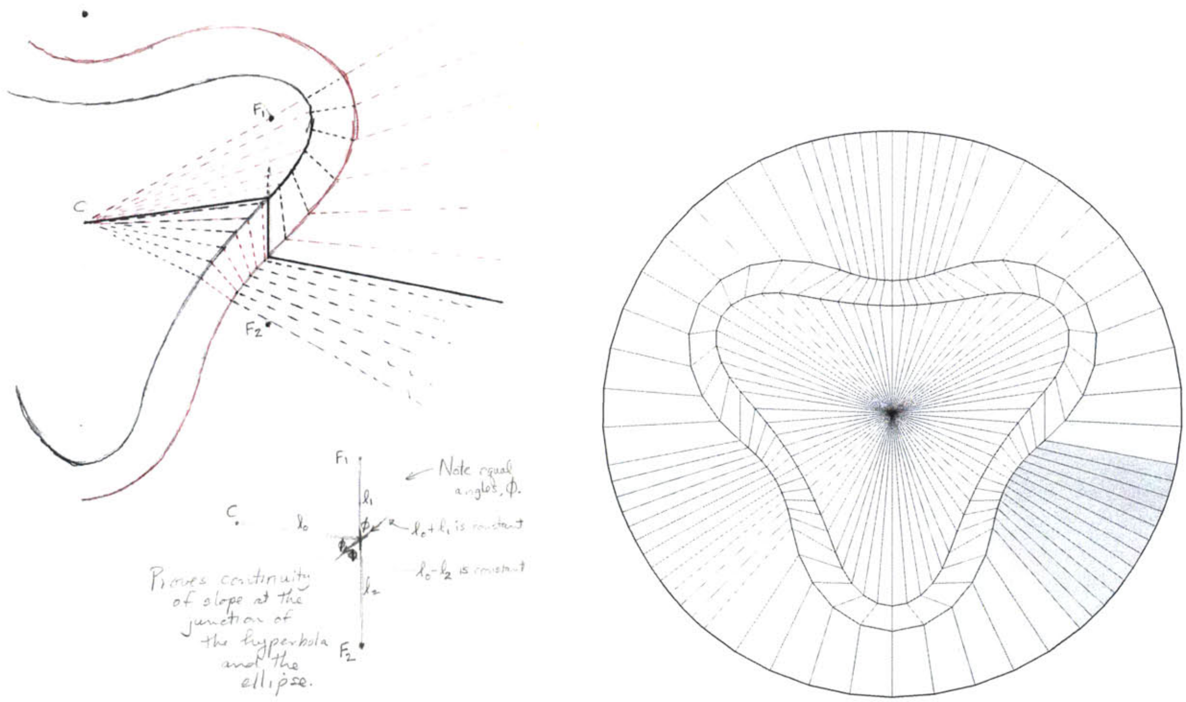


Fig 4.8.25 Sketch (undated, DAH, DK), Simulated model [PC]

Huffman experiments with a new form of quadratic splines consisting of 3 partial ellipses joined with 3 partial hyperbolas. He shows a proof for the smooth connection of the configuration in the above sketch (Fig 4.8.25).

Crease pattern and ruling analysis

The design consists of 2 splines with the above mentioned curves, one as mountain and one as valley (Fig 4.8.26). The rulings form a general cone with alternating curvature. The simulated model folds reasonably well (Fig 4.8.25 right).

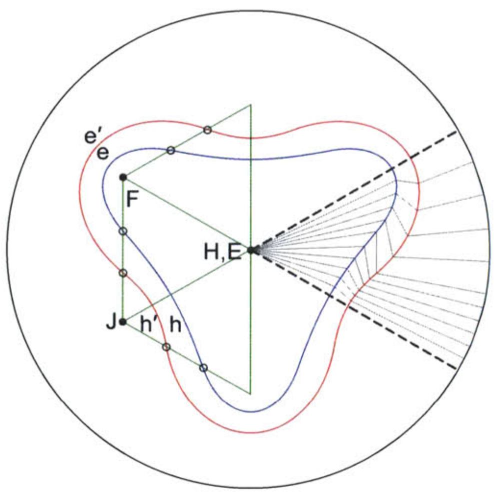


Fig 4.8.26 Crease pattern [DK]

4.9 Gadgets with ellipses, parabolas and hyperbolas

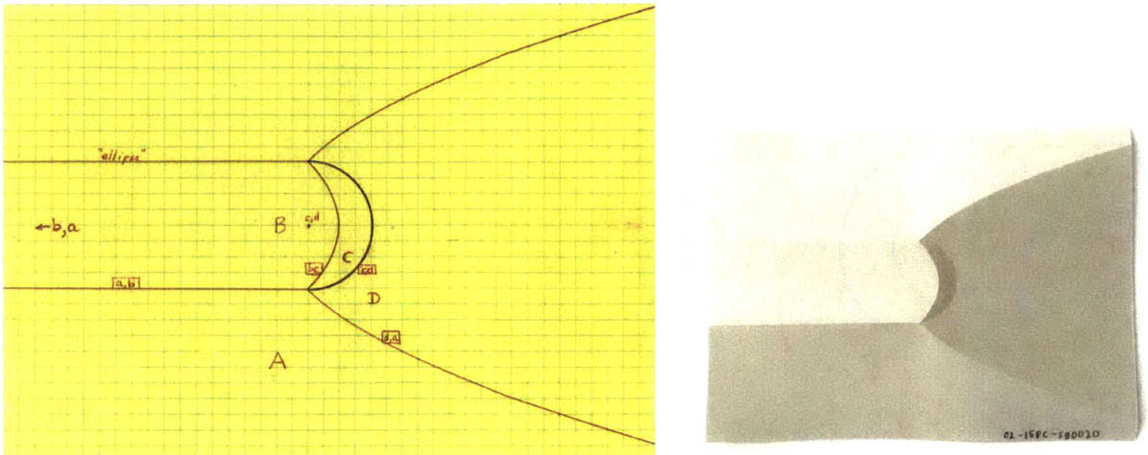
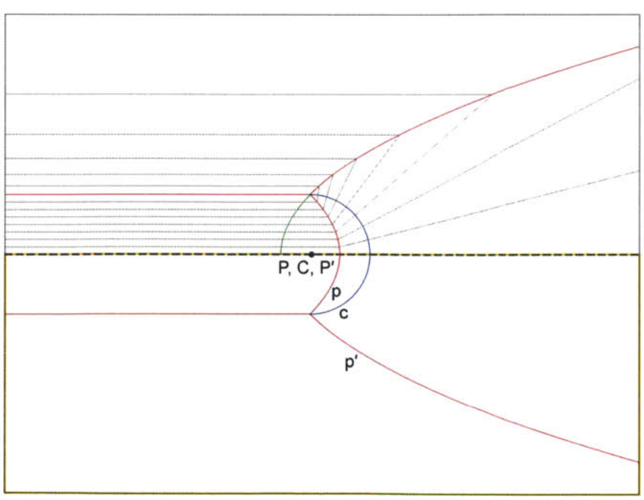


Fig 4.9.1 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The crease patterns in this section combine up to 4 different kinds of curves and all designs have a symmetry of *d1* in common. Huffman generally does not create designs that are only symmetrical along a single line, which makes this a rare series of 4 paper models. He appears to be interested in speculating on 'infinitely stretching' ellipses, which results in the straight line segments on the left side of the following examples. The right halves of the designs show similarities to designs in the previous 2 chapters.

Crease pattern and ruling analysis

The first crease pattern consists of 2 parabolas and a circle. Huffman adds line segments where the 3 curves intersect (Fig 4.9.2). The rulings follow Huffman's frequent base case of a cylinder at the center. Their first refraction remains unaltered when they pass through the circle and they change direction at *p'* to become parallel again. Simulation fails as the model does not fold, but the paper model provides a visual impression of the design (Fig 4.9.1 right).



a = 2 *[6]*
d = 0 *[0]*
a' = 2 *[6]*
l=4 *[12]*

Right side same as (Fig 4.7.2)

Fig 4.9.2 Crease pattern [DK]

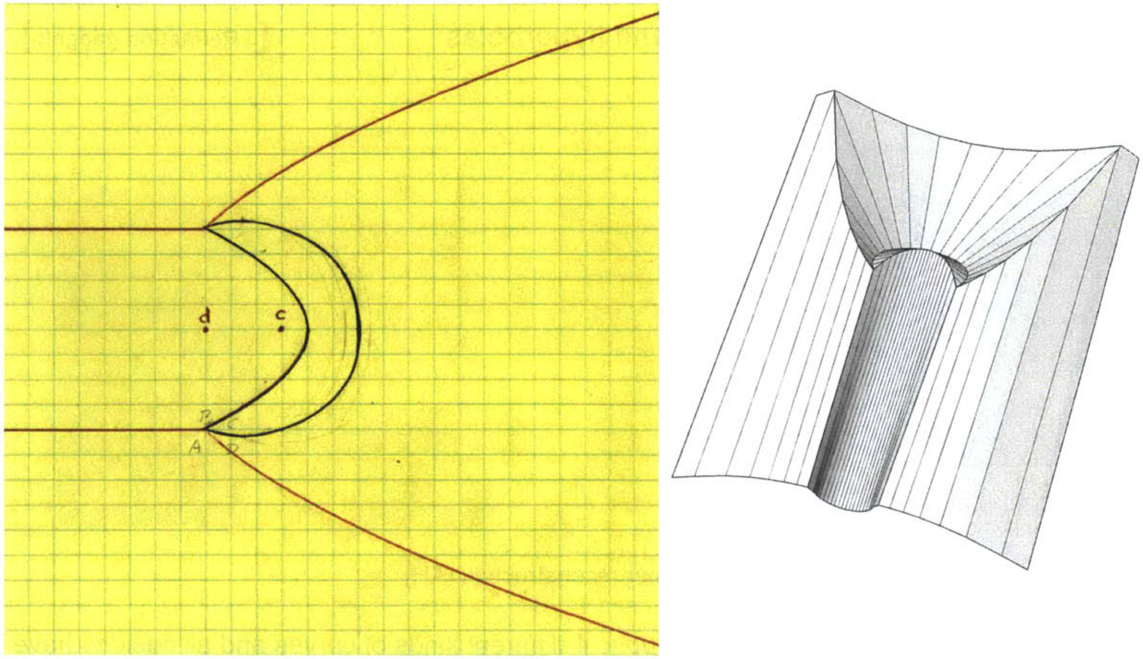


Fig 4.9.3 Paper model (undated, DAH [DK]), Simulated model [PC]

The second crease pattern Huffman draws consists of 2 parabolas and an ellipse combined with similar line segments as in the first design (Fig 4.9.3).

Crease pattern and ruling analysis

The prototile uses 2 partial parabolas and a partial ellipse with similar mountain and valley assignments (Fig 4.9.4). The ellipse has 1 focus on the vertical that passes through the intersecting curves and shares the second focus with *p*. The second parabola, *p'*, shares its focus with the ellipse on the vertical in *E*. The rulings change direction 3 times to become parallel again. Simulation shows inconsistent curvatures in the cylindrical areas (Fig 4.9.3).

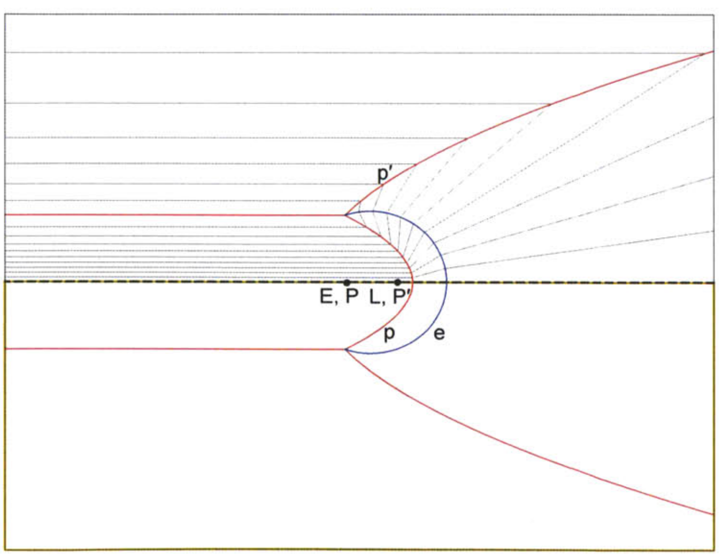


Fig 4.9.4 Crease pattern [DK]

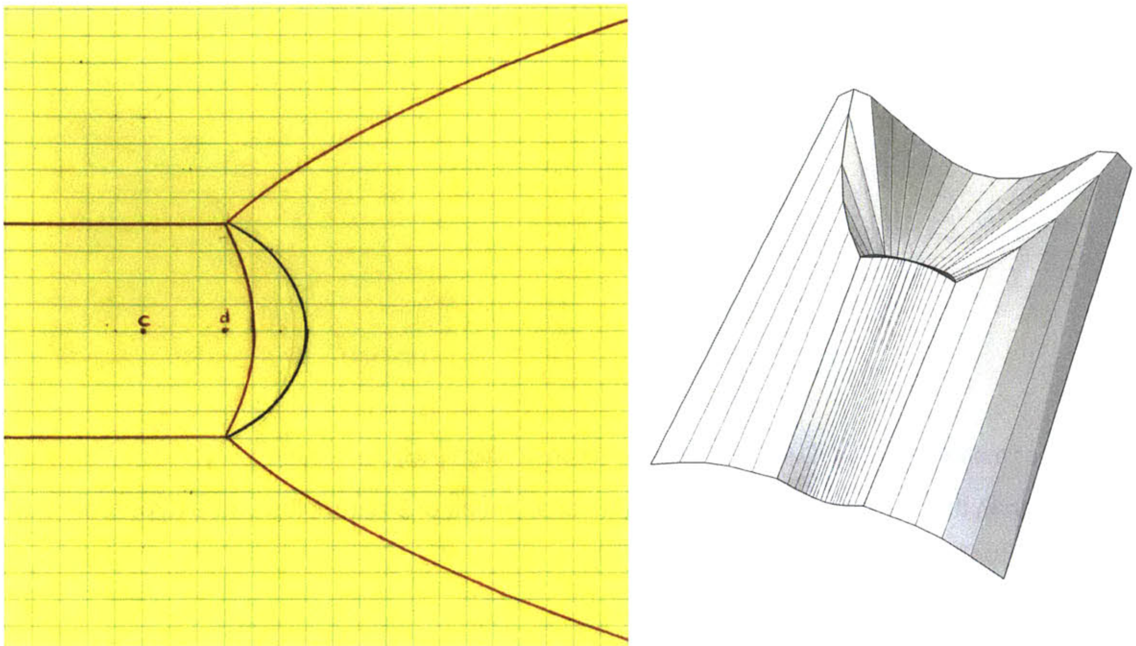


Fig 4.9.5 Paper model (undated, DAH [DK]), Simulated model [PC]

In the third design Huffman combines a similar set of curves, but moves the foci to different locations (Fig 4.9.5).

Crease pattern and ruling analysis

The prototile uses 2 partial parabolas and a partial ellipse with similar mountain and valley assignments (Fig 4.9.6). The ellipse now has its right focus on the vertical that passes through the intersecting curves and shares it with *p'*. The first parabola, *p,* shares its focus with the left focus of the ellipse in *E,P*. The rulings change direction again 3 times to become parallel. Simulation shows inconsistent curvatures in the conical areas (Fig 4.9.5 right).

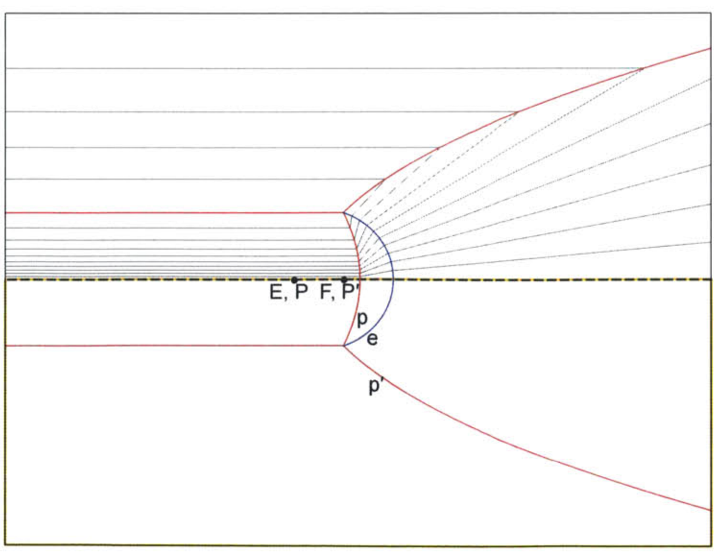


Fig 4.9.6 Crease pattern [DK]

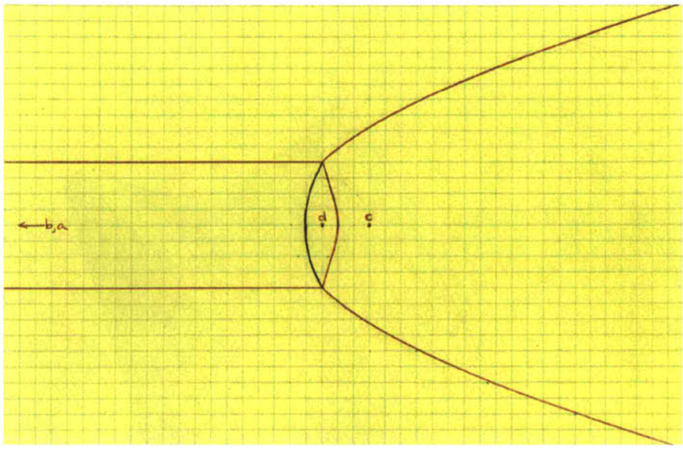
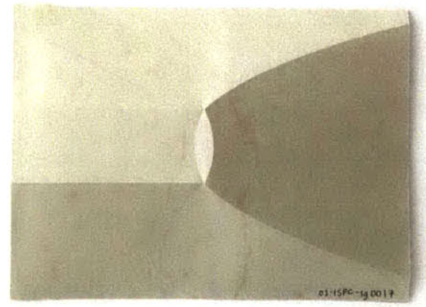



Fig 4.9.7 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The last design in this short section consists of a combination of 2 parabolas and a hyperbola that are connected to the linear segments (Fig 4.9.7).

Crease pattern and ruling analysis

The prototile consists of the first partial parabola Huffman needs for the central cylinder, a subsequent partial hyperbola and lastly the second partial parabola (Fig 4.9.8). The mountain and valley assignments differ from the previous examples. The hyperbola shares both its foci with the 2 parabolas. The first parabola, oriented in the opposite direction from the hyperbola, forces the rulings toward the horizontal center line. The hyperbola redirects them upward and the second parabola aligns the rulings horizontally as in the previous examples.

Simulation does not give any meaningful result and even the paper reconstruction has difficulty assuming the folded configuration (Fig 4.9.7 right).

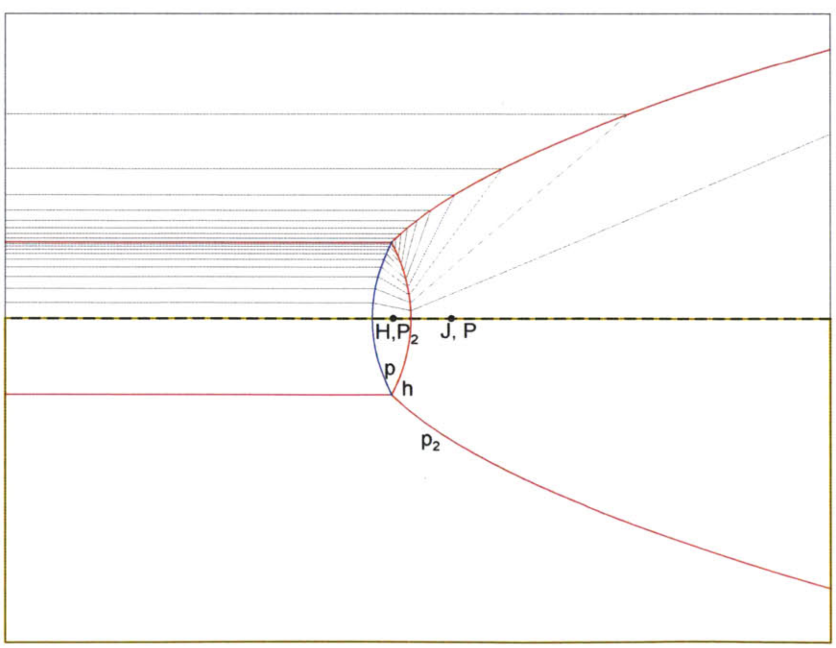


Fig 4.9.8 Crease pattern [DK]

4.10 Cone and cylinder gadget

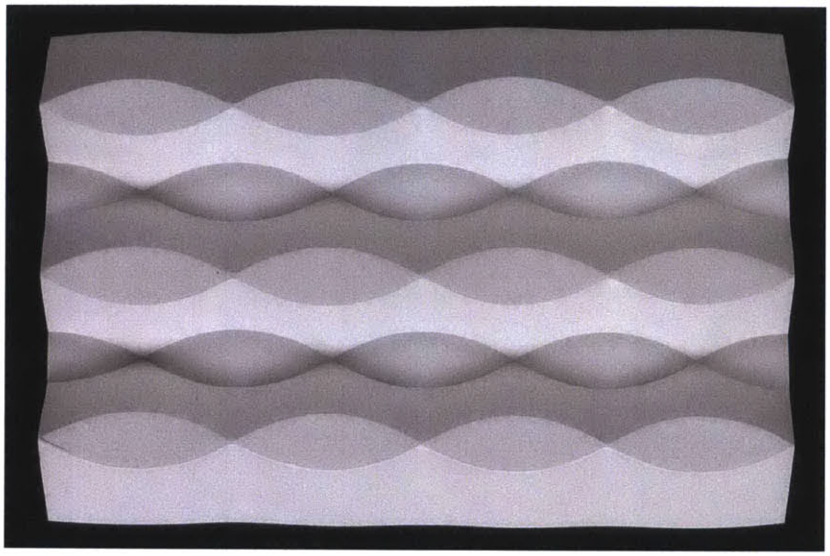


Fig 4.10.1 Vinyl model (1992, DAH [TG])

This section discusses a single example, which Huffman explores as a sketch and as a vinyl model (Fig 4.10.1). He does not reveal his thoughts on the type of curve or location of rulings, but spends the effort to make a vinyl model and photographs it in 1992 and again in 1998.

Crease pattern and ruling analysis

Curve fitting does not suggest any specific curve Huffman might have used. The sketch might be part of a series of drawings that uses scaled sine curves. However circle arcs with their center on the graph paper appear to be a good guess. The upper row of lens-like shapes made of 2 convex curves consists of mountain creases and the next row of valley creases. They are moved horizontally by half the width of one lens. No traces of rulings can be found on the sketch and Huffman

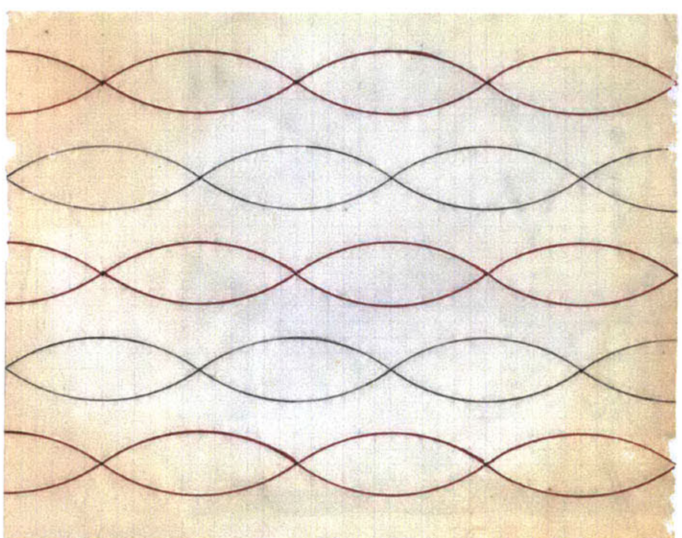


Fig 4.10.2 Sketch (undated, DAH [DK])

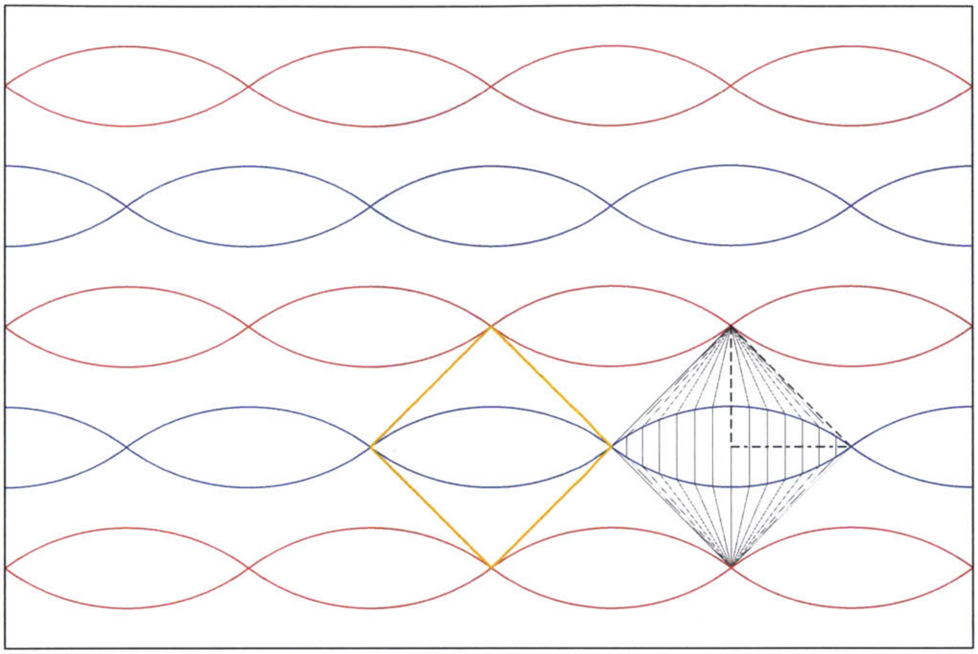


Fig 4.10.3 Crease pattern [DK]

does not appear to fold the sketch (Fig 4.10.2).

One possible solution for the rulings consists of cylindrical segments within the lens, and partial cones above and below. Their apices fall onto the intersection of the 2 neighboring lenses (Fig 4.10.3). The monohedral tiling has a triangular prototile and 4 of them create the diamond-shaped design tile.

This tiling exists with many types of convex curves in the design tile. The proof is based on the characterization of the qualitative behavior of the rulings, while the crease pattern also needs to fulfill certain requirements. A smooth 3d crease can not have incident cone rulings and

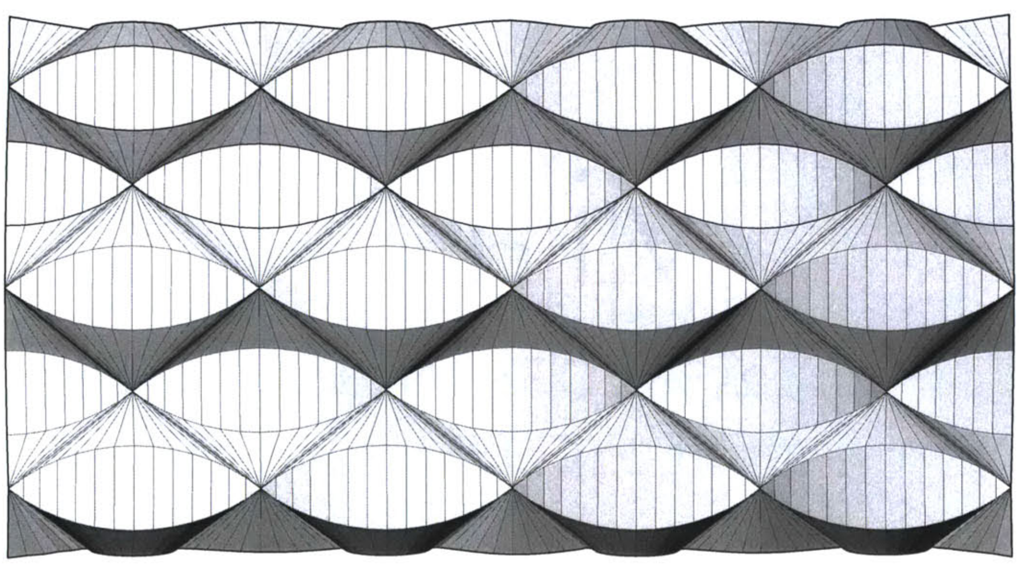


Fig 4.10.4 Simulated model [AH]

2 concave curves cannot be joined if the mountain and valley assignments are different [DDHKT 14].

These properties force the rulings between the 'lenses' to be particular cones with their apices falling on the previously mentioned vertices of the tiling. The tile edges need to have a common tangent to their neighbor. The crease pattern is also rigidly foldable.

The last requirement regarding non-reflecting rulings is curious as Huffman typically does not work with such an assumption. Since he doesn't reveal his thoughts on this design, it remains unclear how he intended to resolve the position of the rulings.

Huffman makes a discrete version of the design (Fig 4.10.5). His paper model might give us a hint about his thoughts regarding the rulings. The discrete tiling shows similar lenses with alternating mountain and valley assignments. The pencil marks indicate that Huffman wants to explore another discrete version with additional creases, but they could also relate to the ruling location of the continuous version.

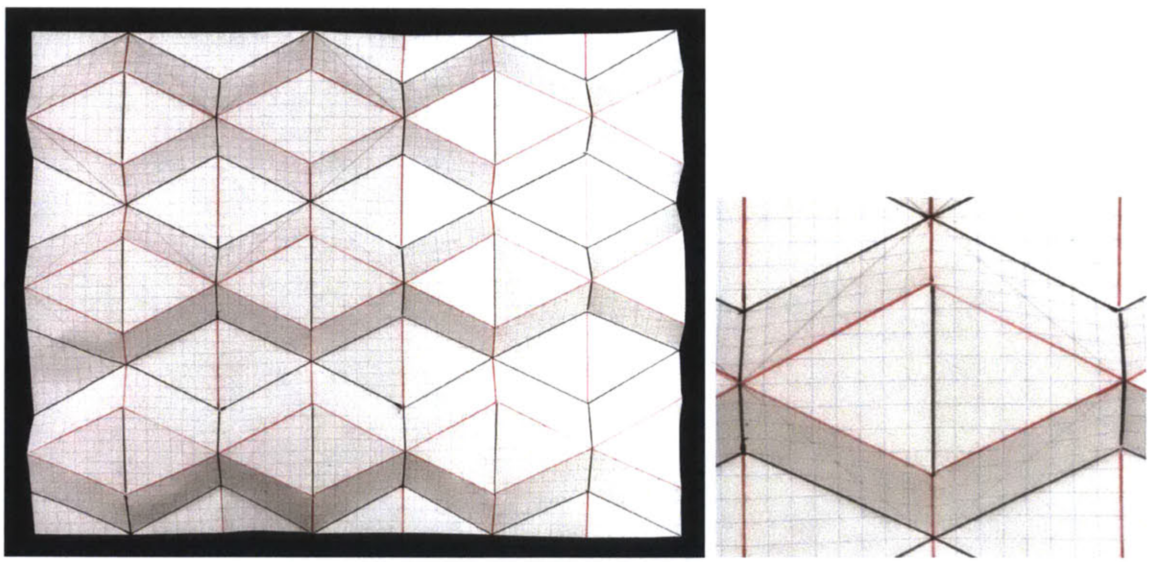


Fig 4.10.5 Paper model (undated, DAH [EAH]), Identical model (undated, DAH [EAH])

4.11 Cyclic tilings with converging curves

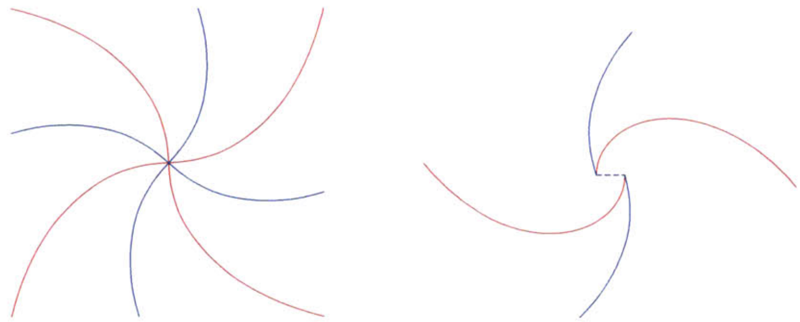


Fig 4.11.1 Cyclic tilings with converging curves [DK]

This section discusses designs by Huffman that convey little to no information on the location of rule lines. No gadgets can be drawn for this and all following sections as the ruling path can not be constructed.

The order in which the designs are presented depends on the kind of tiling Huffman is using. Most examples belong in a category of cyclic finite tilings. The center area around which the prototiles rotate usually consists of a vertex, but Huffman investigates other options that include line segments and polygons (Fig 4.11.1).

Huffman decides to work with converging curves or spirals and produces many sketches of different kinds of curves. Curve fitting suggests that 3 candidates are used in his designs, the pursuit curve, the tangent spiral and a spiral defined by the relationship $r = \Phi$ (Fig 4.11.2). Huffman plots the curves in various sizes and might have scaled the curves in order to fit them within the boundaries of his drawings.

If crease patterns are available, I try to fit these or other curves in his archive. When I find a likely match I also make a paper model and show the reconstruction. In some cases, Huffman keeps a crease pattern, but no image of the folded model exists. I make paper reconstructions of these designs even when fitting curves is not successful in order to provide the reader with

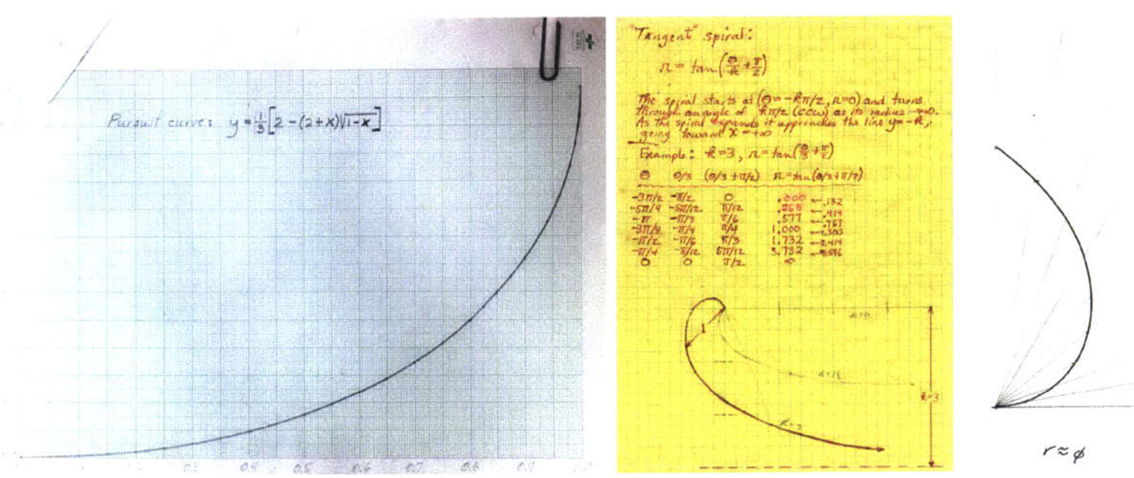


Fig 4.11.2 Notes and drawings (undated, DAH [DK])

Tilings with a central vertex

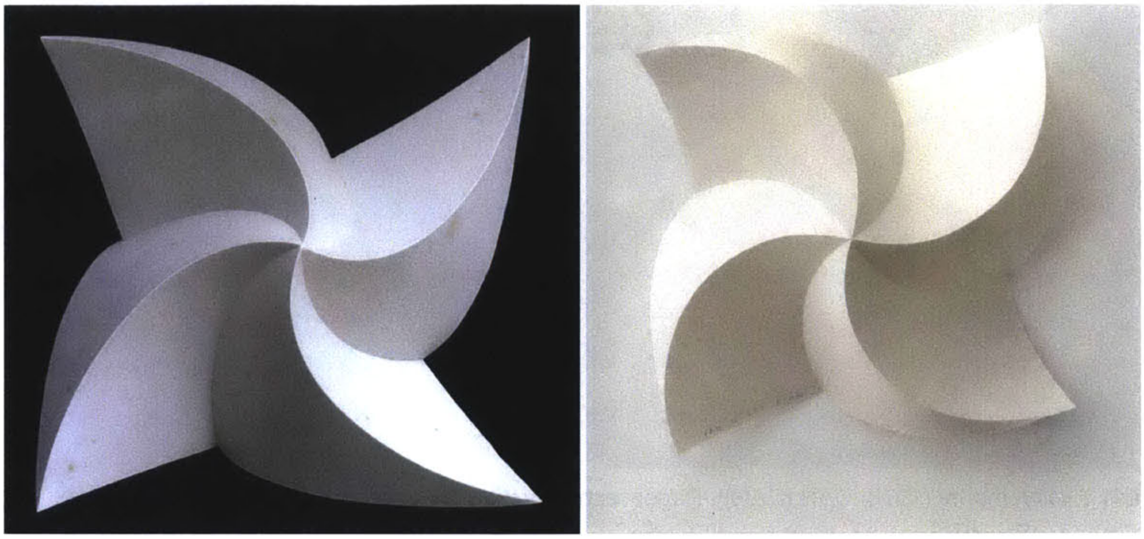


Fig 4.11.3 Vinyl model (undated, DAH [EAH]), Paper reconstruction [AH]

visual feedback on the folded configuration. If a reconstruction needs to be made based on the photograph of a model, I have to revert to a trial and error process, which rarely yields reasonably precise results.

The first tiling in the section, reconstructed from an image of the folded model, most probably consists of 8 pursuit curves. A possible inspiration for the design consists of a small sketch (Fig 4.11.4). Huffman draws on top of a figure on page 143 of his own copy of 'Mathematical Snapshots' by H. Steinhaus. The diagram shows the concept of a pursuit curve based on 4 points that start walking along one edge, but orient themselves toward the point in front of them. He appears to draw possible directions for rule lines.

Crease pattern

The design consists of 4 mountain and 4 valley creases, all identical and rotated around the center with equal rotation angles.

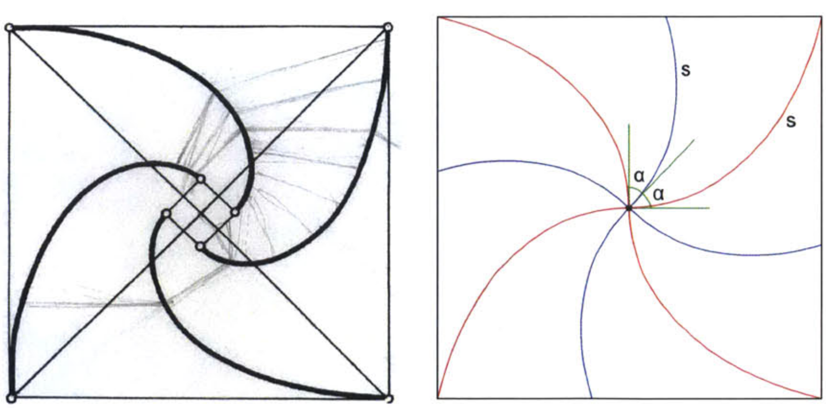


Fig 4.11.4 Sketch on figure (undated, DAH [DK]), Crease pattern [DK]

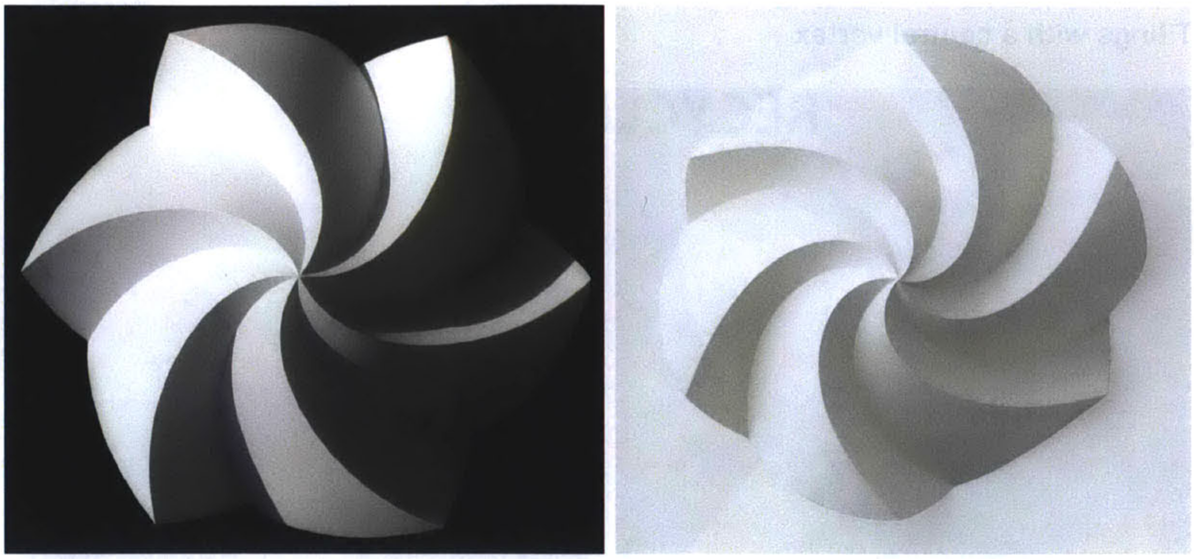


Fig 4.11.5 Vinyl model (1978, DAH [DAH]), Paper reconstruction [AH]

The above vinyl model, also visually reconstructed from an image of the folded artifact, might also consist of pursuit curves (Fig 4.11.5). The reconstruction is folded with smaller folding angles, but displays many similar visual features in terms of surface, shape, light reflection and outline.

Crease pattern and notes

The design consists of 6 mountain and 6 valley creases, probably all identical and drawn as scaled pursuit curves (Fig 4.11.6). The cyclic tiling uses rotation angles of approximately $\alpha = 24°$ and $\beta = 36°$.

Huffman uses a material that can be poured and subsequently hardens similar to resin. He casts a dowel in place, which he then attaches to a wooden disc.

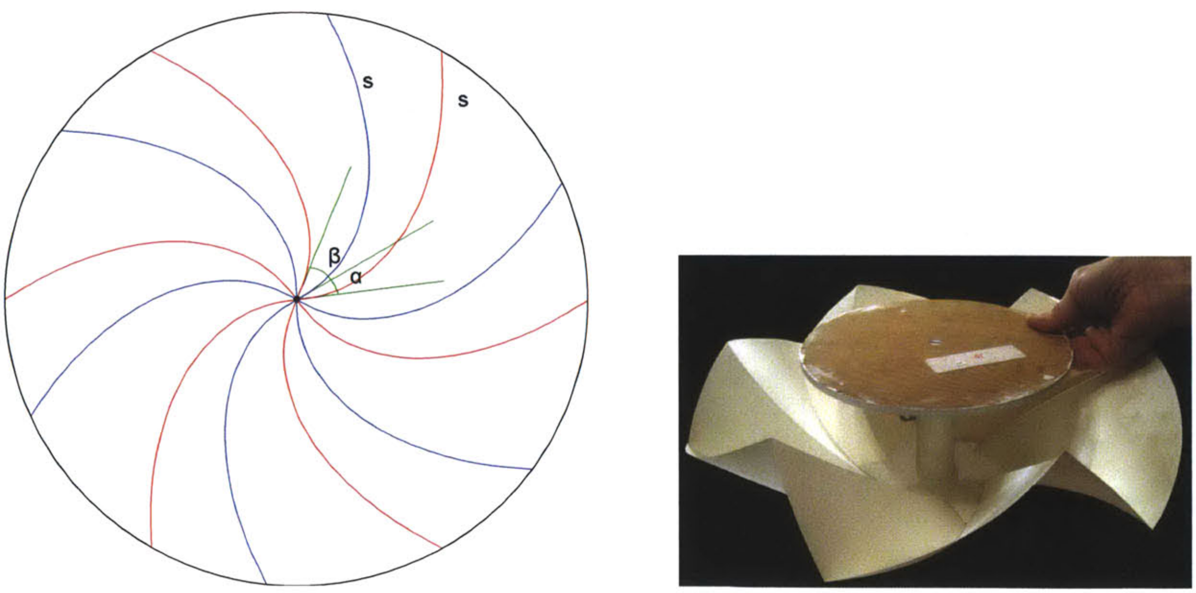


Fig 4.11.6 Crease pattern [DK], Detail of vinyl model [DK]

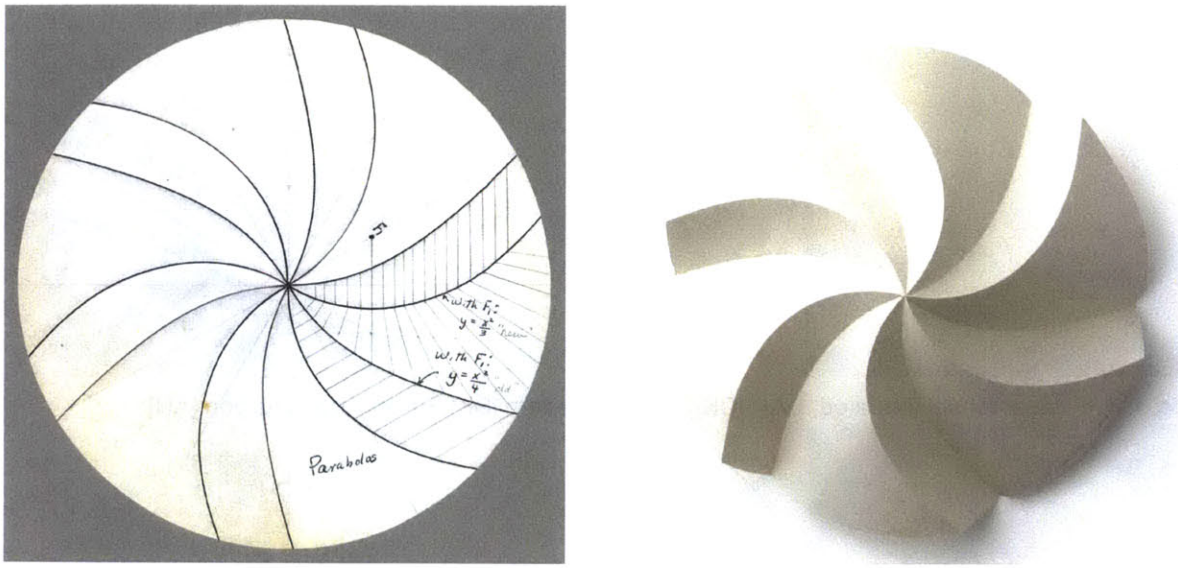


Fig 4.11.7 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The above design, which Huffman only makes as paper model, uses parabolas and is the only example in this section that uses quadratic curves. It is included this section with cyclic tilings with a central vertex, because it is based on similar tiling principles.

Crease pattern and notes

Huffman sets up pairs of confocal parabolas, where the first 'old' parabola has a vertical axis and the second 'new' parabola is scaled and rotated about the focus of the first parabola by 60° (Fig 4.11.8). The rulings, while known, converge toward a circle and make simulation difficult. The paper reconstruction displays the conical and cylindrical areas well.

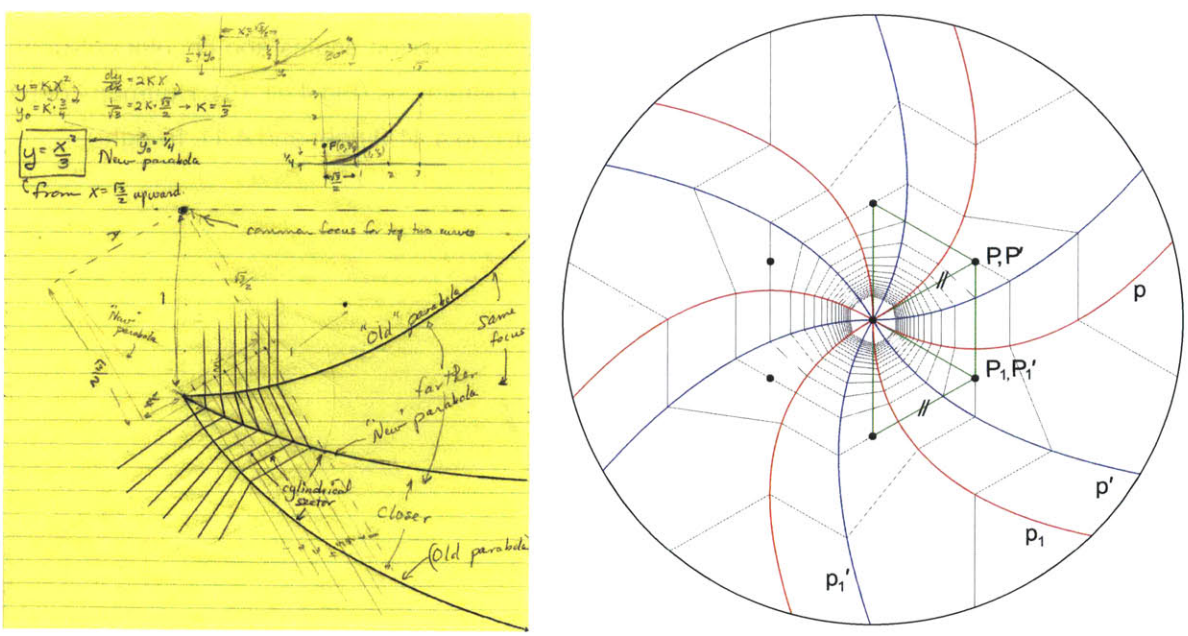


Fig 4.11.8 Sketch (undated, DAH [DK]), Crease pattern [DK]

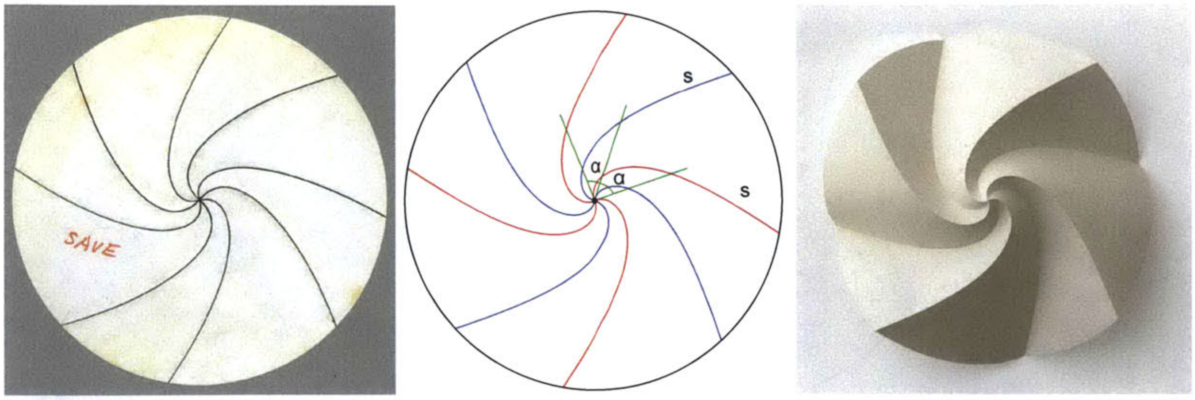


Fig 4.11.9 Paper model (undated, DAH [DK]), Crease pattern [DK], Paper reconstruction [AH]

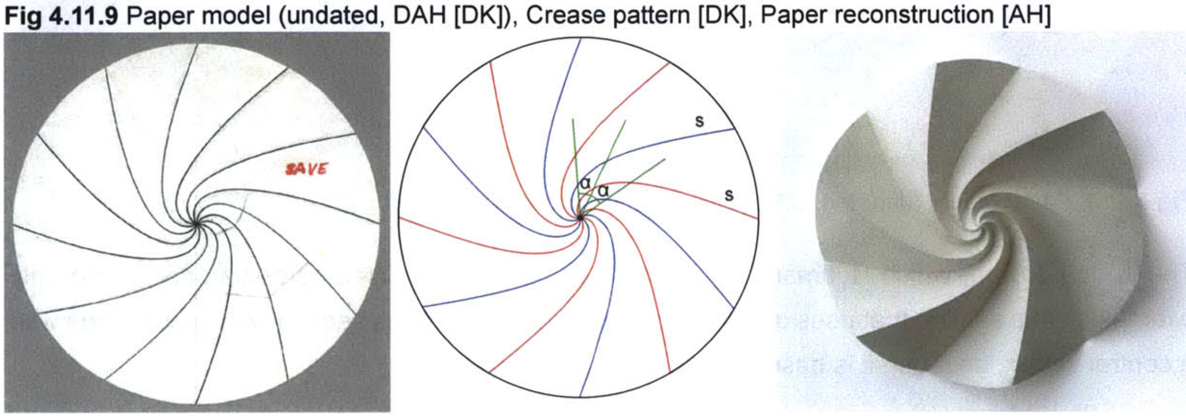


Fig 4.11.10 Paper model (undated, DAH [DK]), Crease pattern [DK], Paper reconstruction [AH]

Crease pattern and notes

The 2 examples above converge in a center and consist of the identical spiral (Fig 4.11.9 left and 4.11.10 left). Both designs are reconstructed via the use of Huffman's tangent spiral, drawn in pencil with a *k* value of $k = 1.5$ (Fig 4.11.11), which matches the creases well. The designs consist of 4 and 6 mountain and 4 and 6 valley creases with equal rotation angles and circular cut-outs. The pencil marks in his design appear to suggest ruling refractions (Fig 4.11.10). Huffman keeps no folded model and the paper reconstructions fold well (Fig 4.11.9 right and 4.11.10 right).

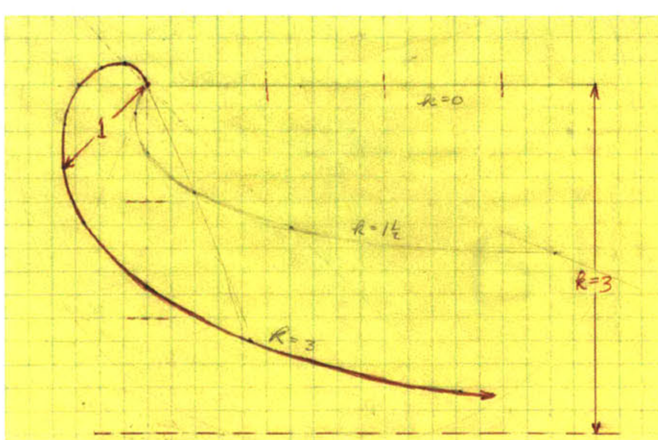


Fig 4.11.11 Sketch (undated, DAH [DK])

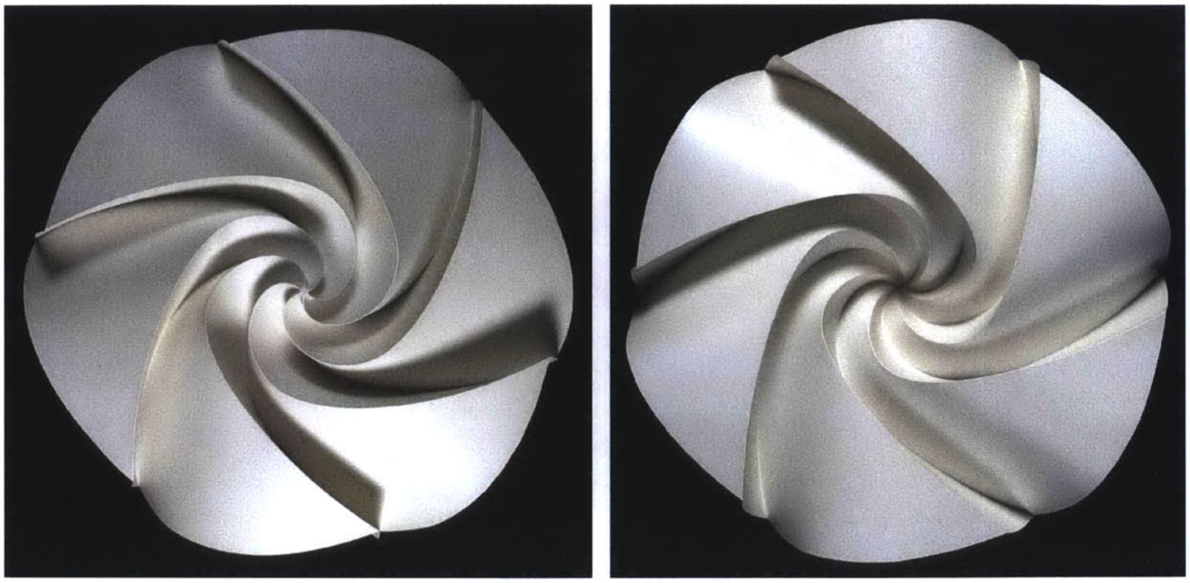


Fig 4.11.12 Vinyl model (undated, DAH [DAH]), Vinyl model (undated, DAH [DAH])

The above elaborate designs, which Huffman makes in vinyl, obtain the nickname 'Lemon juicer' from the family (Fig 4.11.12). The models represent prime examples of Huffman's craft and attention to detail.

Crease pattern and notes

Both designs are based on an identical crease pattern, which does not seem possible at first. Both probably use a scaled version of the tangent spiral. Huffman draws pencil guides in his crease pattern that demonstrate a scaling procedure based on a change in the y-direction, but curve fitting does not provide satisfactory results (Fig 4.11.13 left). The ratio of the of the *60°* rotations of the 6 mountain and 6 valley crease is approximately α = *15°* and β = *45°*. Huffman's crease

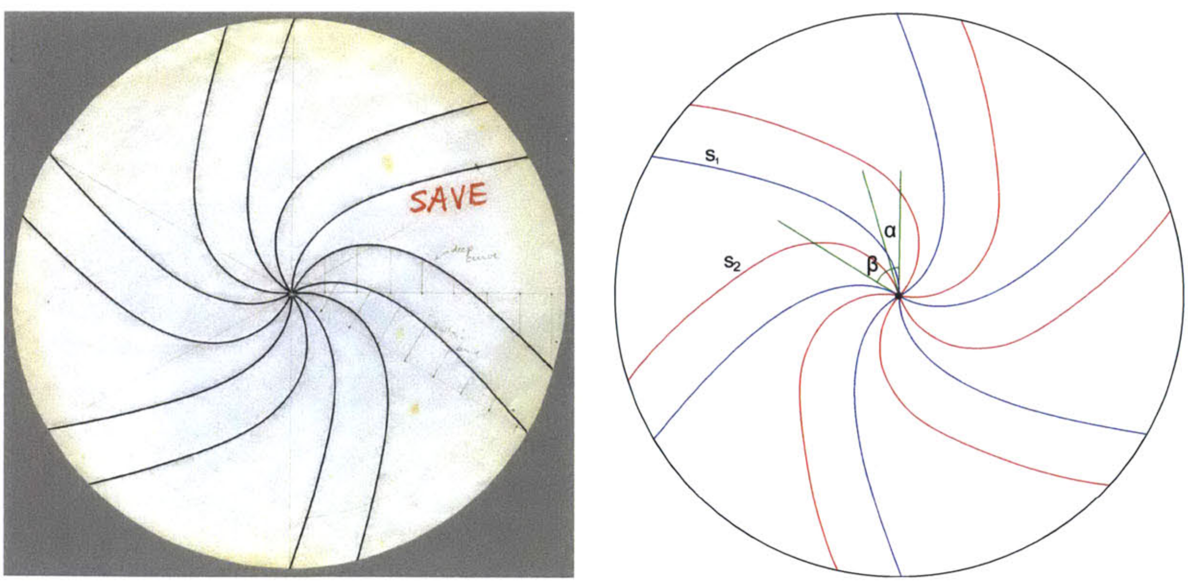


Fig 4.11.13 Paper model (undated, DAH [DK]), Crease pattern [DK]

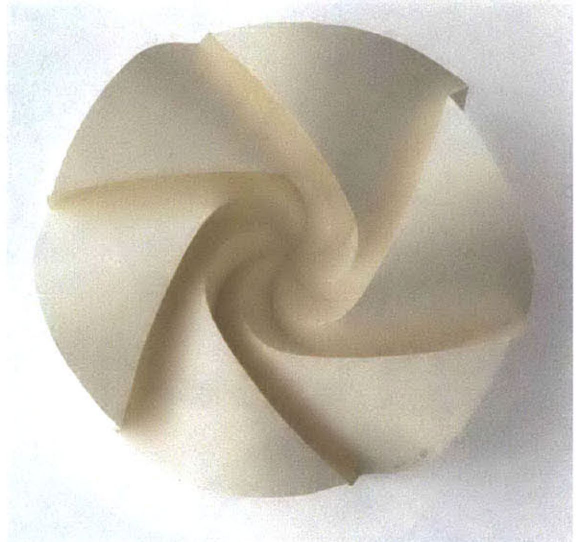
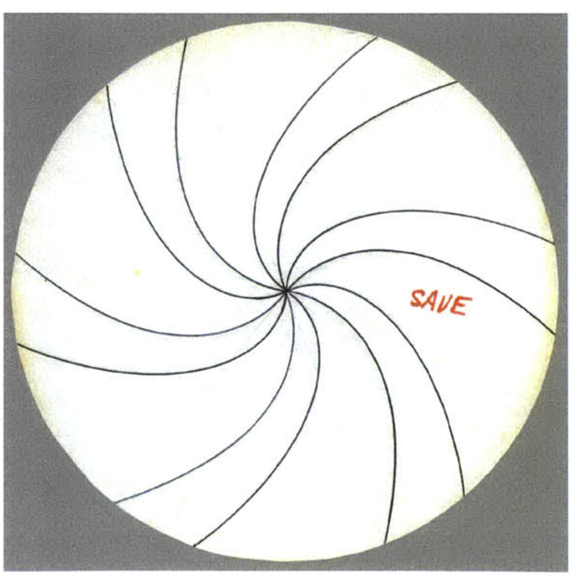


Fig 4.11.14 Paper reconstruction [AH], Paper reconstruction [AH]

pattern is drawn flipped relative to his vinyl model. While both models use identical mountains and valleys one can force the center vertex up or down in the early stages of folding. Subsequent folding produces the 2 reconstructed results in paper (Fig 4.11.14).

Huffman uses a technique similar to the previous design to adhere the vinyl to a wooden disc, but includes a cast cylinder to replace the dowel in this case.

The last example in this series consists of similar spirals, but curve fitting is inconclusive (Fig 4.11.15). The design consists of 6 mountain and 6 valley creases with a similar rotation angles. The paper reconstruction is based on mere tracing of Huffman's digitized crease pattern and displays slight visual differences to the previous designs.



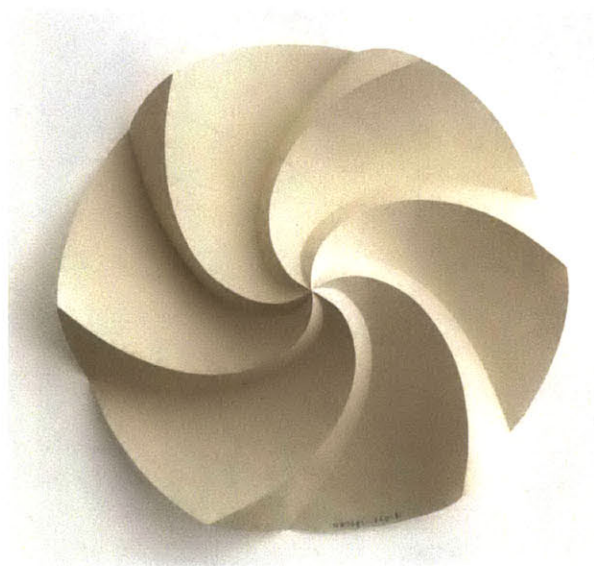


Fig 4.11.15 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Tilings with a central gap or line segment

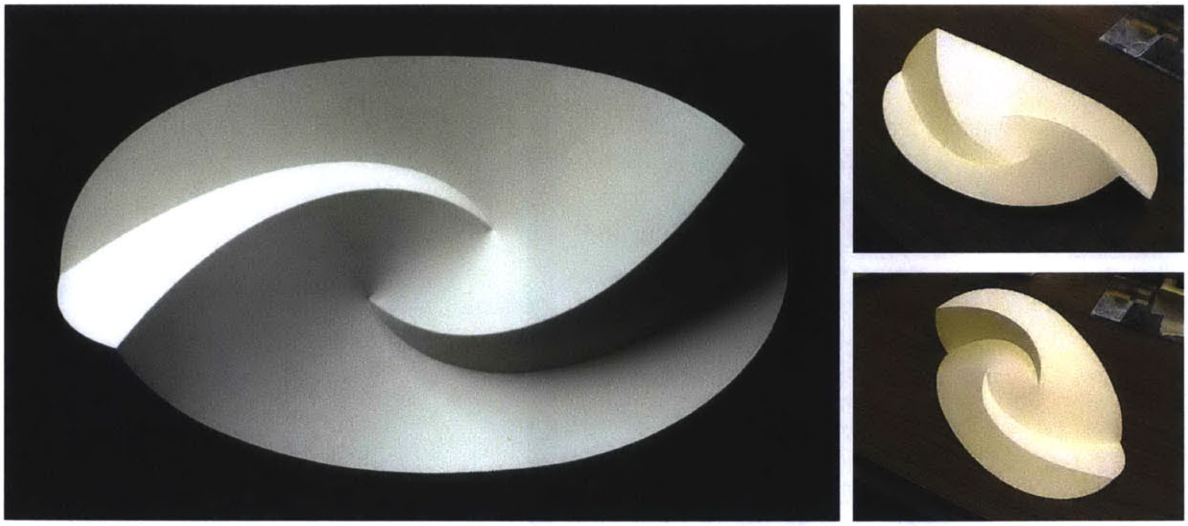


Fig 4.11.16 Vinyl model (undated, DAH [EAH]), possibly identical model (1990s, [PH)

The section now considers cyclic tilings without a central vertex, but with a gap instead. Huffman rarely uses creases that end in the middle of a surface, but does so in the design above which only exists in vinyl. According to Paul Haeberli, a computer scientist, who visits Huffman during the 1990's, this example consist of 4 free hand curves [Hae].

Crease pattern

2 different spirals form the 2 mountains and 2 valleys and visual reconstruction of the curves do not yield a satisfactory result. Huffman probably uses an ellipse to cut out the design.

Huffman appears to be interested in investigating the transition between the 2 vertices, where the spirals meet. He makes several drawings of discrete models with a gap and 4 spiraling creases such as the example below. The curves, however, are very different to the above vinyl model.

The discrete model folds reasonably well during simulation.

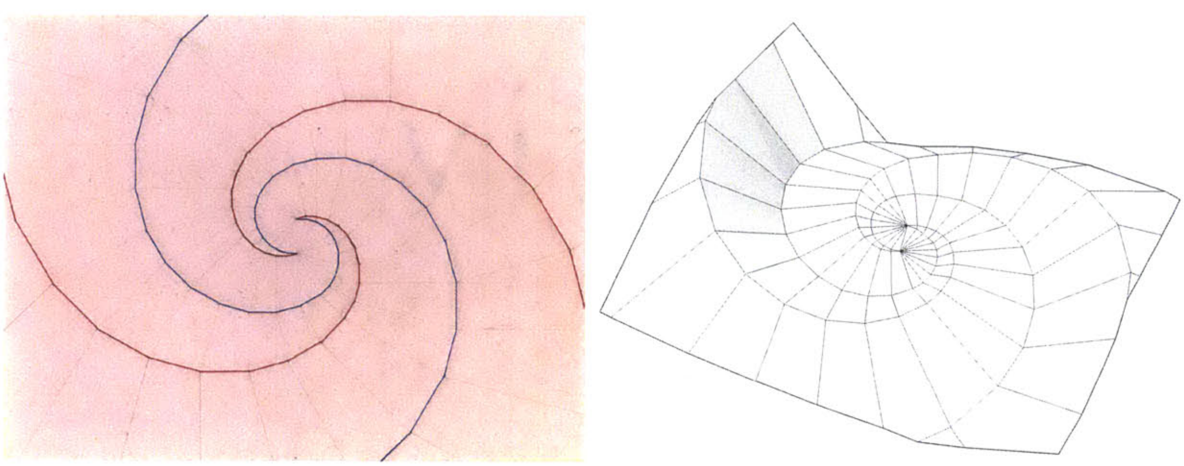


Fig 4.11.17 Paper model (undated, DAH [DK]), Simulated model [JH]

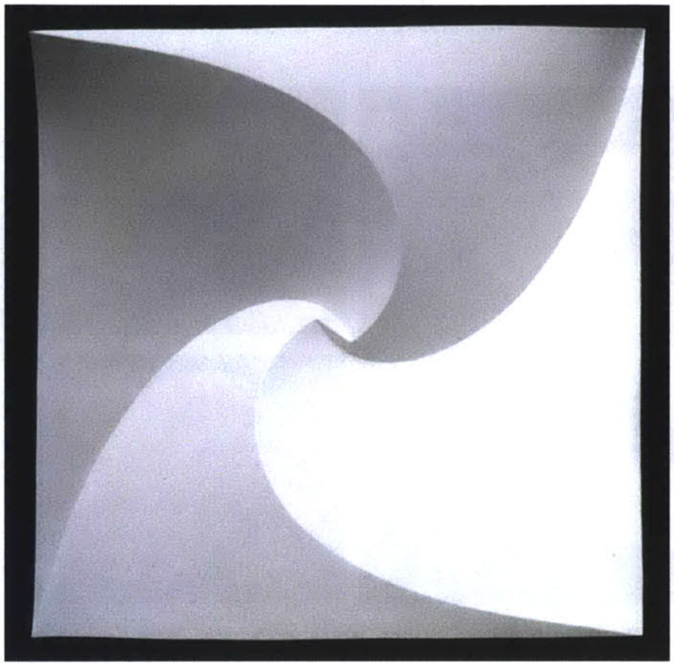
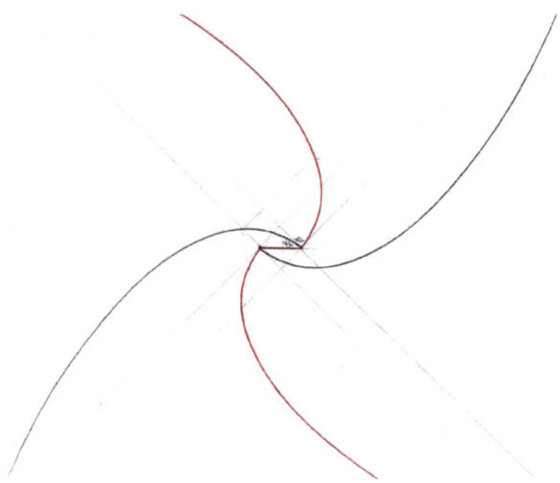


Fig 4.11.18 Vinyl model (undated, DAH [EAH])

The following series introduces a creased connection along a straight line between the 2 vertices such as the vinyl model above (Fig 4.11.18). The few designs Huffman makes with the new feature are finite cyclic tilings with 4 creases. Reconstructions are problematic as very few notes exist beyond his crease patterns.

Crease pattern and notes

The first example uses 2 different curves and the pencil lines suggest that he uses a spiral with the relation $r = \Phi$, but curve fitting is inconclusive (Fig 4.11.19 left). The pairs of mountain and valley creases intersect in the 2 vertices that are connected with a horizontal line segment. The paper reconstruction twists as it folds (Fig 4.11.19 right).



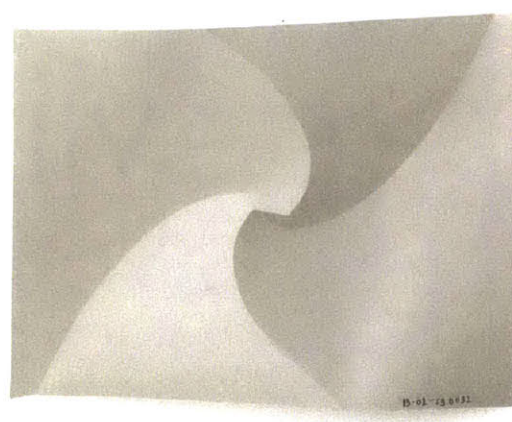


Fig 4.11.19 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

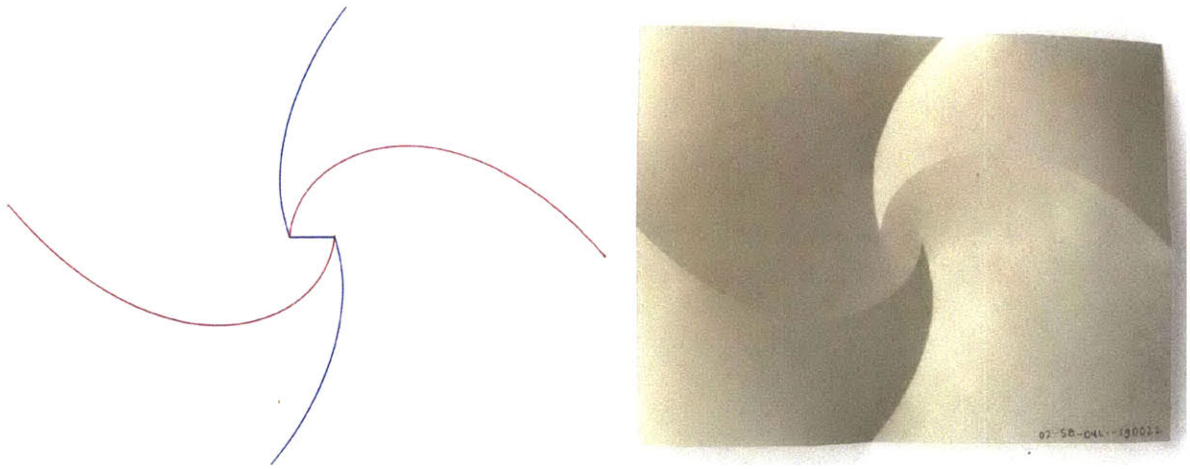


Fig 4.11.20 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman draws another variation of the previous design using the same principles, but lets the curves spiral in the opposite direction (Fig 4.11.20).

Crease pattern and notes

The design uses 2 different curves and curve fitting remains inconclusive. The pairs of mountain and valley creases intersect in the 2 vertices that define the series. Huffman uses the line as valley crease in this example.

The paper reconstruction does not fold well and the paper buckles (Fig 4.11.20 right).

Huffman seems to create a discrete versions (Fig 4.11.21) of the above crease pattern. It is possible that the discrete version precedes the continuous counterpart. Huffman would in that case look for a portion on one of his French curves that aligns with the vertices in the discrete design and draw a continuous 'approximation'. In order to be able to compare the 2 versions I superimpose the 2 crease patterns (Fig 4.11.21 right). The match is not perfect but convincing enough.

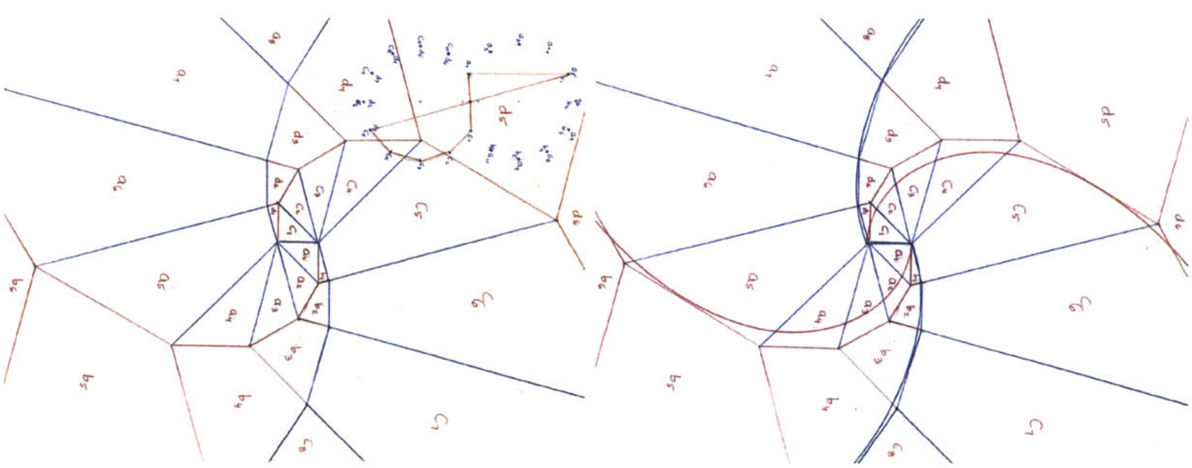


Fig 4.11.21 Paper model (undated, DAH [DK]), Composite [DK]

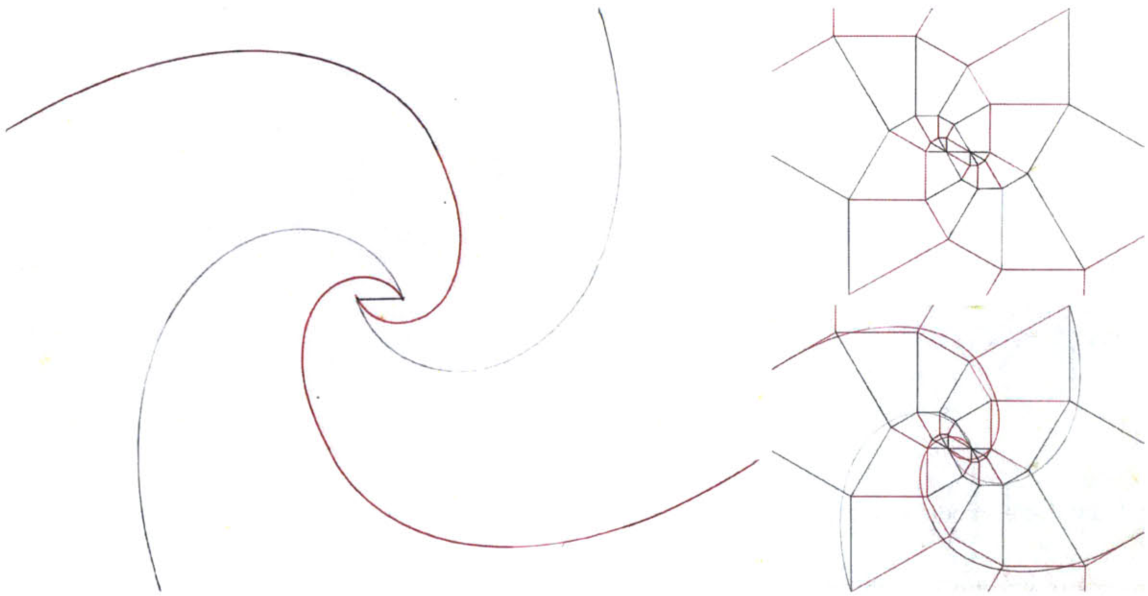


Fig 4.11.22 Paper model (undated, DAH [DK]), Paper model (undated, DAH [DK]), Composite [DK]

Another example with unknown curves fits in similar ways (Fig 4.11.22).

Crease pattern and notes

The long and unusual looking spirals in the above drawing follow the same principles as the previous 2 designs. The continuous crease pattern matches a discrete version surprisingly well (Fig 4.11.22 bottom right).

Tilings with central polygon

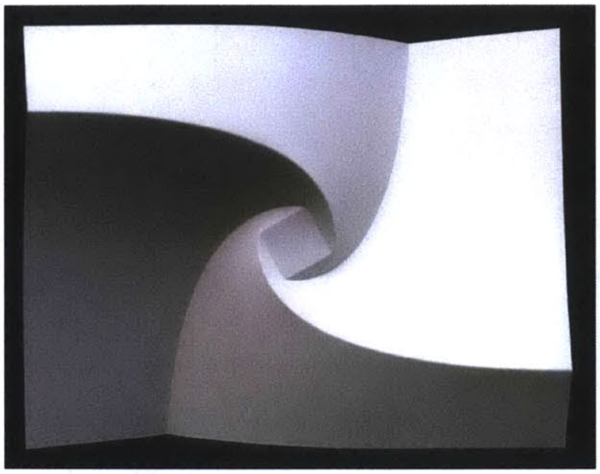


Fig 4.11.23 Vinyl model (undated, DAH [EAH])

Huffman experiments with polygons as an alternative for the center of cyclic tilings regarding the designs in this section. The order in which the designs are presented relates to the number of edges in the polygon. The first examples use squares, then hexagons and so forth.

No sketch or paper model exists of the above design with a square, which Huffman makes in vinyl (Fig 4.11.23). The creases consists of parabolic curves according to Paul Haeberli, who publishes a photo of this model on his web site [Hae]. However, my reconstruction efforts do not confirm the use of parabolas and suggest altered pursuit curves, but the results are not conclusive enough to produce a crease pattern.

Curve fitting for some of the following cases suggests that Huffman uses a spiral he constructs with polar coordinates in the drawing below (Fig 4.11.24) He plots the curve by using a specific relationship between the radius and the rotation angle. The definition of the spiral is $r = \Phi$.

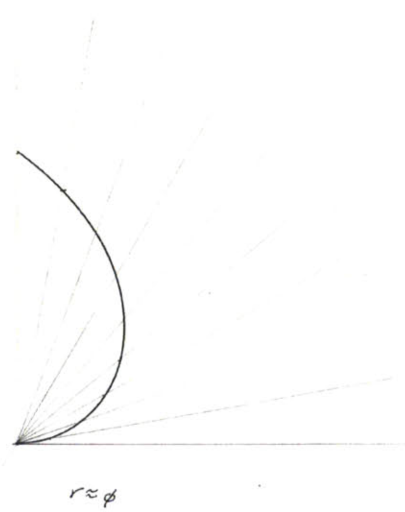


Fig 4.11.24 Sketch (undated, DAH [DK])

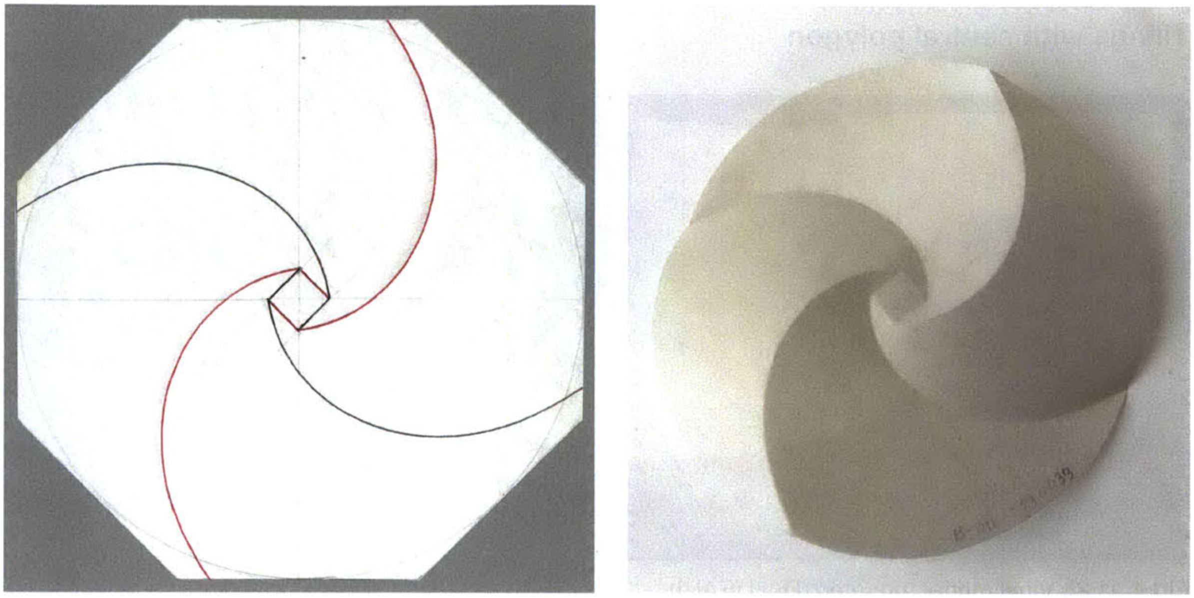


Fig 4.11.25 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

The above paper model is the second example that features a square at the center (Fig 4.11.25 left). He seems undecided about whether to cut it with an octagon or a circle.

Crease pattern and notes

The identical curves in the paper model fit the above mentioned spiral defined by $r = \Phi$ reasonably well and his pencil lines indicate possible construction guides (Fig 4.11.26). The pairs of mountain and valley creases of the previous designs with a single line segment obtain a different configuration as every individual curve now intersects with one corner of the square. The mountain and valley assignment of the line segments of the square alternate relative to the closest spiral.

The paper reconstruction, cut out with a circle in this case, folds well (Fig 4.11.25 right).

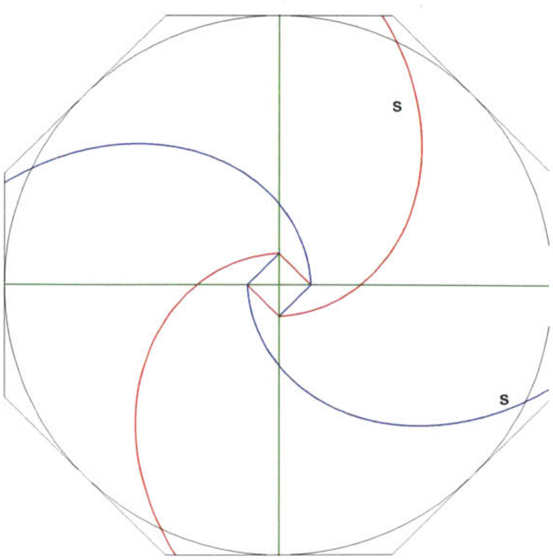


Fig 4.11.26 Crease pattern [DK]

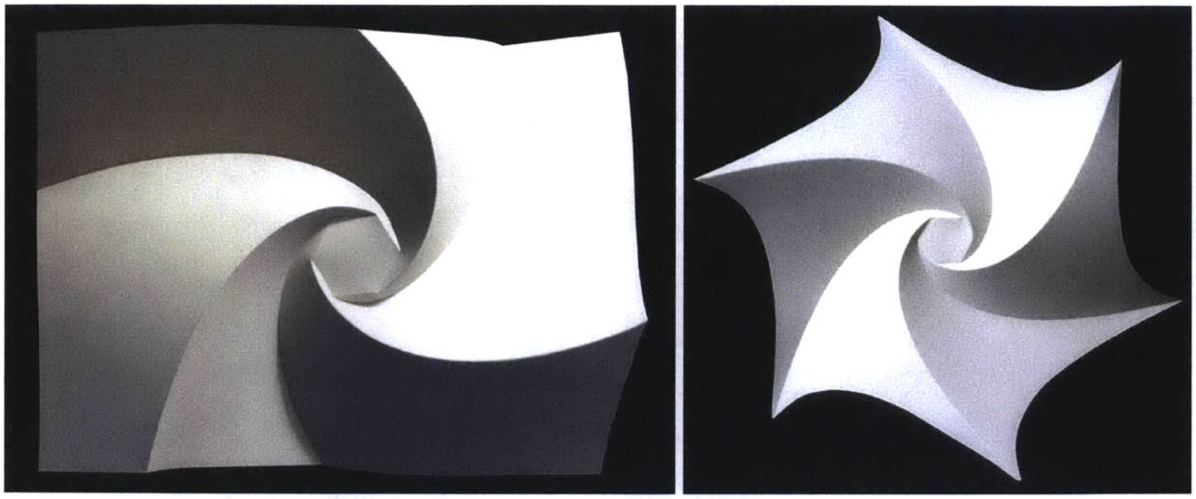


Fig 4.11.27 Vinyl models (undated, DAH [DK])

Huffman replaces the center square with a hexagon in the above designs (Fig 4.11.27). Both vinyl models share an identical set of curves, but the hexagons of the 2 models have different sizes.

Crease pattern and notes

The curves might consist of scaled spirals defined by $r = \Phi$, but curve fitting is inconclusive (Fig 4.11.28 left). The crease pattern consists of 3 mountains and 3 valleys that intersect individually with the corners of the hexagon. The mountain and valley assignment of the line segments of the hexagon follow the previous principle and alternate relative to the closest spiral.

The paper reconstruction, based on tracing Huffman's crease pattern, folds reasonably well (Fig 4.11.28 right).

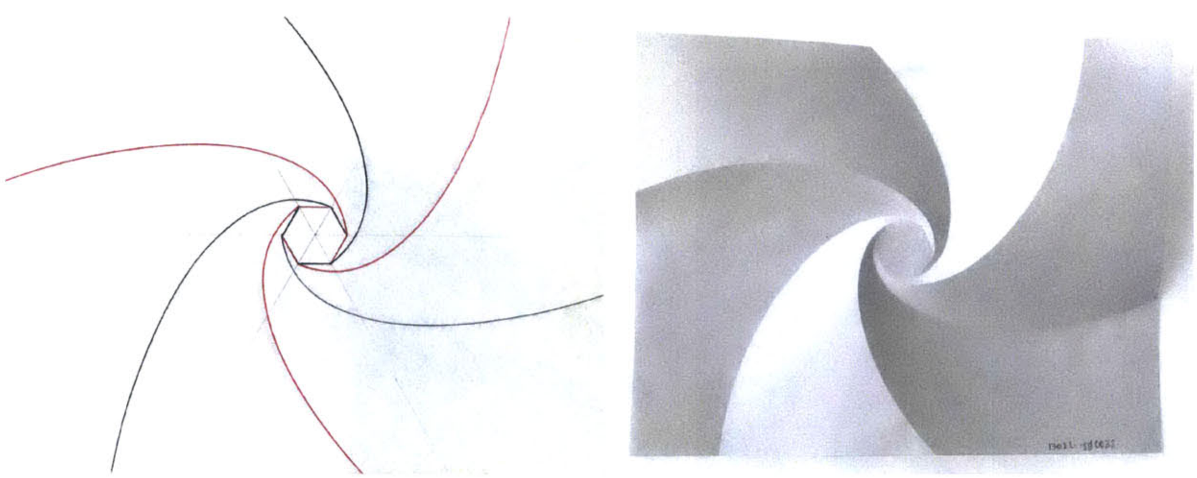


Fig 4.11.28 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

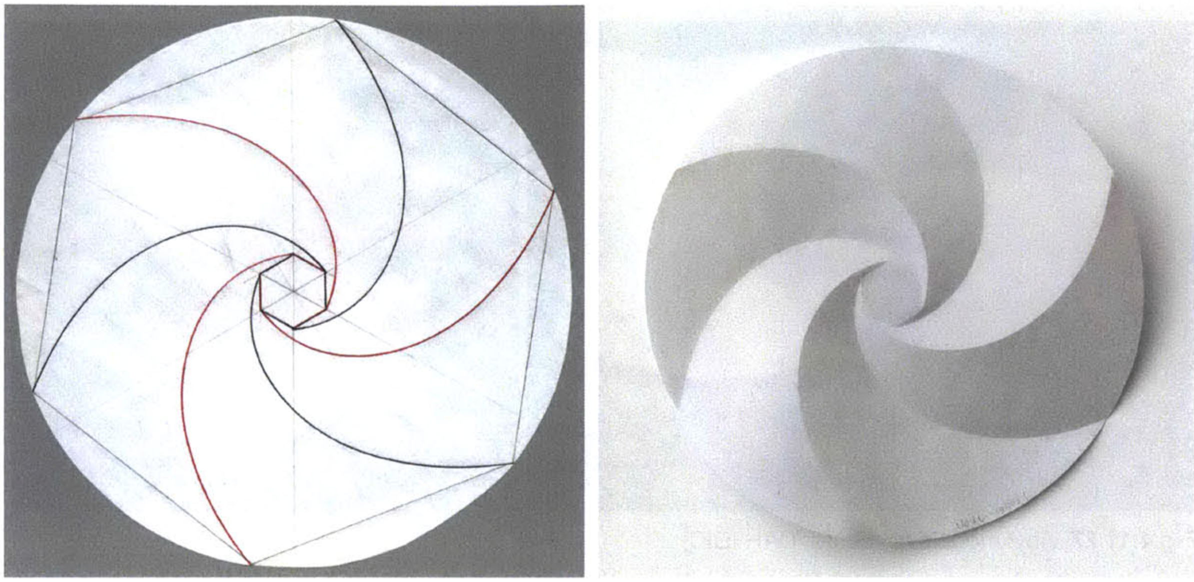


Fig 4.11.29 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman creates a second design with a center hexagon (Fig 4.11.29 left). He draws an outer hexagon in pencil, but decides to use a circular cut out.

Crease pattern and notes

The curves might consist of similar spirals, but curve fitting is inconclusive. The 3 mountains and 3 valleys intersect individually with the corners of the hexagon. The mountain and valley assignment follows the principle of the previous crease patterns.

The paper reconstruction, also based on tracing Huffman's design, folds reasonably well (Fig 4.11.28).

The last vinyl model in this section has an n-gon with 24 edges at its center and is cut along a circle. A diagram of a tiling on page 217 in 'Tilings and Patterns' by B. Grunbaum and G. Shephard

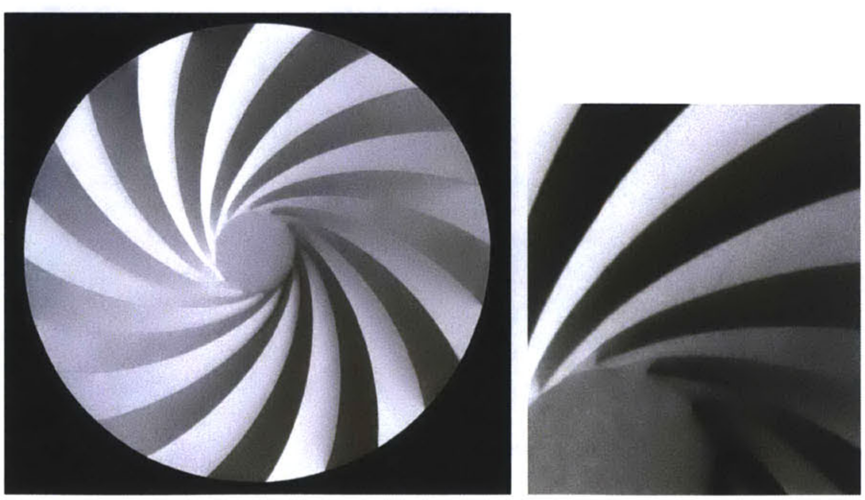


Fig 4.11.30 Vinyl model (undated, DAH [EAH]), Detail of identical model

in Huffman's library might have served as a source of inspiration as it is one of very few examples with spirals (Fig 4.11.31). The tiling in the figure uses 12 black tiles that converge toward a circle in the center.

The vinyl model does not fold well along the polygon as additional creases form during the folding process.

Fig 4.11.31 Figure in 'Tilings and Patterns' (Grunbaum and Shephard)

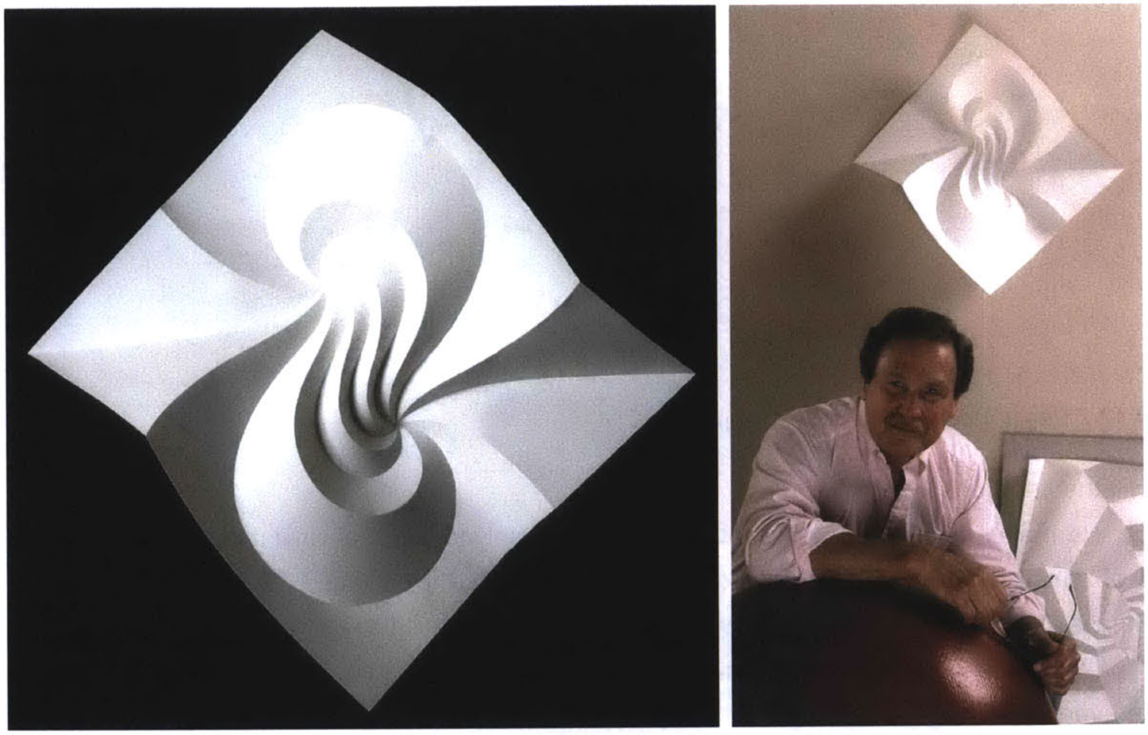


Fig 4.12.1 Vinyl model (undated, DAH [TG]), Huffman in front of 'The Peanut' (1991 [M. Mulbry])

In this section, I discuss the elaborate vinyl model above, which the family casually calls 'The Peanut' (Fig 4.12.1). The design is related to Huffman's investigation of sinks, vortexes and loxodromic spirals. I am presenting archival material by topic in this section since very few dates reveal when Huffman investigates which subject. In terms of trying to date the vinyl model, it is useful to study Matthew Mulbry's photographs. He takes pictures of Huffman for the September issue of Scientific American in 1991 similar to the above image and publishes a cropped version (Fig 4.12.1 right). The model in the background is not cropped in this shot and it hence predates 1991.

Crease pattern

The model consists of loxodromic spirals and I explain their definition later in the section. 9 full spirals go from pole to pole and their mountains and valleys are assigned such that matching curves have the same direction (Fig 4.12.2). On the left and right side of the design 3 creases fold the edges of the paper and the one reaching the corner is a valley crease on either side. Huffman designs the border for this alignment, which means that the paper versions and the vinyl model have different borders. When hanging the model on the wall he orients the vinyl model the way Mulbry and Grant photograph it.

Huffman makes several drawings, in which he decides on the number of creases and distances between curves. The crease patterns are always constructed via a bipolar projection of longitudinals on a sphere. He studies options for the largest s-shaped valley crease in a drawing, in which 5 creases reach the edge of the paper in the corners (Fig 4.12.4). He eventually decides

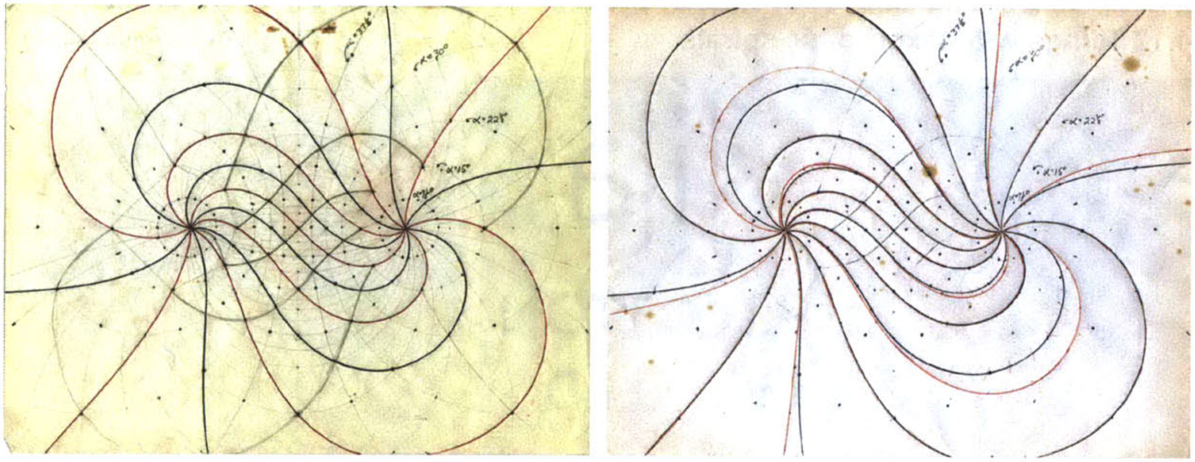


Fig 4.12.2 Paper models (undated, DAH [DK])

to omit the iteration.

He makes the 2 drawings above that relate closely to the final vinyl model. The first consists of constructed pencil circles and red and blue crease lines (Fig 4.12.2 left). The second drawing is a photocopy of the first, on which he alters creases with a red pen (Fig 4.12.2 right). He eventually decides to turn the first drawing into the vinyl model and omits his edits on the photocopied version.

The crease pattern is constructed by using the matching coordinate tables Huffman keeps among his notes (Fig 4.12.5). He traces all diagonals in the bi-polar grid with a 1:2 ratio, which creates the loxodromic spirals (Fig 4.12.3 right). The crease pattern and the paper reconstruction (Fig 4.12.4 right) differ from Huffman's drawing regarding the valley creases that cross the left and right edge of the paper. The crease in Huffman's drawing (Fig 4.12.3 left) has less curvature than

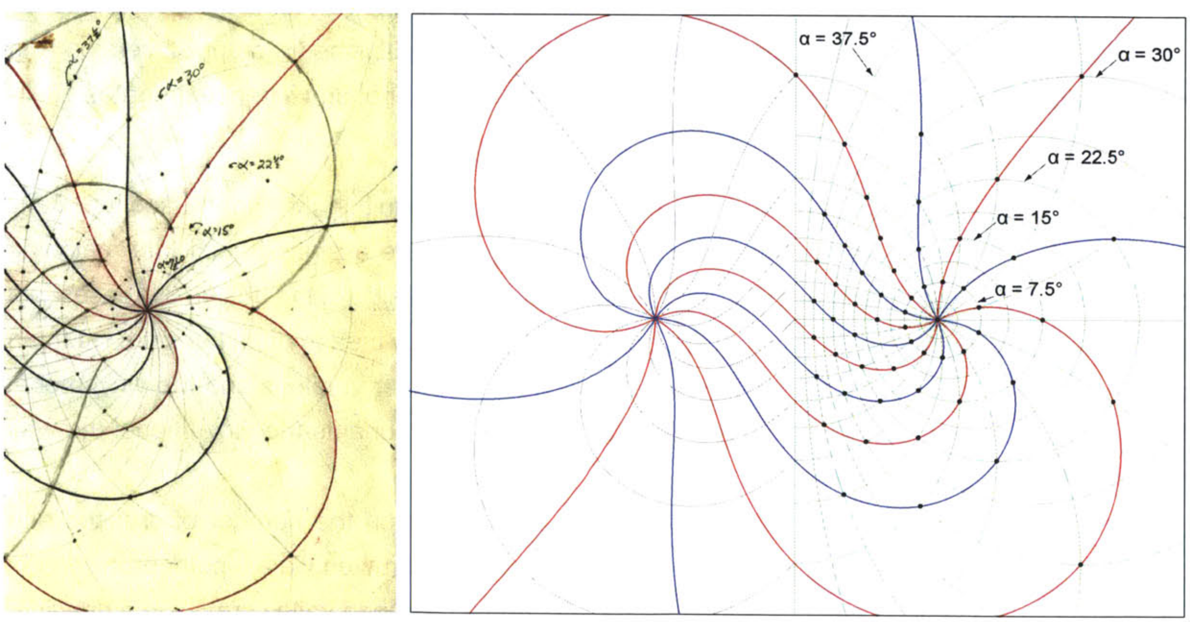
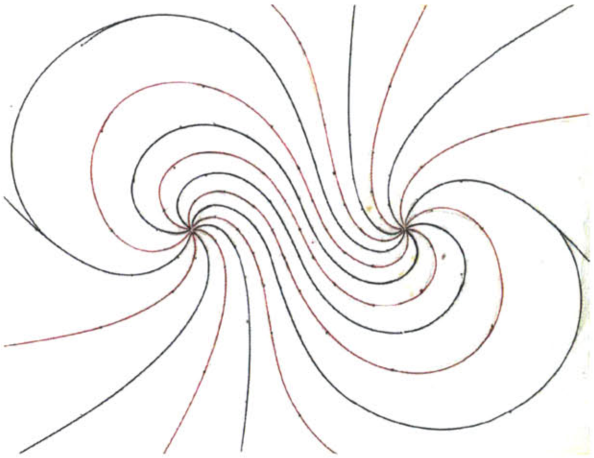


Fig 4.12.3 Detail of paper model (undated, DAH [DK]), Crease pattern [DK]



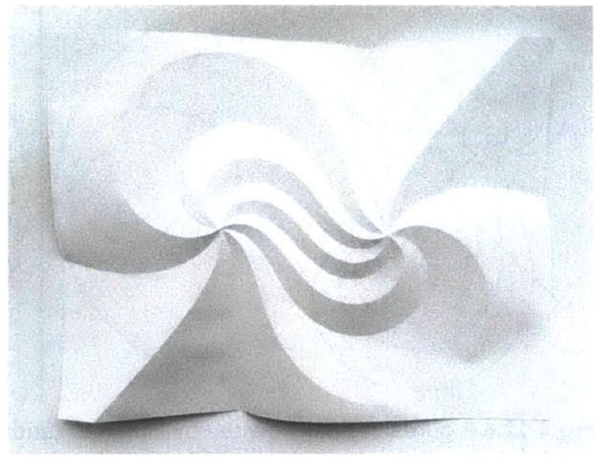


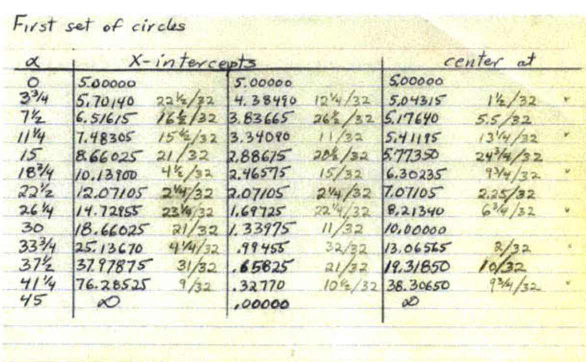
Fig 4.12.4 Paper model (undated, DAH [DK]), Paper reconstruction [UP]

the ones in the crease pattern (Fig 4.12.3 right). It is possible that Huffman has no available grid intersection that far outside of the page and hence estimates the curvature of the crease.

Notes

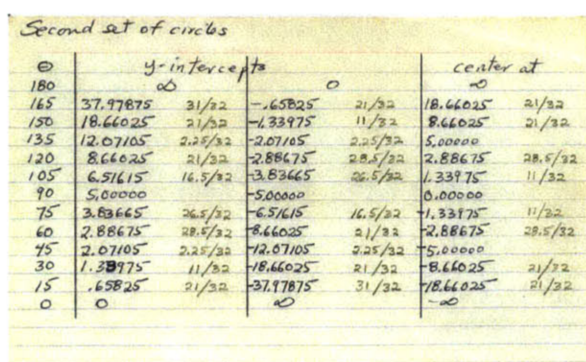
In order to find out how Huffman may have come up with this idea I discuss several sketches and notes of his which provide clues for a possible genesis of this design. A figure in Huffman's copy of 'Vortex Flow in Nature' by Hans Lugt on page 50 reads: 'Line Vortex 41 - Fig 8.2. Streamlines of the motion due to a counteroriented vortex pair' and might have served as inspiration for the design (Fig 4.12.6 left). He studies vortexes and sinks in general and the figure shows strong parallels. Even though the figure states that the vortex pair is counteroriented, from the perspective of someone designing a crease pattern the spirals are rotating in the same direction, something Huffman appears to be interested in. The second figure depicts a set of coaxal circles from 'Mathematical Snapshots' by Steinhaus on page 144 (Fig 4.12.6 right). Huffman draws an additional circle on the diagrams in his book and might have thought of either a bi-polar coordinate system or ruling directions.

In 1981 Huffman takes notes on index cards that relate to a loxodromic spiral, a curve on a sphere that goes from pole to pole. 'Think of a spiral that intercepts all latitude lines at a constant



First set of circles

α	X-intercepts				center at	
0	5.00000		5.00000		5.00000	
3¾	5.70140	22½/32	4.38490	12¼/32	5.04315	1½/32
7½	6.51615	16½/32	3.83665	26½/32	5.17640	5.5/32
11¼	7.48305	15½/32	3.34090	11/32	5.41195	13¼/32
15	8.66025	21/32	2.88675	28½/32	5.77350	24¾/32
18¾	10.13900	4½/32	2.46575	15/32	6.30235	9¾/32
22½	12.07105	2¼/32	2.07105	2¼/32	7.07105	2.25/32
26¼	14.72955	23¼/32	1.69725	22¼/32	8.21340	6¾/32
30	18.66025	21/32	1.33975	11/32	10.00000	
33¾	25.13670	4¼/32	.99455	32/32	13.06565	2/32
37½	37.97875	31/32	.65825	21/32	19.31850	10/32
41¼	76.28525	9/32	.32770	10½/32	38.30650	9¾/32
45	∞		.00000		∞	



Second set of circles

θ	y-intercepts				center at	
180	∞		0		-∞	
165	37.97875	31/32	-.65825	21/32	18.66025	21/32
150	18.66025	21/32	-1.33975	11/32	8.66025	21/32
135	12.07105	2.25/32	-2.07105	2.25/32	5.00000	
120	8.66025	21/32	-2.88675	28.5/32	2.88675	28.5/32
105	6.51615	16.5/32	-3.83665	26.5/32	1.33975	11/32
90	5.00000		-5.00000		0.00000	
75	3.83665	26.5/32	-6.51615	16.5/32	-1.33975	11/32
60	2.88675	28.5/32	-8.66025	21/32	-2.88675	28.5/32
45	2.07105	2.25/32	-12.07105	2.25/32	-5.00000	
30	1.33975	11/32	-18.66025	21/32	-8.66025	21/32
15	.65825	21/32	-37.97875	31/32	-18.66025	21/32
0	0		∞		-∞	

Fig 4.12.5 Index cards with coordinate tables (undated, DAH [DK])

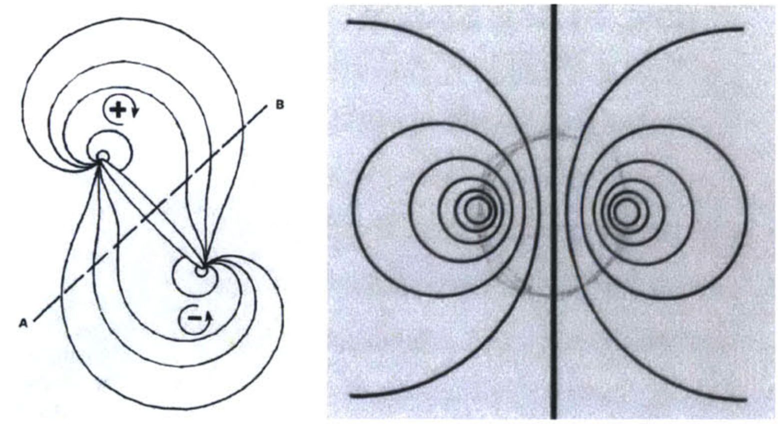
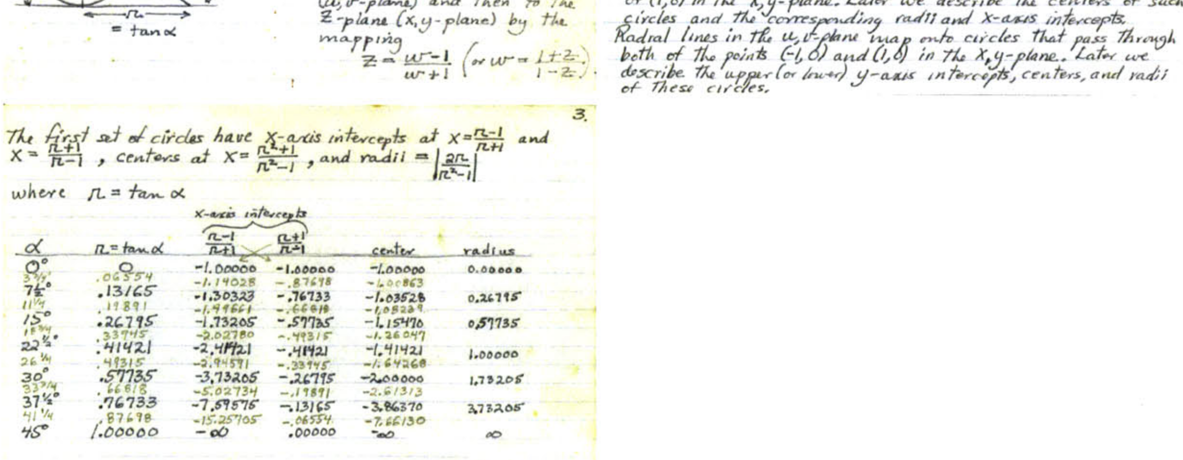


Fig 4.12.6 Figures in books owned by Huffman (undated, DAH [DK])

angle' he writes on the first index card (Fig 4.12.7 top left). He describes the mapping of the curve onto a plane and produces one of his many coordinate tables (Fig 4.12.7 bottom left).

A more elaborate sketch displays two sinks that rotate in the same direction and also seems to show ideas for ruling directions (Fig 4.12.8). It is unclear which mathematical definitions Huffman uses for the drawing. The pairs of spirals share one pair of curves with a common tangent in the center similar to the curves he uses for his final design. It is unclear if Huffman thinks of sinks and loxodromic spirals interchangeably, but both investigations display similar effects, if viewed from the perspective of someone wanting to design a crease pattern.

The projection of a sphere results in a diagram much like another figure he alters with a



March 1981

Mapping associated with pair of spirals that are rotated in the same direction

Think of a spiral that intercepts all latitude lines at a constant angle (loxodromic spiral). Let Θ be the longitudinal angle on a unit diameter sphere and α be the angle shown in the diagram. (It is related to the longitudinal angle φ by the relationship $\alpha = \frac{1}{2}(\phi + \pi/2)$).

side view of the u,v-plane

$r = \tan\alpha$

The loxodromic spirals are of the form $\alpha = \alpha_0 + K\Theta$. We first map them from the sphere to the w-plane (u,v-plane) and then to the z-plane (x,y-plane) by the mapping $z = \frac{w-1}{w+1}$ (or $w = \frac{1+z}{1-z}$).

2.

(Mapping; pair of spirals)

u,v-plane

x,y-plane

Circles in the u,v-plane map onto circles enclosing either (-1,0) or (1,0) in the x,y-plane. Later we describe the centers of such circles and the corresponding radii and x-axis intercepts. Radial lines in the u,v-plane map onto circles that pass through both of the points (-1,0) and (1,0) in the x,y-plane. Later we describe the upper (or lower) y-axis intercepts, centers, and radii of these circles.

3.

The first set of circles have x-axis intercepts at $x = \frac{r-1}{r+1}$ and $x = \frac{r+1}{r-1}$, centers at $x = \frac{r^2+1}{r^2-1}$, and radii $= \left|\frac{2r}{r^2-1}\right|$

where $r = \tan\alpha$

α	$r = \tan\alpha$	x-axis intercepts $\frac{r-1}{r+1}$	x-axis intercepts $\frac{r+1}{r-1}$	center	radius
0°	0	-1.00000	-1.00000	-1.00000	0.00000
3¾°	.06554	-1.14028	-.87698	-1.00863	
7½°	.13165	-1.30323	-.76733	-1.03528	0.26795
11¼°	.19891	-1.49661	-.66818	-1.08239	
15°	.26795	-1.73205	-.57735	-1.15470	0.57735
18¾°	.33745	-2.02780	-.49315	-1.26047	
22½°	.41421	-2.41421	-.41421	-1.41421	1.00000
26¼°	.49315	-2.94571	-.33745	-1.64268	
30°	.57735	-3.73205	-.26795	-2.00000	1.73205
33¾°	.66818	-5.02734	-.19891	-2.61313	
37½°	.76733	-7.59575	-.13165	-3.86370	3.73205
41¼°	.87698	-15.25705	-.06554	-7.66130	
45°	1.00000	-∞	.00000	-∞	∞

Fig 4.12.7 Index cards (1981, DAH [DK])

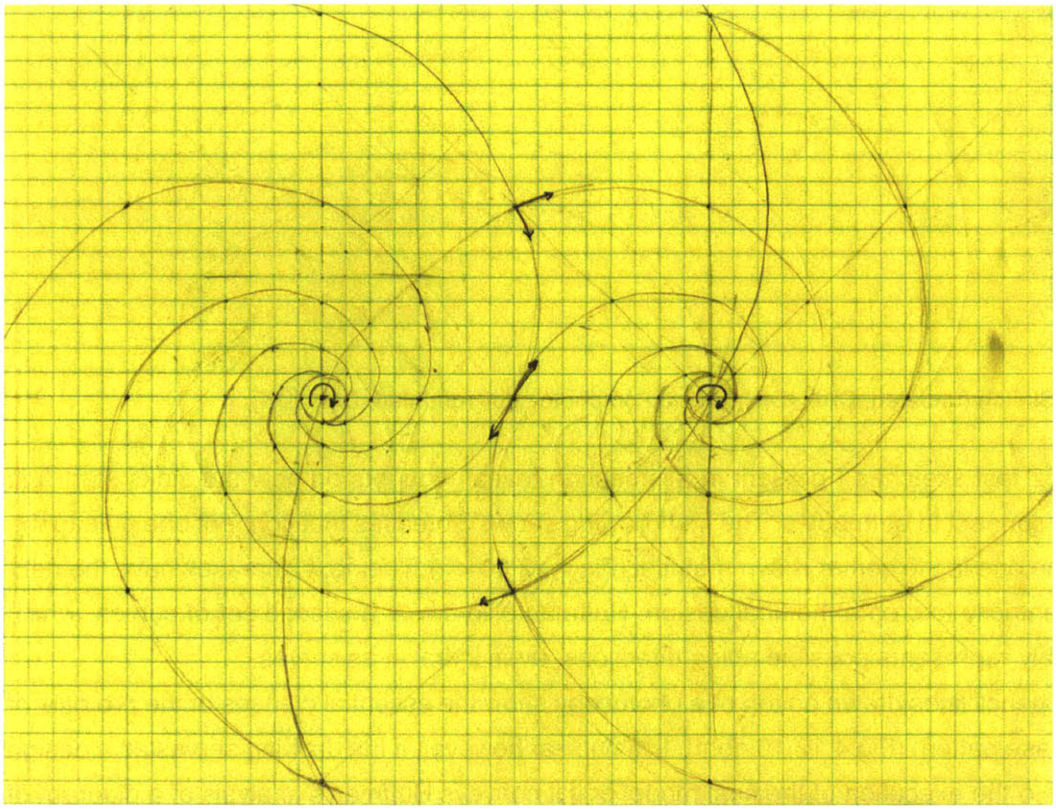


Fig 4.12.8 Paper model (undated, DAH [DK])

pencil in one of Huffman's books from 1971, namely 'Introduction to mathematical fluid dynamics' by Richard Meyer (Fig 4.12.9). The pencil marks, enlarged in the adjacent image (Fig 4.12.9 right), indicate a possible traversal of the grid in 2 directions. It is possible that Huffman is thinking of creases and rulings, which would always intersect in a similar way as the curves are angle preserving on the sphere.

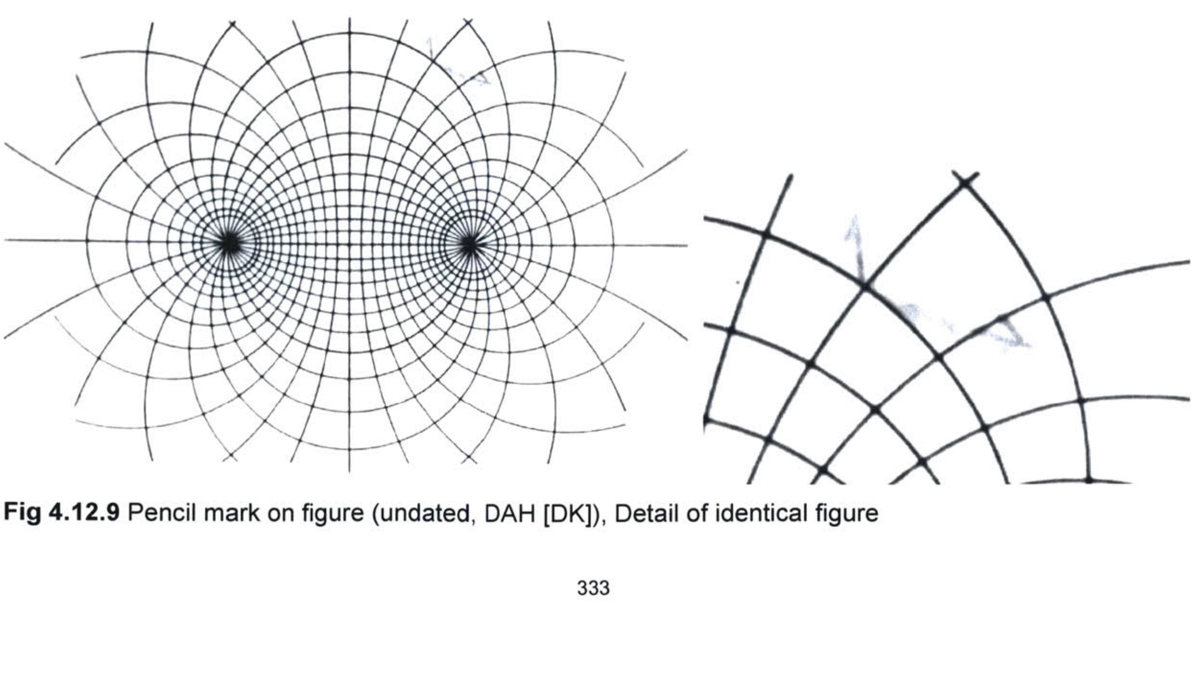


Fig 4.12.9 Pencil mark on figure (undated, DAH [DK]), Detail of identical figure

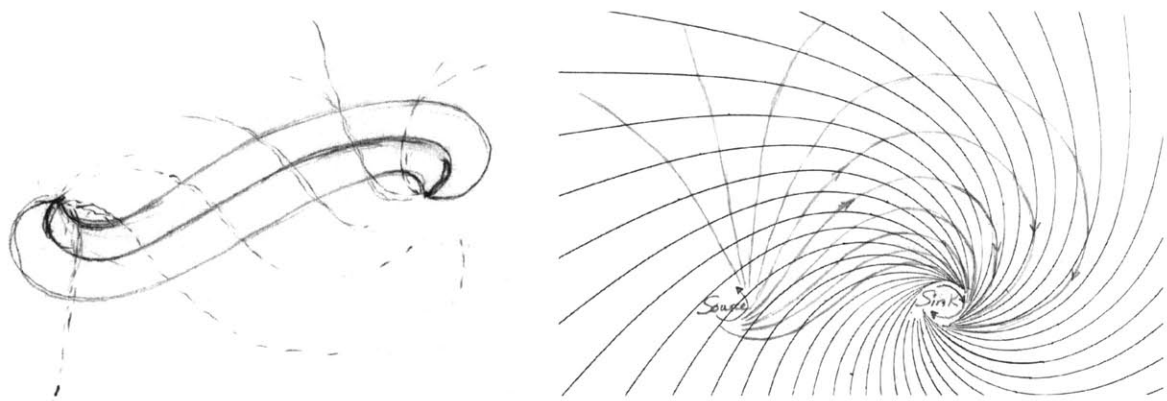


Fig 4.12.10 Sketch (undated, DAH [DK]), Sketch on photo copy (undated, DAH [DK])

The 2 last sketches I present in this section reveal thoughts on how the sinks start and how curves connect to both poles. The first drawing shows 3 creases that connect one pole to the other and the one in the center is symmetrical (Fig 4.12.10 left). The next 2 creases have the necessary eccentricity to create the final design. Huffman also draws a second set of curves, which most probably represents possible ruling directions, even if drawn as curves.

The pencil lines drawn on top of a photocopy show the essential creases of the prototile of the final crease pattern (Fig 4.12.10 right). It is unclear however, if the drawing serves as a design sketch prior to the execution of the final model or if it conveys Huffman's analysis of a diagram of a sink.

4.13 Sinks and Vortexes

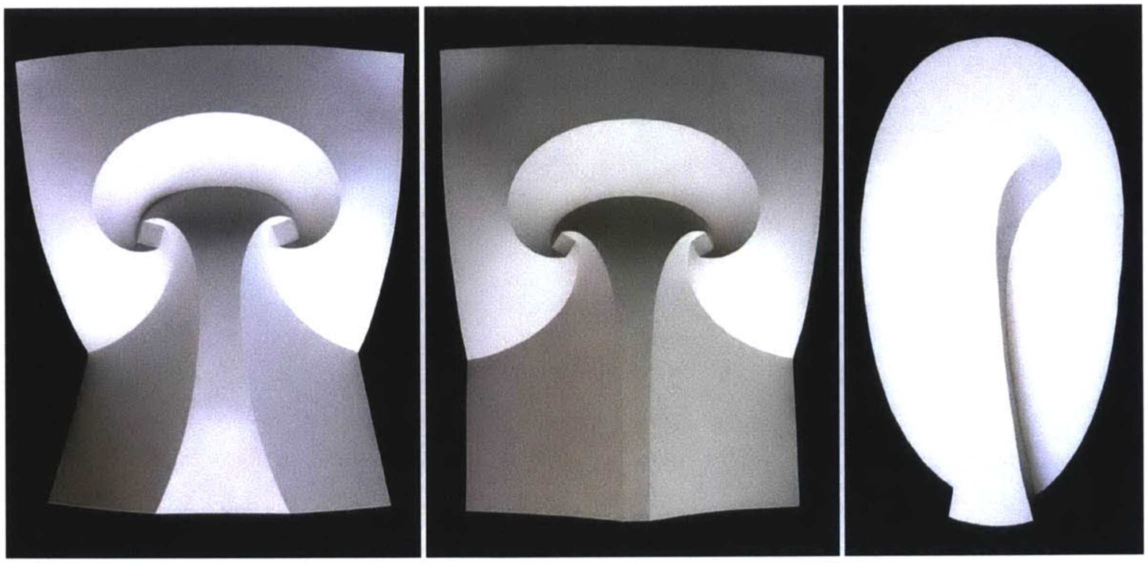


Fig 4.13.1 Vinyl models (undated, DAH [EAH])

The designs in this section appear to relate to Huffman's contemplations on flows of fluids and air. He makes 3 models in vinyl (Fig 4.13.1) and keeps no paper models or crease patterns. They all might be related to a series of sketches and drawings on flow lines. No model could be reconstructed in a way that would elucidate the curve types he uses.

The 2 models on the left consist of 2 prototiles with a square sink and 4 curved creases, 2 mountains and 2 valleys. The upper curved crease is designed such that it smoothly connects to the second prototile similar to connections of splines. The 2 models differ in terms of the size of the square and the creases that reach the bottom edge of the paper. The third model consists of a long spline and 2 creases that end within the area of the paper. The model appears to be cut with an ellipse

The models have only 1 line of symmetry and all have creases that fold the bottom edge of the paper. These folds could be the entering flow lines into a sink. The design on the right can

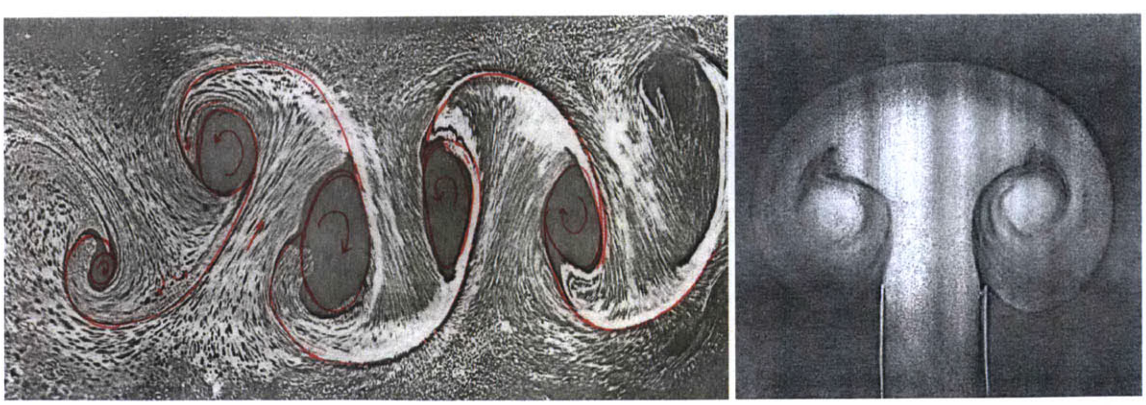


Fig 4.13.2 Photocopies with pen marks (undated, DAH [DK])

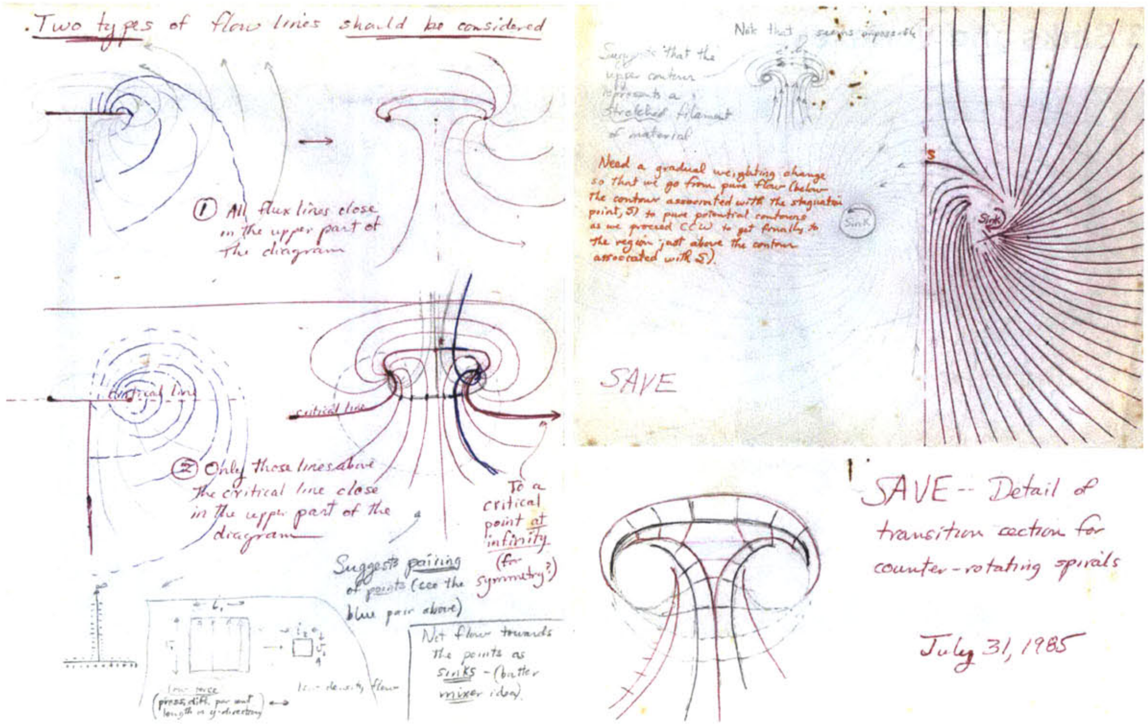


Fig 4.13.3 Sketches (2 undated, 1985, DAH [DK])

be seen as an abstraction of the 2 examples with square centers.

Huffman keeps photocopies of images of sinks and vortexes he finds in a book of his and marks one of them with flow lines (Fig 4.13.2). He elaborates his thoughts in another series of sketches, in which he draws 2 sinks turning in opposite directions (Fig 4.13.3 left). He draws a pair of blue curves that meet in the right sink. The 2 curves may be an indication of crease direction and related ruling direction. His ideas materialize in rough sketches that show similarities to the vinyl models and he dates one of them with 1985 (Fig 4.13.3 top and bottom right).

Further small sketches reveal ideas on curvature of some of the surfaces in the vinyl models (Fig

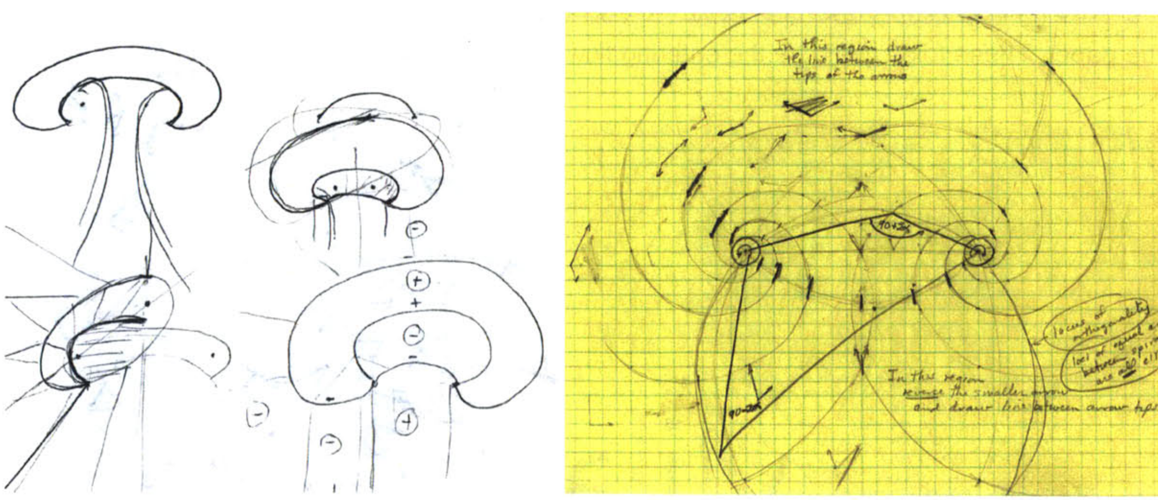


Fig 4.13.4 Sketches (undated, DAH [DK])

4.13.4 left). Huffman notes 'locus of orthogonality' and 'loci of equal angles between spirals are all circles', which also appear to relate to thoughts on crease and ruling directions (Fig 4.13.4 right).

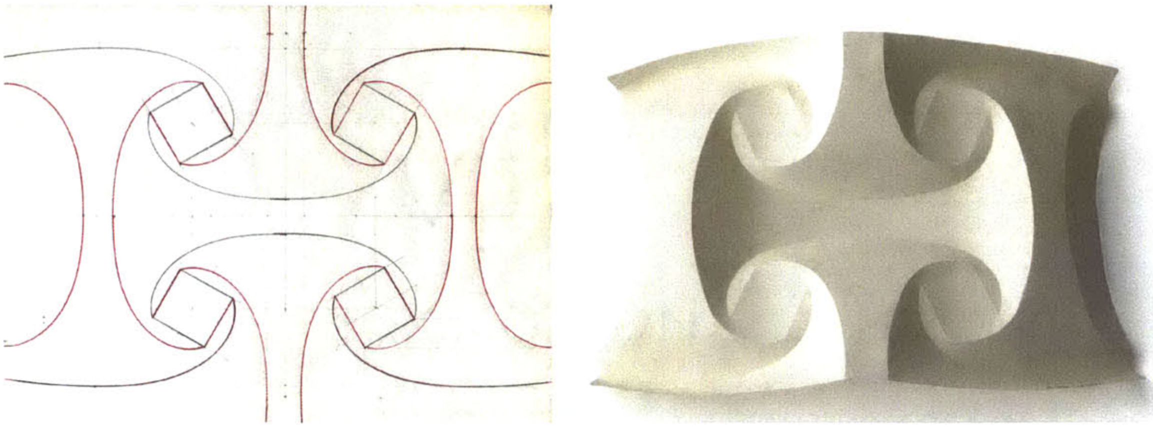


Fig 4.13.5 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

Huffman makes 2 more models that are included in this section, both of which could not be reconstructed in terms of curve types.

The first models uses similar square centers for sinks, but consists of a regular tiling (Fig 4.13.5). The second model has some similarities to the right of the 3 vinyl models. The valley crease ends within the area of the paper, for example. It has a degree 4 vertex, which differs from the vinyl model (Fig 4.13.6).

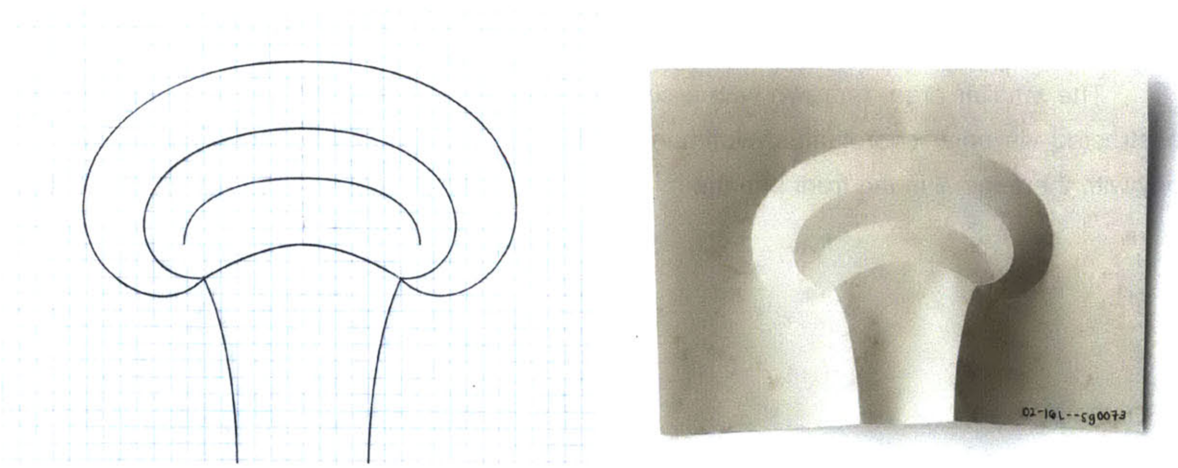


Fig 4.13.6 Paper model (undated, DAH [DK]), Paper reconstruction [AH]

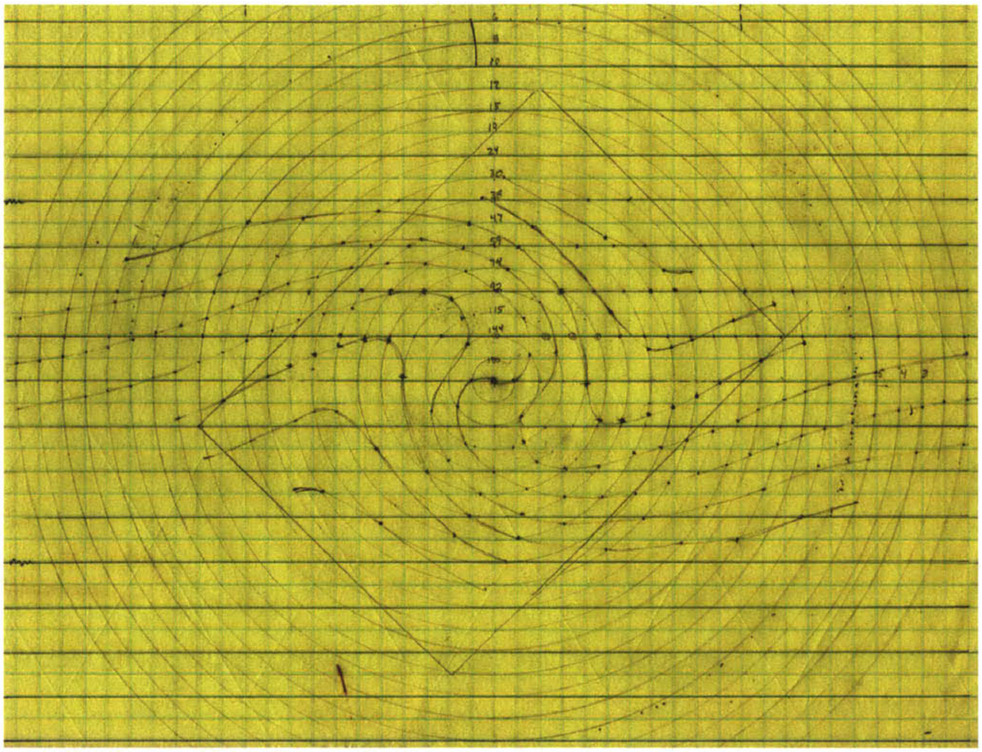


Fig 4.13.7 Drawing (undated, DAH [DK])

The following series of drawings shows Huffman's interest in a particular flow diagram (Fig 4.13.7). He appears to construct the curves via point plotting on polar coordinates. The above drawing operates as template for a smaller cropped version, which he indicates with the rotated rectangle drawn in pencil.

The smaller area, redrawn with a circular boundary (Fig 4.13.8 left) also appears to be constructed via polar coordinates. Huffman photocopies the drawing and then makes another copy with the original in the front and the mirrored copy in the back (Fig 4.13.8 right). He obtains

Fig 4.13.8 Drawing (undated, DAH [DK]), Photo copy of identical drawing (undated, DAH [DK])

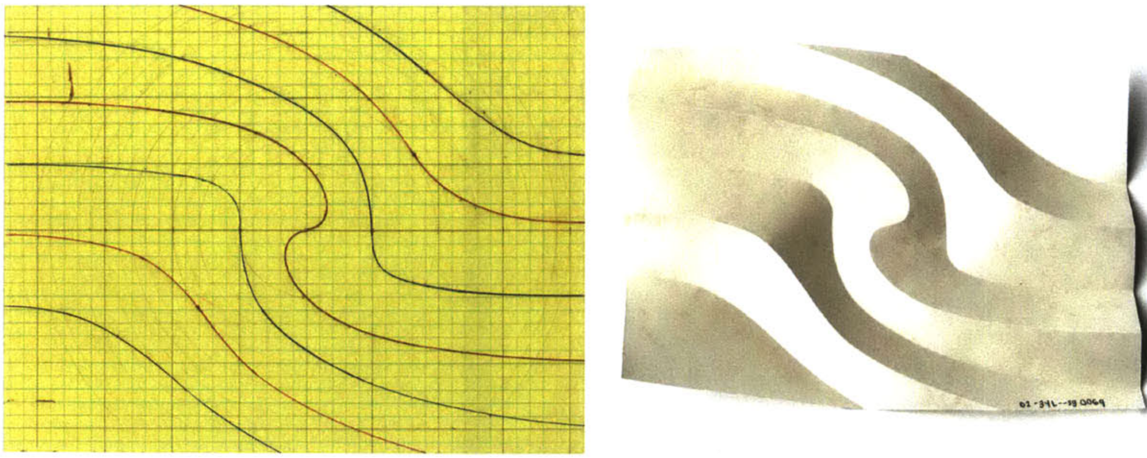


Fig 4.13.9 Drawing (undated, DAH [DK]), Paper reconstruction [AH]

a reflected image of the same flow lines, which probably indicates that he is studying ruling directions.

The above drawing, an even more enlarged detail, consists of 7 creases, 3 mountains and 4 valleys (Fig 4.13.9 left) and has a symmetry of one rotation in the center. It is the only drawing he folds and no further models or notes exist that appear to relate to his investigation. The paper reconstruction provides visual feedback of Huffman's crease pattern but yields no results in terms of curve types (Fig 4.13.9 right).

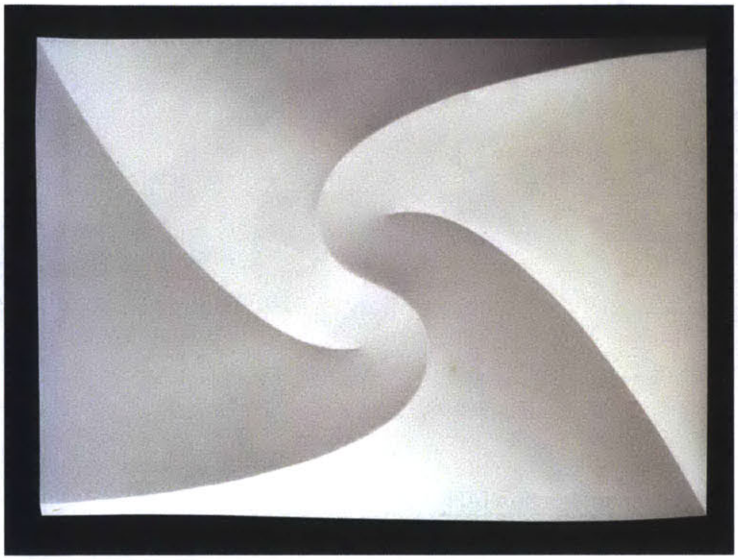


Fig 4.13.10 Vinyl model (undated, DAH [EAH])

The 2 final designs in this section have an s-shaped valley crease and 2 mountain creases that end within the area of the paper. Huffman makes one of them in vinyl (Fig 4.13.10). No notes or sketches exist and neither the visual reconstruction of the first design nor curve fitting of the second design yield any satisfactory result (Fig 4.11.33).

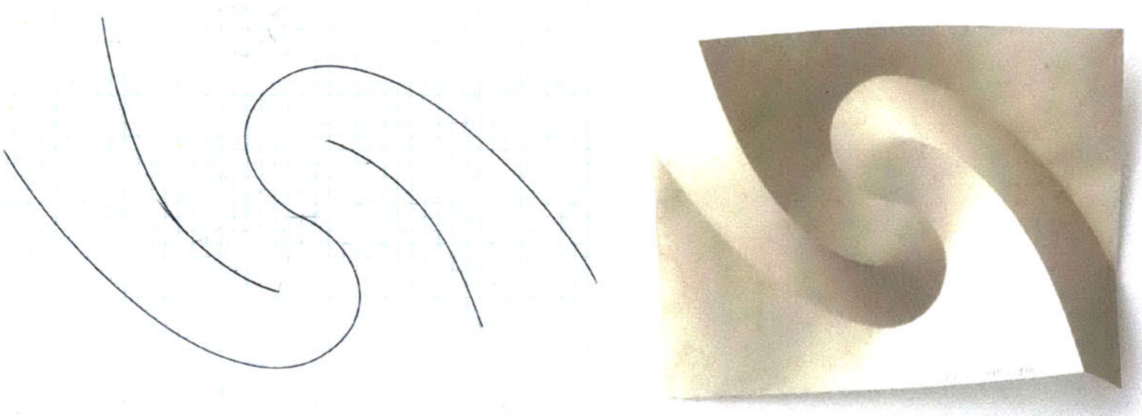


Fig 4.13.11 Paper model (undated, DAH [DK]), paper reconstruction [AH]

These designs conclude the examples of the taxonomy of Huffman's work and I discus general findings in the final section.

Findings of the taxonomy

Huffman's curved crease paperfolding work consists of well over 150 designs, more than half of which are unique. This impressive body of work may well represent the largest number of curved-crease designs by a single author. He shows little of this work in exhibitions and does not publish any of the work in the form of a book or paper. Huffman lectures on the topic and publicly proclaims to only use conics for his designs [Dem 97]. This is true of most of his designs, however, another group show at Xerox PARC in 1997 includes the loxodromic spiral design (Fig 4.12.1), which is not made of conics. Furthermore, the work presented in the chapters on forced rulings and converging curves is not based on conics. This means that Huffman is willing to explore curve types that do not allow him to predict the location of rulings.

As the taxonomy shows most of Huffman's designs can be defined via gadgets while less rigidly defined examples are presented in sections 4.10 to 4.13. This means that he mostly adheres to a very methodical approach throughout his explorations. Our abstraction of his work into gadgets allows geometers and designers to gain fast and comprehensive access to the work. The gadget table includes quantitative information about the 223 implementations of all his gadgets. Since a design can consist of several gadgets the table indicates the frequency of the use of individual gadgets (Fig 4.4 to 4.9). This provides a metric to see how much effort he invests in an idea. The parabolic splines are used close to 80 times and he investigates parabolic splines in combination with straight edges in over 47 variations for example. Combined curve types with parabolas in the center make up over 30 designs.

General findings

Huffman spends more time on some crease patterns than others. In some cases, he draws many versions of a crease pattern, but makes no vinyl model. For example, the entire subsection of section 4.4 on splines and line segments lacks vinyl models (Fig 4.4.114).

In other cases he makes only a paper or vinyl model without keeping any other record at all (Fig 4.4.60, 4.11.3 and 5).

He decides to photograph a small selection of designs and documents the making of the model in even fewer cases (Fig 4.1.10, 4.2.40, 4.3.16, 4.4.22, 4.4.29, 4.4.44, 4.4.50, 4.4.70).

Huffman's work is mostly symmetrical except for 2 designs, one with a single ellipse and one made of parabolic splines (Fig 4.3.3 and 4.4.96).

Many models can fold more than the state he chooses to present. However, it is hard to qualify this as he stores the majority of his work in a flat state. Only the vinyl models tell us when he arrests the folding motion.

Huffman explores designs in which he cannot predict the location of rulings. Sections 4.11 and 4.12 on converging curves provide evidence of him exploring spirals in a playful way, but also show that he is searching for a discrete representation.

DAH's design approach

Huffman thinks of a crease pattern as a constraint propagation problem that guides the rule lines. Once the reflection or refraction scheme is chosen the algorithm takes over.

Huffman's interpretation of ray refraction allows me to deduce gadgets that include a representation of the paper in addition to the curve. This is significant as it provides opportunities for digital folding. The paper and the curve can be represented by line segments. I hence propose to use his gadgets in combination with my categorization of tilings in simulation software as a design approach in chapter 5.

The gadgets he defines can only create cones and cylinders, which means he has to omit tangent surfaces. This reduces the potential for expressive designs as tangent surfaces appear less regular.

Open problems and possible directions for further investigation

Some of Huffman's designs result in ruling paths that are cyclic (Fig 4.7.2 and following) and it is unclear if the rulings never meet the vertex they cycle around. Does the vertex exist?

Huffman's designs that use unknown rulings in section 4.11 with converging curves could pose an interesting problem for future research. The loxodromic spiral design for example suggests curved rulings and their approximation might point to an open problem (Fig 4.12.1).

Some of Huffman's gadgets fold more than others during simulation. For example, the ellipse and hyperbola gadget behaves very differently in several of his designs (Fig 4.1.19 vs Fig 4.1.29, Fig 4.5.7 left and right).Can one qualify how much a specific gadget can fold?

Huffman explores gadgets with 2 curves extensively, but maybe other combinations of 2 curves can be created. Similarly gadgets with 3 and more curves might exist outside of Huffman's repertoire.

5. Design approaches for curved creases in comparison

The behavior of curved creases may be poorly understood mathematically speaking, but we have seen many examples of this geometry in art, design, mathematics, education and this dissertation. In order to be able to provide a broad perspective on the use of this subset of geometry in design, I propose 5 design approaches that summarize the design knowledge behind the work in the taxonomy and the work of other artists, designers and geometers. The goal is to distill 5 design approaches that rely on specific aspects of protagonists in the field and then compare them.

Huffman's accomplishments are embedded in several of the proposed design approaches. I choose this format as it allows me to expand on Huffman's approach and also provides an opportunity to show the limitations of his methods in some cases. Each section begins with the relevant source of the design approach. The sections include commentaries relative to Huffman's work in order to provide context and an opportunity for critique.

This introduction serves as an opportunity to formalize design approaches and define characteristics of a design approach. I expand on existing ideas and introduce digital tools in each design approach. Digital discrete representations of crease patterns only exist in a few cases.

Ways of designing

Apart from work mentioned in the historical section, works by contemporary artists such as Robert Sweeney [Sch 09], Yuko Nishimura, and Matthew Shilan [Smi 09] might elucidate how artists design with curved creases. Sculptors and paper artists, in general, tend to expose personal aspects of their creative process, but finding information on their use of geometry and specific abstract design techniques is difficult [Kla 07] [Nol 95].

Nishimura states that she has a connection to her art through folding day to day commodities from kimonos to wrapping goods in gift shops. She does not focus on any specific area of origami tessellations and is interested in expressing the Japanese soul through form [Smi 09]. Such an artistic goal is difficult to translate into a design approach for example.

Shilan, who works with what he calls 'systems', says that he does not know what the result will be. Once a system of folding is initiated, the outcome is unknown, led - as it is - by the qualities of the material. His process consists of exploration and invention [Smi 09]. This statement hints at the possibility to derive a design approach, but is still vague.

Paul Jackson's work, specifically his 'one fold models', are omitted here as they consist of curved paper but straight creases in 2d [Tho 01]. This is unfortunate as the prolific paper artist and educator would have been a formidable source. Expanding a search to other design disciplines unfortunately yields no results. Industrial designers and architects hardly ever publicize their methods as authorship and trade secrets provide motivations for non-disclosure. It is hence surprisingly difficult to find out how contemporary artists and designers design with curved creases. The study of related work by mathematicians, geometers, computer scientists and educators provides useful information that is included in the following design approaches.

Defining a design approach and its 4 characteristics

I define a design approach in the most general way as: the way to design with curved creases, which ultimately results in a physical model. While this definition addresses very basic notions of designing with a specific kind of geometry, further parameters and qualities need to be identified.

A way to design with curved creases can range from narrowly defined to fairly open. A designer can, for example, simply score paper with a curve and fold it. The questions that arise are, which design goals can s/he define before starting, how can s/he evaluate the work, and how is one supposed to evaluate the design process itself? In order to be able to describe and also evaluate qualities of a design approach I use characteristics as a heuristic. I discuss 4 characteristics that can help in answering such questions. Characteristics can overlap as their boundaries can become blurry.

The first characteristic distinguishes between designing in a top-down or bottom-up way. In the case of a bottom-up approach, the designer reacts to what the curved crease gives him or her. The logics of the crease pattern and rulings drive the design process and the designer evaluates the result. This could include the behavior of physical paper, but for the purpose of characterizing a design approach I use material constraints as a separate characteristic.

A top-down approach demands decision making beforehand, meaning that the designer has to have made up his or her mind about what the model is supposed to look like before beginning the design process. This can in some cases, represent a strong design goal, but can also become less relevant in the process.

The second characteristic relates to a priori knowledge of geometry, the reasoning related to previously acquired knowledge in geometry. It can play a large role in a design approach in terms of making decisions. A designer's decision might depend on or be influenced by the knowledge of mathematical models, similar to Huffman's ray refraction for example. The knowledge is different to the knowledge obtained from observation or experience and can help in characterizing a design approach.

The third option for characterization lies in exploiting material properties. Paper resists when one folds it and the pressures in the material can guide design decisions. A designer can start to manipulate paper or felt and 'let the material fold the way it wants to fold'. The material driven process requires little mathematical knowledge, but relies heavily on the tacit tactile knowledge that comes with practice. The necessary craft a designer needs to obtain is based on knowledge that is transferred via the repeated physical manipulation or folding of paper. This can also be described as a bottom-up approach based on material logics.

Lastly, the fourth characteristic relates to the use of digital tools. The 2 subcategories relate to the previous bottom-up and top-down characteristics. The first relates to the direct geometric manipulation of static 3d objects in software, which means that a designer looks at a static model while working on it. The second is related to the simulation of folding creases. A designer can see how a discrete version of a curved crease folds in real-time.

A designer can also use digitally controlled vinyl cutters to pre-crease paper in order to get more precise results.

The 5 proposed design approaches

The proposed design approaches, presented in 5 sections, draw on works in the curved crease history chapter and categories I have established for the taxonomy. I also introduce new examples which I have not discussed previously.

For ease of comparison I use the same headlines in every section. I discuss the nature of the used geometry and options for discrete representation under 'Geometry and representation'; relevant sources, authors and designers with examples of their work under 'Historical references and precedents'; the new design approach under 'Definition of the design approach'; and commentaries on Huffman's work under 'DAH'.

The design approaches, briefly introduced here, are discussed in the following 5 sections that describe them via the use of the introduced characteristics.

5.1 Cylinder and cone reflection - Digital reflection in 3d describes design opportunities with geometrically well known configurations of cones and cylinders. The designer can make decisions in 3d as s/he can directly manipulate a digital 3d model. The process allows for one reflection at a time.

5.2 Closed curved creases - 'The Bauhaus model' expanded further develops the work by students in Albers's foundation course from the late 1920s. A designer can engage with a specific crease pattern and explore its many variations without exactly knowing what the outcome will be.

5.3 Curved crease gadgets - Huffman's approach and forward simulation takes ad-

	top-down / bottom-up	knowledge of geometry	material driven	digitally driven
5.1 **Cylinder and cone reflection**	partially top-down	medium	no	yes
5.2 **Closed curved creases**	bottom-up	low	yes	no
5.3 **Curved crease gadgets**	both	high	no	can be
5.4 **Step by step evaluation**	neither	medium	yes	no
5.5 **Sculpting and digitizing**	can be top-down	low	yes	partially

Fig 5.1 Design approaches in comparison

vantage of the largest part of the taxonomy chapter and is based on gadgets. The gadgets can be represented in a discrete way which allows for the use of simulation software in the design process. Some knowledge in geometry will be required for a designer to use this approach.

5.4 Step by step evaluation - Iwaki's approach abstracted builds on a rare example of an artist sharing his way of working and relies in part on tacit tactile knowledge. A designer will need to mostly rely on evaluation 'on the fly'.

5.5 Sculpting and digitizing - Ron Resch's crinkling with curved creases represents the least restrictive approach that does not require any a priori knowledge of geometry. A designer will explore design by sculpting paper of more flexible materials and will need to define a notational system.

The table above shows relationships between the design approaches and the discussed characteristics from above (Fig 5.1).

Field testing design approaches

I have tested the use of the proposed design approaches in workshops and seminars as a proof of concept. I have taught a preliminary and informal 1-day workshop at the Massachusetts Institute of Technology, Cambridge, a 2-day workshop at Southern Polytechnic State University, Atlanta and have held a seminar in the Architecture Department at the Pratt Institute, Brooklyn. The chapter includes examples of my students' work.

5.1 Cylinder and cone reflection - Digital reflection in 3d

This section describes the possibilities for designers to work with reflections of developable surfaces. Mostly cylinders and cones have been used in this context and the surfaces can be manipulated in 3d with CAD software.

Geometry and representation

We can imagine a general cylinder that is cut with a plane and subsequently mirrored as we have seen in the chapter on cylinder reflection (Fig 4.1.1). The rulings are all parallel in the 2d crease pattern. Cones can be manipulated in a similar way, but their rulings converge in an apex (Fig 4.2.2 - 4). Models can easily be made with paper, but CAD software can be used just as effectively. As the geometry is well known, digital 3d models can be made.

Historical references and precedents

A student of Josef Albers, this time in Ulm, is working on a large model (Fig 5.1.1) similar to Huffman's 'Horizontally-fluted column' (Fig 4.1.10). It appears to have a longer bay on the left, which makes it less symmetrical than Huffman's version. Both examples have the same design goal, namely to create cylindrical shapes or enclosures, which can be achieved with cylinder reflection.

Haresh Lalvani provides many variations of such shapes in the work that has been built by Milgo/Bufkin using large sheets of steel (Fig 5.1.2 left) [Lal 03]. The tangent points, where the curvature of the 2d crease changes, remain on the same ruling.

Since the rulings have to be parallel to each other Tim Herok and Markus Schein's 'Lieg-

Fig 5.1.1 Students of Josef Albers, Ulm (1950s [Hans G. Conrad])

Fig 5.1.2 Metal column covers and wall panel (Haresh Lalvani)

engenerator' can use a discretized approach [Her 02]. Their process starts by defining the constraints for the 2 edges of the bench that touch the ground (Fig 5.1.4 right). The folded surface can be defined via its 'plan spline' on the floor (Fig 5.1.3 left) [Tre 08]. Schein sets up a digital model with user inputs for height and undulation that uses a genetic algorithm [Sch 02]. The resulting plan spline is used to construct rectangular sections. As the width of the crease pattern is known (Fig 5.1.4 left), the program can easily compute the heights of the discrete sections for the final shape (Fig 5.1.3 center and right). This case demonstrates that the designers were willing to accept a constrained base premise in order to realize their project.

Regarding cones, a drawing by a student of Josef Albers demonstrates how to construct a conic section by plotting the points of the resulting curve (Fig 5.1.5). It is unclear if Albers taught how to make this construction out of one piece of paper, but many examples of student work used

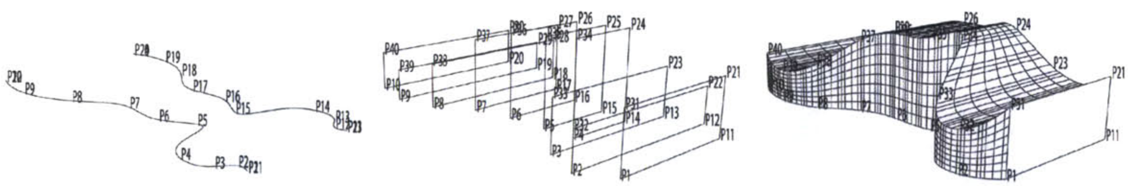


Fig 5.1.3 Plan splines, Discrete sections, 3d shape [Schein and Herok]

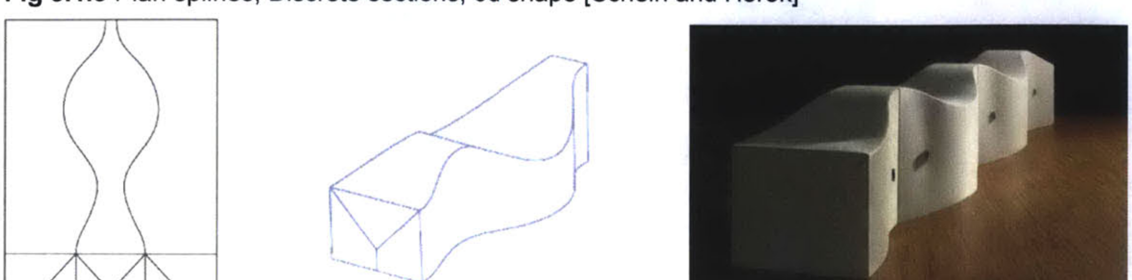


Fig 5.1.4 Crease pattern, 3d model, Bench prototype [Schein and Herok]

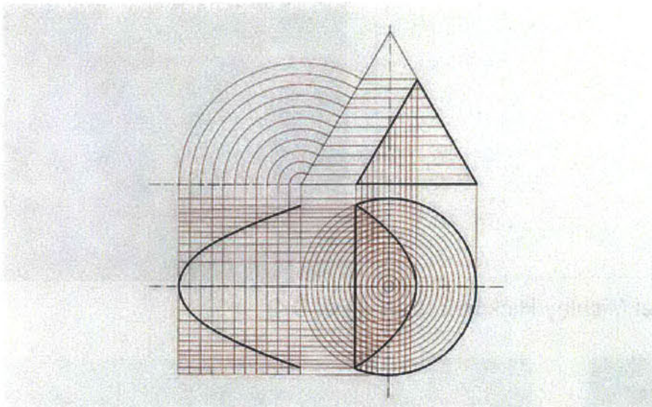


Fig 5.1.5 Student work, Hochschule fuer Gestaltung, Ulm (1950s)

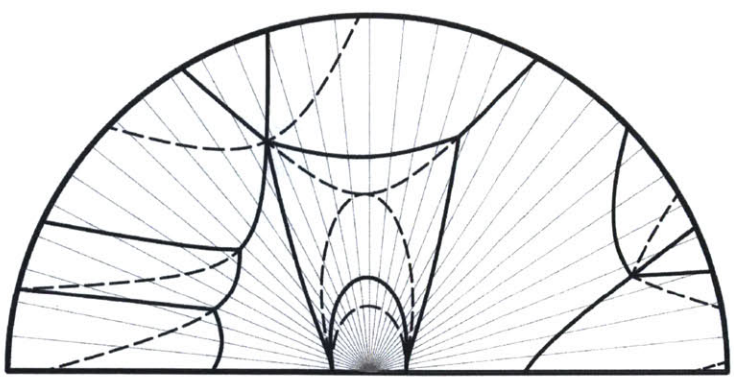
partial cones.

The examples with cones by Huffman are based on reflections between 2 planes, and designs that gradually rotate the main axis (Fig 4.2.20 - 22). Poul Christiansen also uses cone reflections in one of his lamp designs (Fig 2.2.9 bottom right). Ron Resch works with 2 cones without cutting the paper in 'Yellow Folded Cones: Kissing' (Fig 4.2.17). Lastly, Tomohiro Tachi uses cone reflection in his architectural design curriculum (Fig 5.1.6).

Definition of the design approach

Regarding cylinders, the 3d approach begins with making a model of a general cylinder, meaning a curved or undulating surface, and cutting it with a plane. Any contemporary CAD software can be used for this operation. The subsequent step consists of reflecting or mirroring the cut-off part and placing it on the resulting curved crease. A designer can now continue the process to her or his liking. The approach can also be applied to cones and a designer can manipulate a digital 3d model in much the same way.

I used the software 'ORI-REF: A Design Tool for Curved Origami based on Reflection' by the Japanese geometer Jun Mitani [Mit 11] in my seminar. The tool can help to speed up the



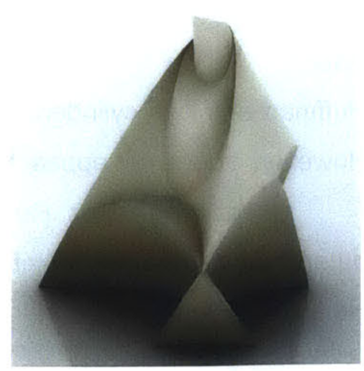


Fig 5.1.6 Crease pattern, 3d model (Tomohiro Tachi)

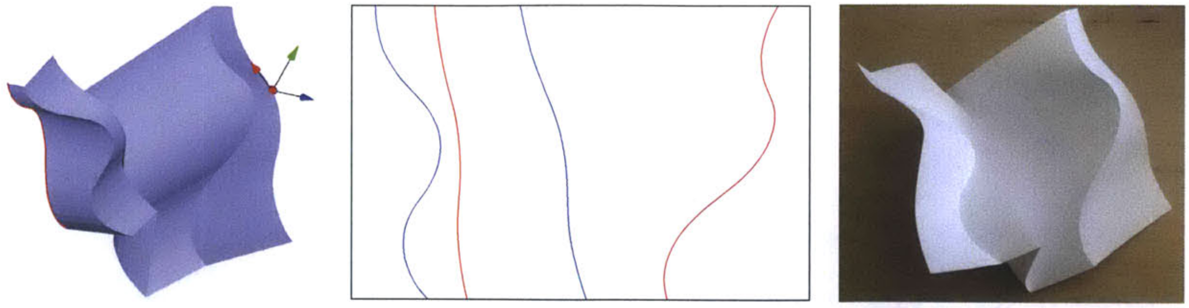


Fig 5.1.7 3d model, Crease pattern, Paper model (Ashley Hickman, student of DK)

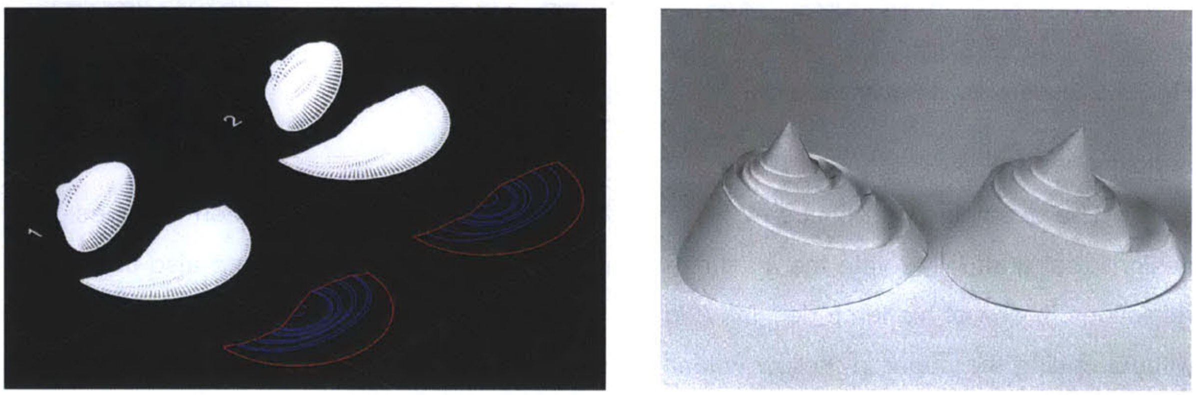


Fig 5.1.8 3d model, Paper models (Juan Sala, Georgios Avramides, students of DK)

process as it constructs the cut with the plane and reflects the remaining part in a single step. The software tool can import cylinders, cones and tangent surfaces as input surface.

The red spline in the above example with several reflections by Ashley Hickman indicates the input surface (Fig 5.1.7). The small coordinate system represents the cutting tool to place the reflections. The corresponding example with cones by my students Juan Sala and Georgios Avramides was made using Rhino 3d as software tool (Fig 5.1.8).

A designer is visually confronted with the result of every cut while working on the computer, which provides static visual 3d feedback during this step-by-step process. There is no element of surprise as the result is evident. Physical models may not always follow the 3d configuration easily.

DAH

Huffman employs cylinders and even includes tucking with expertise and finesse (Fig 4.1.24). However, it does not appear to be the case that Huffman thought of cylinder reflections in 3d.

Regarding cones, Huffman could not observe a digital 3d model as he does not use computers. He instead manipulates the 3d configuration via the use of projected sections (Fig 4.2.6). He also expands this genre rather elegantly by adding tuck folds (Fig 4.2.50 - 55).

5.2 **Closed curved creases** - 'The Bauhaus model' expanded

The Bauhaus model discussed in the history of curved creases provides an opportunity for further exploration for designers (Fig 2.2.2). While the approach may seem very constrained it qualifies as an opportunity to teach many aspects of curved creases.

Geometry and representation

The 2 relevant elements of the Bauhaus model consist of concentric circles and a circular hole in the center. Its 3d configuration is not fully understood, although, recent results by M. Dias and C. Santangelo suggest that constant pleating angles result in a helicoid [Dia 13]. There exist no practical discrete representation for software, so the design approach has to be partially analog.

Historical references and precedents

Josef Albers introduces 'The Bauhaus model' in his design foundation course in 1927. Kunihiko Kasahara displays his version with more creases on the cover of his own publication in 2002 [Kas 03]. Erik and Martin Demaine start to explore the model in 1998 and have made variations of it since then. The model serves either as single module (Fig 5.2.1 left), in an aggregation (Fig 5.2.1 center) or expands into a disc beyond 360° (Fig 5.2.1 right). Multiple discs, in this case 3, joined together transform the model into a very dynamic variation [Dem 09]. The sculptures are part of the permanent collection of the Museum of Modern Art in New York [Ald 08].

In 2008, the Demaines and I published a paper on further variations of the model that uses quadratic curves in addition to circles [KDD 08]. The work investigates closed curved creases and their symmetry in the flat and folded state (Fig 5.2.2).

The first iteration at the top of the figure consists of ellipses in a crease pattern with only one axis of symmetry. The folded configuration exhibits a similar symmetry along a plane through the horizontal axis.

The next design in the same figure below uses alternating circles and ellipses in the flat

Fig 5.2.1 Single module, Aggregation and extended discs (Erik and Martin Demaine)

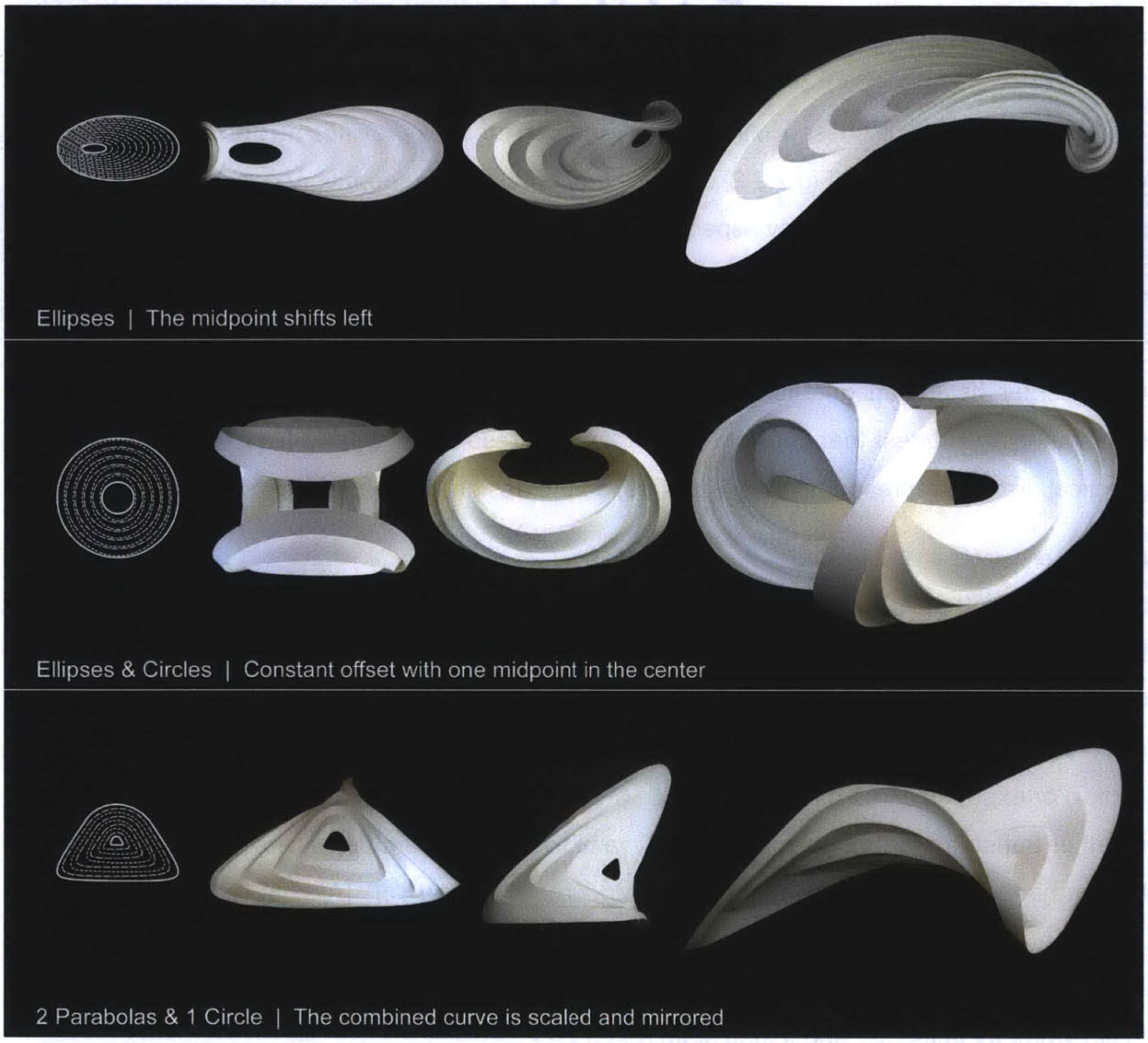


Fig 5.2.2 Studies on 'the Bauhaus model' (DK)

state. The folded model curls rapidly in comparison to the Bauhaus model. Both 2d and 3d configurations have symmetries similar to the previous example.

The last example at the bottom is composed of quadratic splines that consist of 2 parabolas and a circular arc. Scaling the splines achieves the required concentric crease pattern. The symmetry axis in the flat state has no corresponding symmetry in the folded configuration as the model is asymmetrical.

Definition of the design approach

A designer can draw any set of non-intersecting self closing curves in CAD software that can be pleated. In other words the configuration has to consist of a crease pattern with some kind of concentric curves. It is hard to predict the folded state and this element of surprise can be productive

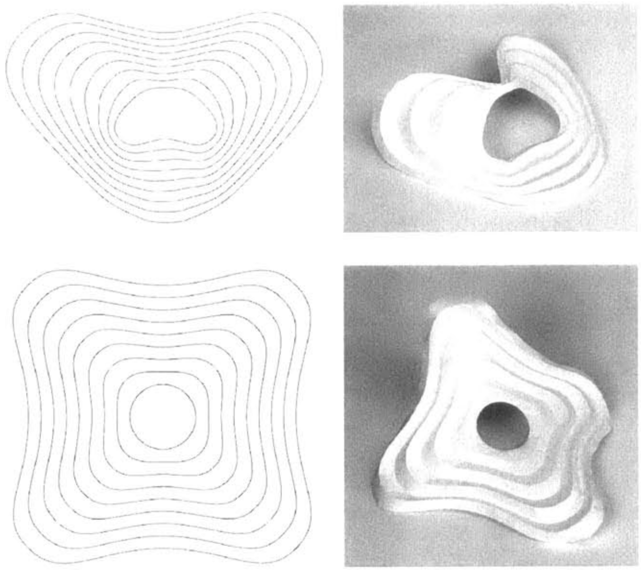


Fig 5.2.3 Crease patterns and paper models (Tobi Lieberson, student of DK)

in the context of pedagogy. It raises curiosity and makes the process of making a model engaging for a student.

Tobi Lieberson, one of my students, investigates undulating curves with concave and convex moments with 2d CAD software (Fig 5.2.3). The final pre-creased models have to be made by hand

An important characteristic of this design approach allows designers to explore the degree to which a model can be folded. Some crease patterns fold very well and provide excellent examples to study folding motion and moving rulings.

DAH

Huffman does not explore this model to my knowledge. The closest example to a closed crease model uses a full ellipse, but the mountain and valley assignments change at the intersections with the straight creases in the crease pattern (Fig 4.3.3).

5.3 Curved crease gadgets - Huffman's approach and forward simulation

The taxonomy of Huffman's work in chapters 4.1 to 4.9 provide many examples for his design approach that makes use of gadgets. It is worth summarizing Huffman's use of conics in this section in order to elucidate how the gadgets operate.

Geometry and representation

Huffman's gadgets determine the position of curves and parallel or focused lines. The rule lines in the gadget can be used to create a discrete version of the crease pattern made of line segments. The opportunity for designers lies in using the discrete crease pattern to simulate the folding process in real-time. The end of chapter 2.3 elaborates on the technique.

DAH as precedent

Huffman uses 2 main kinds of gadgets for his designs, namely reflection gadgets and refraction gadgets.

Regarding reflection gadgets Huffman uses them by drawing sin curves to design cylinder reflections, for example (Fig 4.1.6 - 4.1.9). The ruling is a straight line in the flat state.

Regarding refraction Huffman uses conics and interprets rays as rulings, because he can exploit the refractive properties of the curves to redirect the rulings. The 3 available curves are ellipses, parabolas and hyperbolas and he draws all available refraction cases (Fig 5.3.1). The diagrams on the left show how rulings get refracted if they start in the focal point of a hyperbola, parabola or ellipse. The cases on the right display inversions of the corresponding examples.

The foci of a hyperbola can be used to refract rays on either side of the curve on the left. Rulings can be made to fan out on both sides of the curve on the right.

Parallel rays on the concave side of a parabola refract toward its focus on the left. The inverse configuration on the right pushes rulings away from the focus.

Rays converging towards one focus of an ellipse get refracted through the second focus on the left. The roles of the foci are inverted on the right.

A rare hand drawn sketch reveals that Huffman has the rulings in mind when designing a new model (Fig 5.3.2 left). Huffman uses the corresponding gadget for the inner ellipse (Fig 5.3.2 right) 4 times in one of his final designs (Fig 5.3.3).

Definition of the design approach

I expand on Huffman's design approach by letting a designer use any of the gadgets and subsequently simulate them. A designer can also alter the gadgets to his or her liking. As the gadget provides the opportunity for a discrete representation a designer can use simulation software to

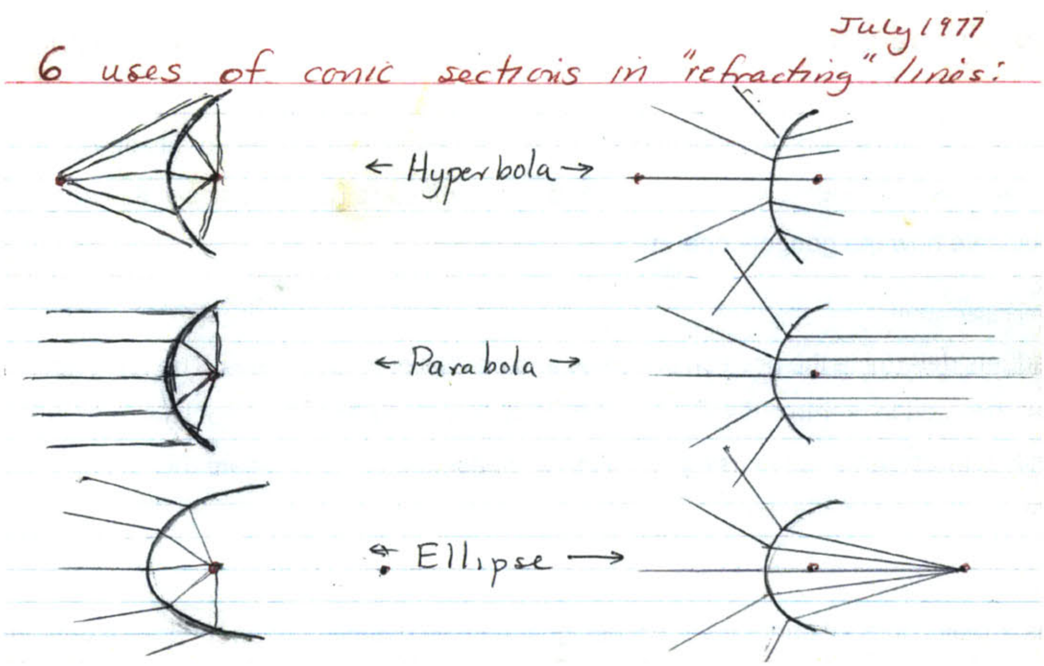
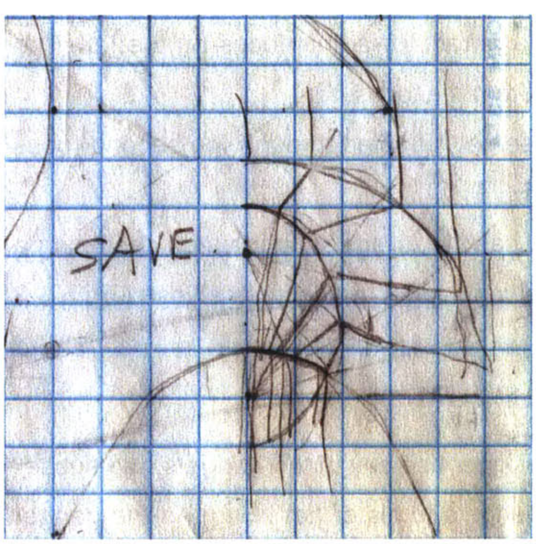


Fig 5.3.1 Refractive properties of conic sections (1977, DAH [DK])

observe the folding motion of the design in real-time.

The design produced by Ashley Hickman, one of the students in my seminar, consists of a combination of a reflection gadget and a refraction gadget (Fig 5.3.4). The result is surprisingly different from a Huffman design, aesthetically speaking, as it combines gadgets in two different meta-categories of the taxonomy.

Even the cases in which Huffman might have had a wrong mathematical conjecture can be used. Folding physical paper is more forgiving than its mathematically defined equivalent. This



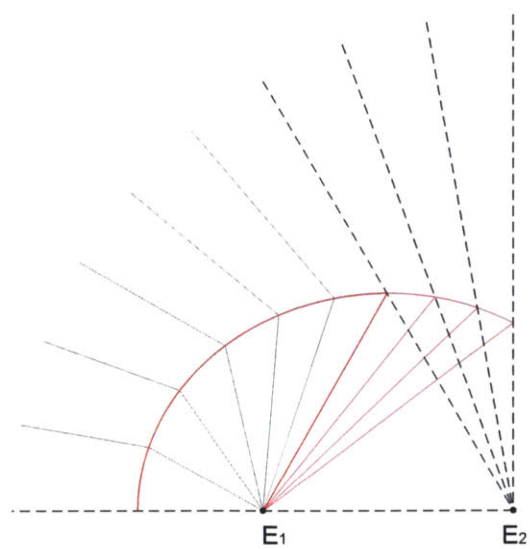


Fig 5.3.2 Sketch (undated, DAH [DK]), Gadget [DK]

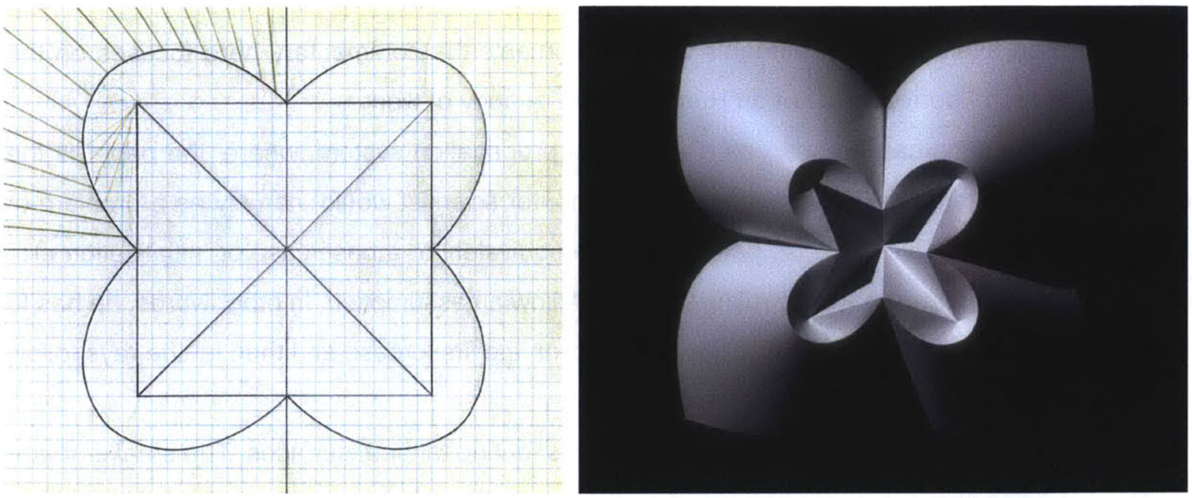


Fig 5.3.3 '4-lobed, cloverleaf, design' (undated, DAH [DK]), Vinyl model (1977, DAH [DAH])

is also sometimes the case in simulation as the discrete representation is an approximation. Thus, all his gadgets are available for designers.

Further design explorations consist of tiling the gadgets. The tilings of gadgets must comply with the specific requirements I explain at the beginning of chapter 4.

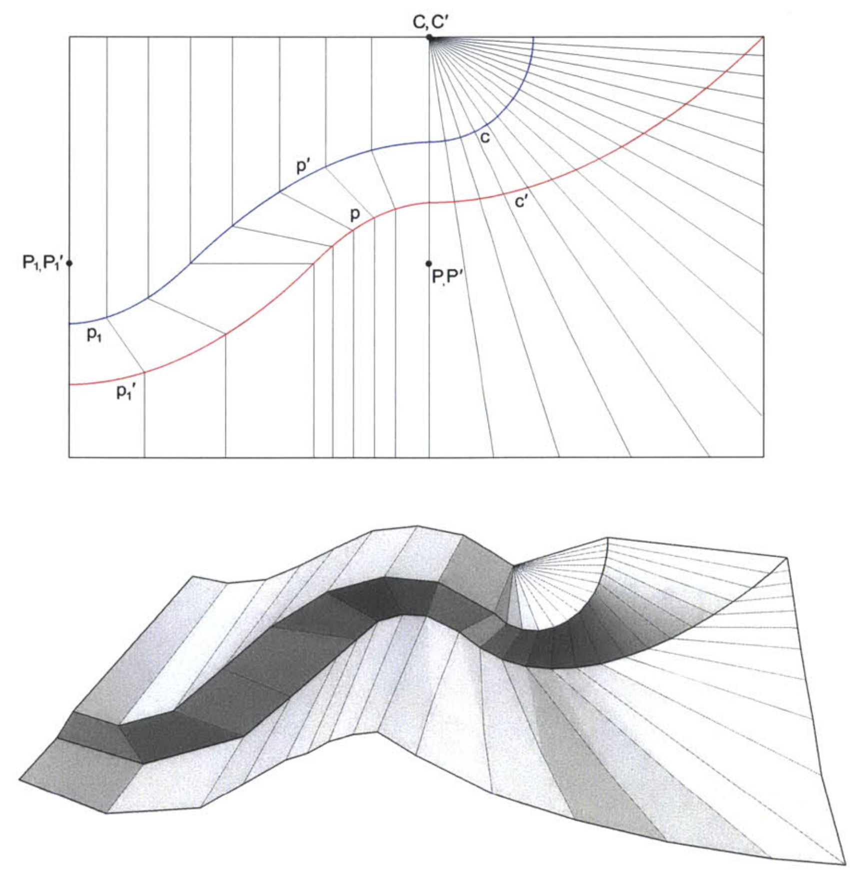


Fig 5.3.4 Combinations of gadgets in crease pattern, Simulated model (Ashley Hickman, student of DK)

In terms of an evaluation of Huffman's accomplishments the commentary here focuses on the design approach I derive, and not on Huffman's entire body of work.

His approach can be seen as a constraint propagation system, where 'the paper is the computer'. He sets up his designs the way a computer scientist would, namely as a problem or puzzle to be solved. Every crease pattern has the equivalent to a base case or initialization procedure of an algorithm, where the rulings have to follow an assumption. In other words, he has to assign the refraction scheme that will get executed on the first crease and then let the rays follow the constraints given by all consecutive curves.

The constraints produce cylinders and cones, but no tangent surfaces. The smaller mathematical solution space reduces opportunities for expressive designs as the most general surface type, the tangent surface, is omitted. The car design by Epps and its post rationalized crease pattern consists mostly of tangent surfaces, for example (Fig 2.3.11).

This design approach is systematic and demands a moderate level of knowledge of geometry. As a result,a mastering the approach might only be suitable for a more computationally-inclined designer. The exploratory potential of the design approach lies in trying out different kinds of tilings and in some cases using more or less curvature for the folded model.

5.4 Step by step evaluation - Iwaki's approach abstracted

Roy Tokuhichi Iwaki, born in 1935, creates a small booklet and provides a rare instance of an artist sharing his methods [Iwa 10]. He invents a loosely structured approach for his animal masks, which forms the basis of a surprisingly varied body of work (Fig 5.4.1).

Geometry and representation

Iwaki's designs are based on freely hand-drawn, or rather, hand-scored curves. As a result neither the curves nor the surfaces are known or precisely definable in any practical way. The models could be 3d scanned and post-rationalized using the process by M. Kilian et al. (Fig 2.3.11), but this might prove to be cumbersome if it was to be part of a design process.

Historical references and precedents

Iwaki defines a 'base mask', which he then modifies to design specific facial features of the animals he wants to portray. He also invents 3 small basic shapes that form parts of the masks (Fig 5.4.1 right). The first consists of concentric circles similar to 'The Bauhaus model'. The second is made of 2 spirals. The third is comprised of a circle intersected by a line, which he uses to form eyes. He designs the masks by combining his base shapes and often adds 2 almost parallel pleated creases such as for the ears of his base mask (Fig 5.4.2).

Iwaki works from one crease to the next and allows the paper to assume a 'natural' folded state before making a decision about the next step. He might have forced the paper on one side of the crease, but not the other, for example. A series of images from a video of one of his talks shows the process. He scores the first crease, folds it, and then pauses to evaluate what he gets (Fig 5.4.3).

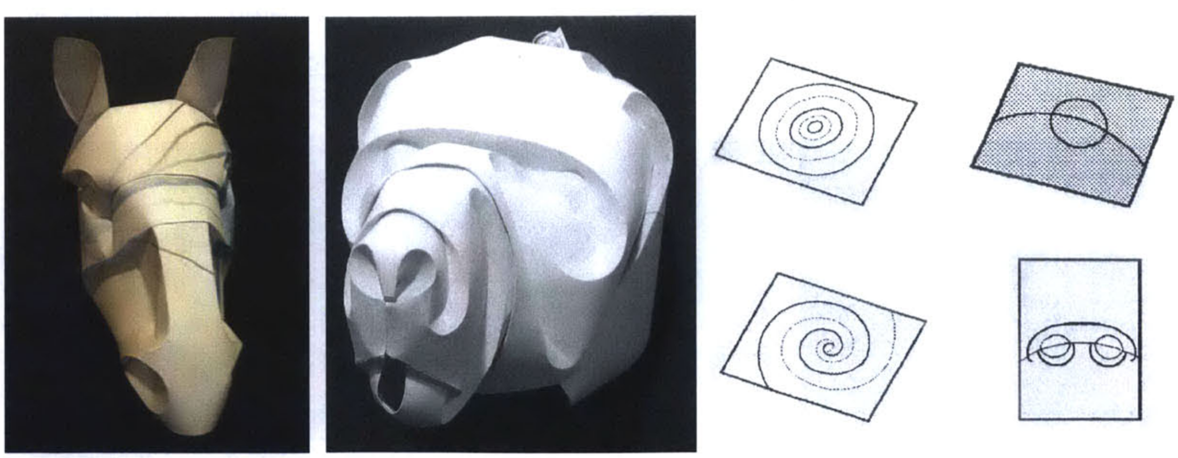


Fig 5.4.1 Horse and Dog mask, Basic shapes (T.R. Iwaki)

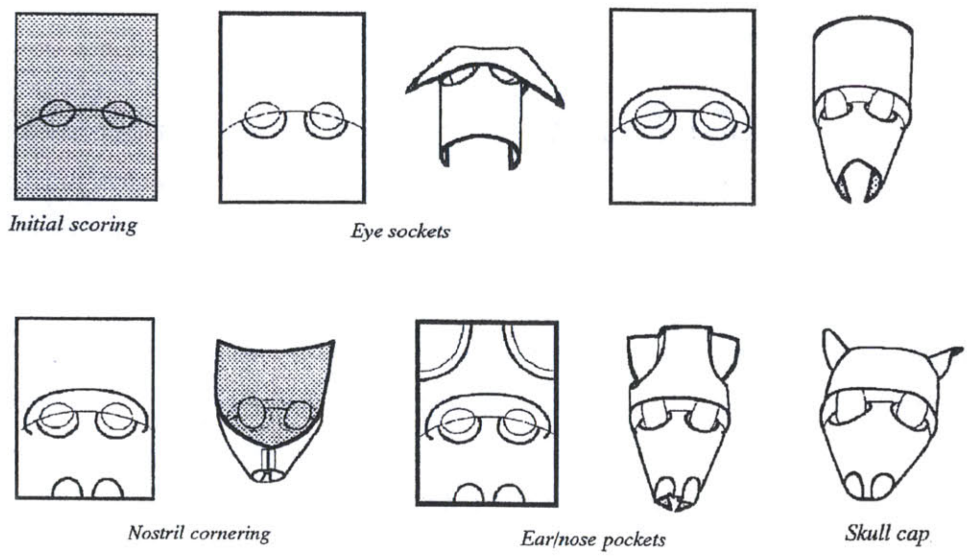


Fig 5.4.2 Instructions for a 'base mask' (T.R. Iwaki)

Definition of the design approach

The proposed approach simply consists of scoring, folding and evaluating the result. A designer can work from one crease to the next until the result is satisfactory. The approach suits designers who are interested in tactile interaction with paper.

Iwaki has a top-down design goal, namely the representation of an animal, but that could be replaced by a more abstract design goal. For example, Ashley Hickman, a student in my seminar, uses the design approach and simply starts by making a model with several creases without any figurative representation (Fig 5.4.4). She marks new decisions about altering regions of paper that are wrinkled by marking options for creases in red. She subsequently folds the marked creases in the next iteration and repeats these steps until the result is satisfactory.

DAH

Iwaki's constraint propagation occurs in less restricted ways when compared to Huffman's approach. Iwaki uses a small catalog of base shapes similar to Huffman's gadgets, but is interested

Fig 5.4.3 Images of a video of a lecture (T.R. Iwaki)

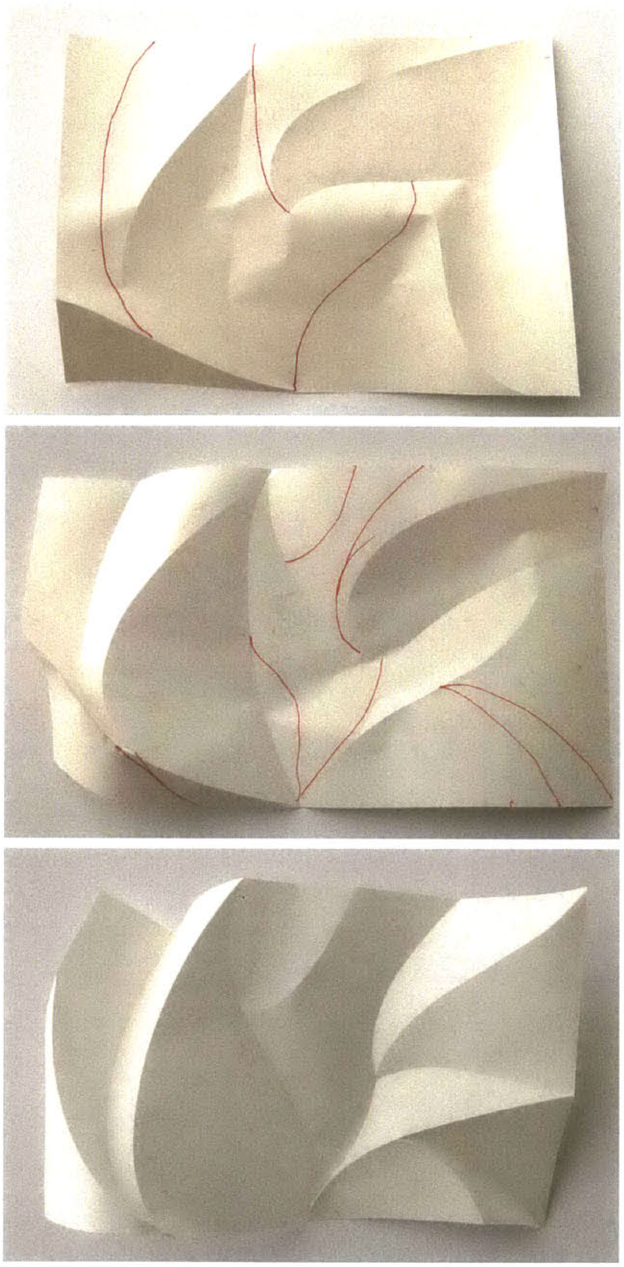


Fig 5.4.4 Paper models (Ashley Hickman, student of DK)

in their effect not their correct mathematical behavior.

The process does not require any a priori knowledge in geometry, but a designer will need to be very patient in order to master this design approach. Probably many hundreds of hours of practice are needed to gain the experience to control the shapes and their precision. To my knowledge Huffman did not engage in this sort of design approach.

5.5 Sculpting and digitizing - Ron Resch's crinkling with curved creases

The Ron Resch Paper and Stick Film features a sequence in the beginning, in which Resch explains how he discovered paperfolding for himself. He derives straight edges and creates the patterns for which he later holds patents. I propose to extend his process to include curved creases via the use of a notational system. Some steps of the process can be achieved by using digitally controlled vinyl cutters.

Geometry and representation

As the creases and paper surfaces are created through an iterative analog process similar to Iwaki's approach, no curve types or surface types can be defined in any practical way. Similarly, using the process by M. Kilian et al might be unpractical. Scanners can be used to digitize crease patterns and vinyl cutters can aid in pre-creasing sheets of paper.

Historical references and precedents

When making his film in 1968 Resch describes the process through which he discovered that paper can assume regular wrinkled configurations. He starts with a sheet of brown paper on a table and slowly crumples it while holding it down with his fingers. Once he has achieved a desired folded shape, he flattens it, and draws simplified versions of the creases with a pen (Fig 5.5.1). He modifies his observations and regularizes them to eventually create tilings made of straight creases.

Gregory Epps, British designer and curved folding expert, has published similar ways of post rationalizing creases [Vys 08].

Definition of the design approach

I propose to extend Resch's approach by using curved creases and developing a notational system to keep track of changes. The design approach can be thought of as a post-rationalized free style method for designing with curved creases. It starts with crumpling or rather sculpting a

Fig 5.5.1 Images from The Ron Resch Paper and Stick Film (1968, Ron Resch)

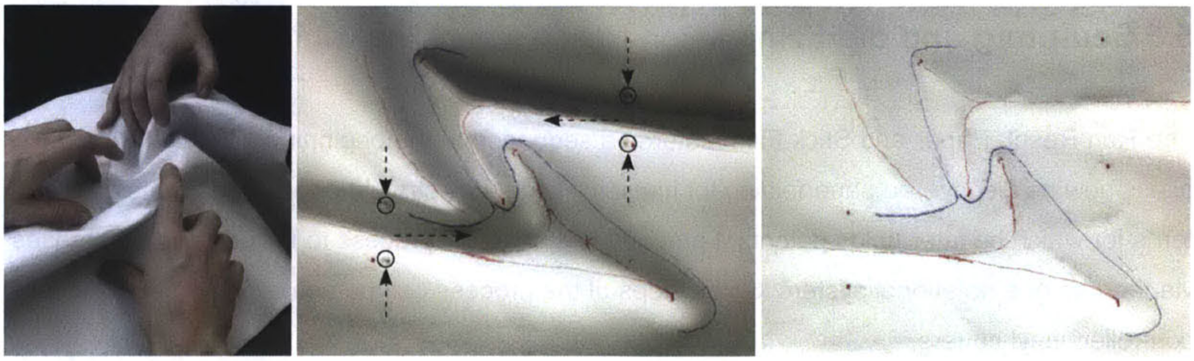


Fig 5.5.2 Sculpting, Notation of pinching, Crease pattern (UnJae Pyon, Lauren Greer, students of DK)

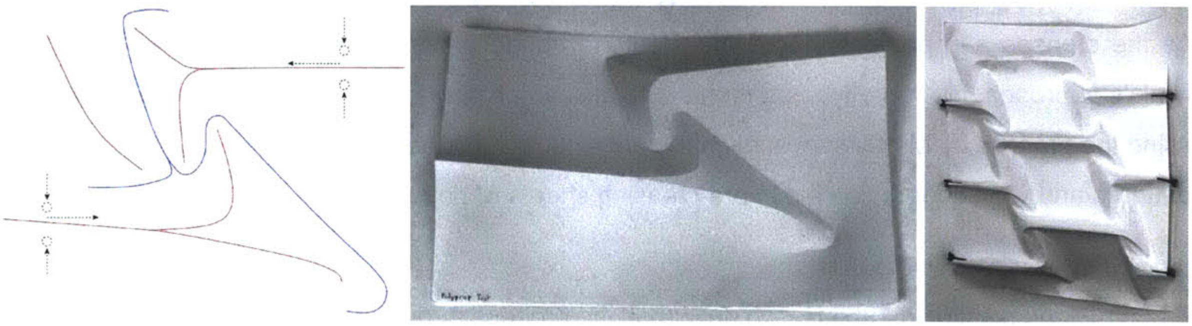


Fig 5.5.3 Digitized crease pattern, Paper model, Tiling (UnJae Pyon, Lauren Greer, students of DK)

sheet of soft paper or felt into a desired shape (Fig 5.5.2 -5.5.3). A designer then marks desired mountains and valleys and subsequently flattens the sheet to digitizes the crease pattern. The digital image needs to be traced with vector-based software. The designer can use vinyl cutters to pre-crease paper with the digitized version of the marked scan.

As the model may not fold the way a flexible material might, a designer might have to go through several iterations of this process until s/he obtains a desirable result.

During my seminar, UnJae Pyon and Lauren Greer sculpt a desired shape (Fig 5.5.2 left) and then mark mountain and valley creases with 2 different colors (Fig 5.5.2 center). I proposed to use a notational system that would allow them to record or document the necessary finger motion, if they needed to recreate the model. They photograph the drawn crease pattern in its flat state (Fig 5.5.2 right).

The next step consists of transferring the crease pattern with CAD software (Fig 5.5.3 left). Further steps involve making new models in paper or plastic and continuously refining the crease pattern until the paper finds an (almost) ideal configuration (Fig 5.5.3 center).

My students then decide to create a tiling with a more regularized design for the final iteration (Fig 5.5.3 right).

The resulting shapes of this design approach tend to be irregular and display great expressive potential. A designer can sculpt with only one restriction, which is to not tear the paper

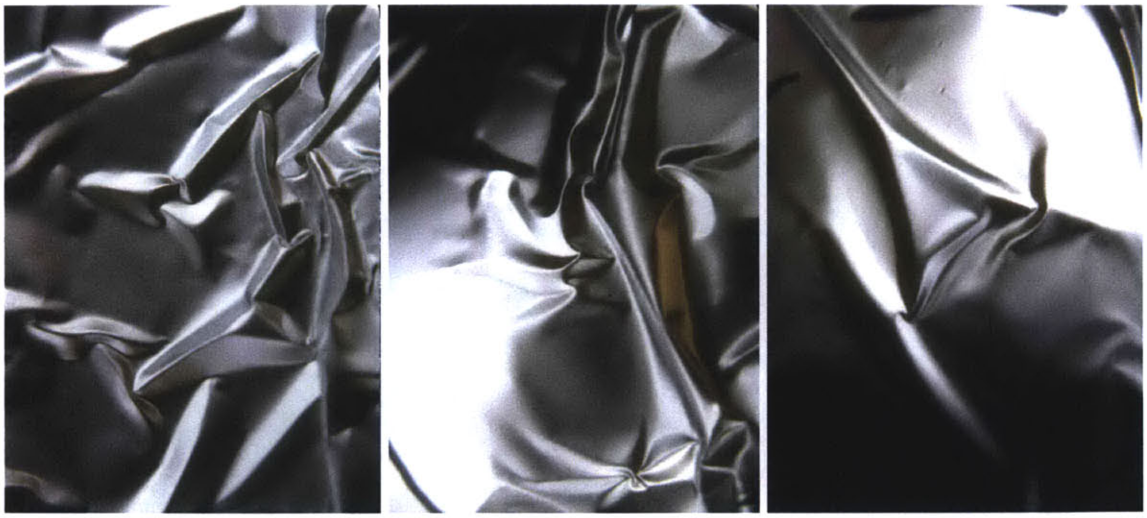


Fig 5.5.4 Studies of crumpled foil (1977, DAH [DAH])

or felt. The analog post-rationalization appears to be open and leaves adequate room for playful interaction.

DAH

Huffman's crumpled foil photography from 1977 gives us insight about his curiosity of similar shapes, but there appears to be no evidence of further investigation (Fig 5.5.4). The foil might not have provided him with the kind of regularity other natural phenomena had that inspired him.

5.6 Summary of findings

I present the following observations as summary of this chapter.

Paperfolding with curved creases can be taught in several ways. The mathematical behavior of curved creases can not be fully described yet, which results in a lack of available digital tools. When proposing alternative design approaches adhering to categories in geometry is useful in some cases. In other cases, it is beneficial to embrace the process of making models with one's hands using paper or flexible material such as felt or sheets of foam.

Huffman worked in several different ways, depending upon which subset of geometry he was interested in. His cone and cylinder explorations are significant accomplishments even if the used geometry is well known. Huffman's refraction gadgets necessitate a base case or initialization. His approaches do not facilitate a lot of room for exploration. His algorithmic approach tends to produce symmetrical designs.

I evaluate the new design approach I distilled from Huffman's work and compare it to approaches I have been able to derive from other artists, designers, educators and computer scientists. The design approaches range from rigidly defined to very open. Iwaki, for example, offers alternatives to the question of 'how to start and continue to the next crease' by not determining the direction in which a designer should proceed. He also evaluates a design at every step and then moves forward from there. Open approaches that provide maximum exploration on a pedagogical level rely on tacit knowledge rather than a priori geometric knowledge.

The proposed collection of design approaches for curved creases helps to situate Huffman's work and contributions in the context of curved crease geometry that has been explored in mathematics, education, art, and design.

6. General conclusion

David Huffman's work covers many areas of paperfolding and his promising insights into the behavior of curved creases might motivate further investigations for several different disciplines.

I elaborate on archival material by Huffman, which has not been published previously, and I structure his work in the form of a taxonomy that relies on gadgets I have derived from his examples. The seemingly eclectic work becomes legible and the categories and gadgets structure the work in geometric genres.

I present his biography in terms of his visual engagements, which provides the reader with relevant context regarding his sense of aesthetics and attitude towards designing with curved creases.

The final chapter contextualizes Huffman's work amidst relevant works of artists and designers that utilize the same geometry. I thereby situate Huffman's contributions in the field on a conceptual level for paperfolding and pedagogy.

Contributions

Due to the interdisciplinary nature of the dissertation it is productive to describe contributions in fields individually.

History of curved crease paperfolding

The writing of an appropriate history of curved crease paperfolding has posed challenges, but the history included in this dissertation elucidates the interdisciplinary nature of the geometry. I write its history by writing of the history of straight-crease folding by region and contextualize curved creases relative to that. The history also addresses the role of curved creases in disciplines such as mathematics, art, design and pedagogy and defines curved crease paperfolding as a field of knowledge.

Design and computation

The dissertation aspires to further the field of design and computation by expanding it to include curved crease paperfolding. I provide ways of describing and evaluating curved creases and formalize design approaches with digital representations for geometry that is not yet fully mathematically describable.

The rigor in my analysis of Huffman's work paired with my proposed design approaches, some of which rely on little a priori knowledge of geometry, may inspire future work that is concerned with computational characterizations and grammars for this geometry.

Architecture and design

This subset in geometry has been used by designers and artists and has become attractive to architects as the manufacturing of curved creases is becoming more feasible. Many building materials rely on the practicality of developable surfaces for shipping and handling. Forming sheet goods into a curved configuration in fabrication facilities directly benefits from furthering knowledge in this domain.

Designers and artists benefit from making design approaches for curved creases accessible as they can expand their canon of forms and shapes while benefitting from inherent efficiencies in manufacturing.

Design pedagogy

The geometry and the constraints posed by the behavior of paper provide an opportunity to develop design approaches that employ rule based systems. Design pedagogy might benefit from my proposed design approaches as they provide a basis for design exploration with rigor that can be taught in constrained and open ways.

Mathematical and computational origami

The contribution to mathematical origami consists of pointing to Huffman's conjectures that we can assess as being wrong. The analysis of the crease patterns include a new proposed convention for descriptions of tilings in paperfolding. The tile in folding terms has its boundaries in the paper, not in the crease, as the ruling behavior needs to be taken into account. The convention might help future analytic work in this domain.

A soon to be published paper co-authored by Erik Demaine, Martin Demaine, Tomohiro Tachi, me and Huffman posthumously proves that the design in chapter 4.10 exists by characterizing the tiles and ruling behavior.

The general findings of the taxonomy that conclude chapter 4 provide open problems for curved creases and mathematicians and folding experts may perhaps be able to further knowledge in the field.

Establishment of a new archive

The Huffman family has agreed to donate the collection to the MIT Museum. The dissertation marks the beginning of a new archive at the MIT Museum and will serve as initial record of the estate. This includes all material digitized by me over the course of the past 3 years.

Next steps

The history of paperfolding requires further academic investigation and much refinement is necessary due to the eclectic nature of its sources and references. More events may exist in the history that relate to curved creases than I have not been able to find.

Huffman's work itself needs further consideration. Some of his designs require deeper investigation and evaluation. Erik Demaine, Martin Demaine, Tomohiro Tachi, me and possibly Huffman posthumously will publish a paper on the 'Hexagonal tower with cusps' for example.

Huffman's regular straight crease tesselations have been published, but I am omitting examples of discrete crease patterns that appear to be approximations of spirals. This body of work should be investigated as separate endeavor.

As Huffman's life work will be gifted to the MIT Museum, a retrospective is currently planned to take place in a few years. I will assist in fund raising efforts via grant applications and will help to curate his work for an exhibition. The effort might include a catalog-like publication with Huffman's work.

Further work on design approaches for curved creases should address the use of tangent surfaces and a general formalization of hand-drawn creases in the form of grammars.

A proposal for a novel manufacturing process for curved-crease folding of metal is currently in progress.

The list of proposed open problems at the end of chapter 4 need further investigation.

Image Index

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CL	**C**	**CR**
BL	**BC**	**BR**

a	**b**	**c**	**d**	**e**

a
b
c
d

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Fig 4.4.127	R	02-21F-02L-sg0028	detail
Fig 4.4.131	L	13-IMG_7584	
Fig 4.4.133	L	02-21F-01PC-sg0009	
Fig 4.4.134	L	02-21F-02L-sg0023	
Fig 4.4.135	L	13-01L-sg0053	
Fig 4.4.137	L	02-38L-sg0033	
Fig 4.4.138	L	02-38L-sg0032	
Fig 4.4.139	L	02-38L-sg0027	
Fig 4.4.140	L	02-38L-sg0026	
Fig 4.4.141	L	02-38L-sg0024	
Fig 4.4.142		02-57-01L-sg0114	
Fig 4.4.144	L	02-57-01L-sg0113	

Fig 4.4.146	**L**	02-57-01L-sg0112	
Fig 4.5.1		01-23PC-01L-sg0247.	
Fig 4.5.3	**L**	02-16L-sg0050	
Fig 4.5.5	**L**	DAH_angel wings_TG	
Fig 4.5.5	**R**	EAH_4_133scan	
Fig 4.5.8		Ogawa, Hiroshi. 1972. *Forms of Paper*. Van Nostrand Reinhold (Trade). 46-47.	
Fig 4.5.9	**L**	02-57-01L-sg0119-folded	
Fig 4.5.9	**R**	02-57-01L-sg0119	
Fig 4.5.10		02-22L-sg0000	
Fig 4.6.1	**L**	EAH_4_032scan	
Fig 4.6.1	**R**	EAH_4_129scan	
Fig 4.6.4	**L**	02-16L--sg0039	
Fig 4.6.6.	**L**	DAH_oval in circle,flat_TG	
Fig 4.6.7	**L**	02-16L-sg0071	
Fig 4.6.8	**a**	02-16L-sg0042	
Fig 4.6.8	**b**	02-16L-sg0039	
Fig 4.6.8	**c**	02-16L-sg0041	
Fig 4.6.8	**d**	02-16L-sg0071	
Fig 4.6.8	**e**	02-65-04L-sg0048	
Fig 4.7.2		02-16L-sg0035	
Fig 4.7.4	**L**	02-16L-sg0031	
Fig 4.7.5	**L**	02-20L-sg0004	
Fig 4.7.7		DAH_2_088scan	
Fig 4.7.8	**L**	02-20L-sg0003	
Fig 4.7.9		EAH_4_094scan	
Fig 4.7.10	**L**	02-65-04L-sg0045	
Fig 4.7.11	**a**	02-20L-sg0004	
Fig 4.7.11	**b**	02-65-04L-sg0053	
Fig 4.7.11	**c**	02-16L-sg0035	
Fig 4.7.11	**d**	02-20L-sg0003	
Fig 4.7.11	**e**	02-65-04L-sg0045	
Fig 4.7.12	**L**	02-65-04L-sg0059	
Fig 4.7.14	**L**	02-16L-sg0037	
Fig 4.7.16	**L**	DAH_3_126scan	
Fig 4.7.17	**L**	13-01L-sg0044	
Fig 4.7.18	**a**	13-01L-sg0044	detail
Fig 4.7.18	**b**	02-65-04L-sg0055	
Fig 4.7.18	**c**	02-16L-sg0038	
Fig 4.7.18	**d**	02-16L-sg0037	
Fig 4.7.18	**e**	02-65-04L-sg0052	
Fig 4.7.18	**f**	02-65-04L-sg0046	
Fig 4.7.18	**g**	02-65-04L-sg0049	
Fig 4.7.19		02-34L-sg0076	
Fig 4.7.21		EAH_4_079scan	
Fig 4.7.22	**L**	02-20L-sg0005	
Fig 4.7.23	**L**	EAH_4_080scan	
Fig 4.7.23	**R**	05-05PC-sg0025	detail
Fig 4.7.24	**R**	05-05PC-sg0017	
Fig 4.7.25	**L**	Box4_DAH_missing1	
Fig 4.7.27	**a**	Box4_DAH_missing2	
Fig 4.7.27	**b**	04-01L-sg0004	
Fig 4.7.27	**c**	05-05PC-sg0015	
Fig 4.8.1	**L**	02-15PC-sg0022	
Fig 4.8.3	**L**	02-15PC-sg0016	
Fig 4.8.5		02-34L-sg0078	
Fig 4.8.6		02-15PC-sg0010	

Fig 4.8.8	L	02-65-04L-sg0071	
Fig 4.8.9	L	02-65-04L-sg0084	
Fig 4.8.11	L	02-65-04L-sg0077	
Fig 4.8.13	L	02-65-04L-sg0078	
Fig 4.8.15	a	02-65-04L-sg0072	detail
Fig 4.8.15	b	02-65-04L-sg0084	detail
Fig 4.8.15	c	02-65-04L-sg0077	detail
Fig 4.8.15	d	02-65-04L-sg0078	detail
Fig 4.8.15	e	02-65-04L-sg0071	detail
Fig 4.8.15	f	02-65-04Lsg0073	detail
Fig 4.8.16	L	02-65-04L-sg0083	
Fig 4.8.19	L	01L-sg0117	
Fig 4.8.21	L	02-34L-sg0080	
Fig 4.8.23	L	DAH_rounded starburst_TG	
Fig 4.8.23	R	13-01L-sg0003	detail
Fig 4.8.25	L	02-34L-sg0086	
Fig 4.9.1	L	02-15PC-sg0020	
Fig 4.9.3		02-15PC-sg0021	detail
Fig 4.9.5	L	02-15PC-sg0019	detail
Fig 4.9.7		02-15PC-sg0017	
Fig 4.10.1		03-01-5-rows-waves_TG_1	
Fig 4.10.2		15-sg0000	
Fig 4.10.5	L	Photograph is courtesy of EAH	
Fig 4.10.5	R	Photograph is courtesy of EAH	detail
Fig 4.11.2	a	IMG_7793	
Fig 4.11.2	b	02-28L-sg0040	
Fig 4.11.2	c	13-04PC-sg0029	
Fig 4.11.3	L	EAH_4_124scan	
Fig 4.11.4	L	Steinhaus, Hugo. 1983. Mathematical Snapshots. Oxford University Press, New York. 143.	
Fig 4.11.5	L	DAH_1_041scan	
Fig 4.11.7	L	13-01L-sg0058	
Fig 4.11.8	L	13-04PC-sg0026	
Fig 4.11.9	L	13-01L-sg0056	
Fig 4.11.10	L	13-01L-sg0059	
Fig 4.11.11		02-28L-sg0040	detail
Fig 4.11.12	L	DAH_lemon juicer1_TG	
Fig 4.11.12	R	DAH_lemon juicer 2_in _TG	
Fig 4.11.15	L	13-01L-sg0060	
Fig 4.11.16	L	EAH_4_092scan	
Fig 4.11.16	R	Haeberli, Paul. 2010. "Geometric Paper Folding: Dr. David Huffman". Collected Computer Graphics Hacks. http://www.graficaobscura.com/huffman/index.html.	
Fig 4.11.17		02-25PC-sg0002	
Fig 4.11.18		EAH_4_096scan2	
Fig 4.11.19	L	13-01L-sg0032	
Fig 4.11.20	L	02-58-04L-sg0022	
Fig 4.11.21	L	02-58-04L-sg0029	
Fig 4.11.22	L	02-16L-sg0057	
Fig 4.11.22	TR	02-16L-sg0054	
Fig 4.11.23		EAH_4_021scan	
Fig 4.11.24		13-04PC-sg0029	
Fig 4.11.25	L	13-01L-sg0033	
Fig 4.11.27	L	IMG_1410	
Fig 4.11.28	L	13-01L-sg0035	
Fig 4.11.29	L	13-01L-sg0034	
Fig 4.11.30	L	EAH_4_086scan	

Fig 4.11.30	**R**	EAH_4_086scan-close	
Fig 4.11.31		Grunbaum, B. and Shephard, G. 1986. 'Tilings and Patterns'. W.H. Freeman and Company New York. 217.	
Fig 4.12.1	**L**	DAH_peanut2X_TG	
Fig 4.12.1	**R**	Owned by EAH. [Photograph by Mulbry, Matthew.]	
Fig 4.12.2	**L**	02-27PC-01L-sg0026	
Fig 4.12.2	**R**	04--01-L-sg0007	
Fig 4.12.3	**L**	02-27PC-01L-sg0026	detail
Fig 4.12.4		02-27PC-01L-sg0027	
Fig 4.12.5	**L**	02-53F-01L-sg0039	
Fig 4.12.5	**R**	02-05PC-sg0076	
Fig 4.12.6	**L**	Lugt, Hans J. 1995. *Vortex Flow in Nature and Technology*. Malabar, Fla: Krieger Pub Co. 50.	
Fig 4.12.6	**R**	Meyer, Richard E. 2010. *Introduction to Mathematical Fluid Dynamics*. New York: Dover Publications. 41.	
Fig 4.12.7	**TL**	02-05PC-sg0072	
Fig 4.12.7	**BL**	02-05PC-sg0074	
Fig 4.12.7	**R**	02-05PC-sg0073	
Fig 4.12.8		02-22L-sg0002	
Fig 4.12.9		Meyer, Richard E. 2010. *Introduction to Mathematical Fluid Dynamics*. New York: Dover Publications. 24.	
Fig 4.12.10	**L**	13-04PC-sg0018	detail
Fig 4.12.10	**R**	02-27PC-01L-sg0028	
Fig 4.13.1	**L**	EAH_4_015scan	
Fig 4.13.1	**C**	EAH_4_073scan	
Fig 4.13.1	**R**	EAH_4_019scan	
Fig 4.13.2	**L**	02-27PC-01L-sg0030	
Fig 4.13.2	**R**	13-03PC-sg0013	
Fig 4.13.3	**L**	02-34L-sg0075	
Fig 4.13.3	**TR**	02-27PC-01L-sg0022	
Fig 4.13.3	**BR**	02-29PCsg0043	
Fig 4.13.4	**L**	02-16L-sg0077	
Fig 4.13.4	**R**	02-22L-sg0003	
Fig 4.13.5	**L**	13-01L-sg0026	
Fig 4.13.6	**L**	02-16L-sg0073	
Fig 4.13.7		02-34L-sg0067	
Fig 4.13.8	**L**	02-28L-sg0039	
Fig 4.13.8	**R**	02-34L-sg0072	
Fig 4.13.9	**L**	02-34L-sg0069	
Fig 4.13.10	**L**	EAH_4_026scan	
Fig 4.13.11	**L**	02-65-04L-sg0102	
Fig 5.1.1		Students of Joseph Albers. 1950s. Ulm. http://www.hfg-archiv.ulm.de/english/the_collections/hfg_collection/photos_photos.html [Photographed by Conrad G. Hans]	
Fig 5.1.2		Lalvani, Haresh. Metal column covers and wall panel. http://www.metropolismag.com/June-2003/Bend-the-Rules-of-Structure/	
Fig 5.1.3		Schein, Markus. 2002. "Applied Generative Procedures in Furniture Design." In *Proceedings of the 5th International Conference GA 2002*.	
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Fig 5.1.5		1950s. Hochschule fuer Gestaltung, Ulm. http://www.hfg-archiv.ulm.de/english/the_collections/hfg_collection/photos_photos.html	
Fig 5.1.6	**L**	Tachi, Tomohiro. Crease pattern.	

Fig 5.1.6	**R**	Tachi, Tomohiro. 3d model.	
Fig 5.2.1	**L**	Single module. [Photograph by DK]	
Fig 5.2.1	**C**	Aggregation and extended disks. http://erikdemaine.org	
Fig 5.2.1	**R**	Demaine, E. D, M. L Demaine, V. Hart, G. N Price, and T. Tachi. 2009. "(Non) Existence of Pleated Folds: How Paper Folds Between Creases." *Graphs and Combinatorics*, 1–21.	
Fig 5.3.1		01-23PC-01L-sg0247	
Fig 5.3.2	**L**	02-15PC-sg0003	detail
Fig 5.3.3	**L**	02-57-01L-sg0116	
Fig 5.3.3	**R**	1977-08--DAH_2_055scan	
Fig 5.4.1	**L**	Iwaki model . [Photograph by Robert Lang]	
Fig 5.4.1	**C, R**	Iwaki, T. Roy. 2010. *The Mask Unfolds.* Cavex Round Folding. Artisans Gallery.	
Fig 5.4.2		Iwaki, T. Roy. 2010. *The Mask Unfolds.* Cavex Round Folding. Artisans Gallery.	
Fig 5.4.3		Stills of one of Iwaki's talks. 2010. Roundfolding Show and Tell. Artisans Gallery.	
Fig 5.5.1		Resch, Ron. 1968. Paper and Stick Film. http://vimeo.com/36122966	
Fig 5.5.4	**L**	DAH_1_136scan	
Fig 5.5.4	**C**	DAH_1_138scan	
Fig 5.5.4	**R**	DAH_1_137scan	

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